Bounds on Achievable Rates for Cooperative Channel Coding

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***Abstract*—We address the problem of cooperative coding among pairs of transmitters and corresponding pairs of receivers. This may be used, for example, to overcome gaps in a multihop networks. We derive upper bounds on the achievable rates for several cooperation scenarios. We show that transmitter cooperation provides significantly more improvement in rate than receiver cooperation. Based on the theoretical results, we also discuss the possible practical implementation techniques.**

I. Introduction

One problem in sensor networks is to overcome the gap between two clusters (or group of nodes). That is, to communicate the data to a distant cluster. Consider a wireless sensor network deployed to measure some phenomenon [1–4]. The sensed data by some group (or cluster) of sensors has to be relayed to the destination. It is very well possible that the clusters of sensors are widely separated, demanding cooperation to achieve communications at the desired rate. Such a scenario is depicted in Figure 1 and is easy to envision for sensor networks.

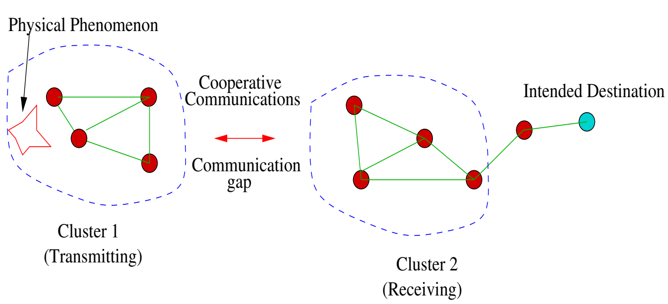

Fig. 1. Cooperative Communications in Sensor Network

The transmitting cluster, in Figure 1, senses the phenomenon and the measured data needs to be transmitted to the destination. In many cases, the data from more than one sensor needs to be transmitted. For example, the sensors could be sensing different perspectives of the phenomenon and hence the information stream contained by each sensor could be different. The most likely scenario for sensor network is to have 4-node cooperative situation, i.e. two independent transmitters and two independent receivers. Note, that the distance between the two clusters is much larger than the nodes within a cluster. This paper deals with the information theoretic aspects of such a channel model and explicitly derives the data rates for each transmitting node and considers the effect of transmitter and/or receiver cooperation on the rate region.

In this paper, we consider the 4-node scenario as described above. Each of the two transmitters is required to transmit not only their information, but also the information of the other transmitter to both the receivers. We explicitly derive the data rates for each transmitting node and consider the effect of transmitter and/or receiver cooperation on the rate region. We also suggest a practical implementation based on the theoretical results. Clearly, this problem is different than the classic multiple access channel, relay channel, broadcast channel, interference channel (Chapter 14, [5]) and multiple input multiple out (MIMO) system. These can however be considered as special cases of our formulation. Apart from overcoming the gap in the sensor networks, the sensors cooperate with each other to achieve more reliable and higher rate communications. Note, that power rather than bandwidth is the main constraint. Also, multiple sensors occupy the same channel and hence, standard multiplexing techniques like TDMA, FDMA, CDMA, and OFDM may not be readily employed. The cooperation scenario is not limited to sensor networks and holds for any multihop networks.

Rest of the paper is organized as follows: Section II discusses the related research followed by the problem statement in Section III. The solution is presented in Section IV. A practical approach to implement cooperation is presented in Section V with the conclusions in Section VI.

II. Related Work

Network information theory has always sprung surprises and excitement among information theorists. Although it has been around for half a century, it has a large set of unsolved problems [5, 6]. For instance, the capacity for the relay channel [7] is still unknown, although some special cases such as the degraded relay channel [8] are solved. Various other channels such as multiple access channels, broadcast channels, and interference channels along with their variations (such as introducing feedback) are either solved or tightly bounded [5].

In recent times, the notion of cooperative transmission has been considered. The system consisting of three nodes was considered in [9–13]. The achievable rates for the channel model

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with two cooperative transmitters and a receiver is derived in [9, 13]. Various permutations of the channel model that could be possible with three nodes, like two cooperating receivers and a single transmitter, are considered in [11, 12]. The four nodes scenario with two nodes acting purely as relays is considered in [14]. A channel with two cooperating transmitters and non-cooperating receivers is considered in [15, 16]. However, the concentration was on outage and diversity. The behavior for fading channel is considered in [15] and for non-fading channel but with a complicated transmitter cooperation scheme involving dirty paper coding in [16]. In more recent work, two cooperating receivers along with the two cooperating transmitters are considered in [17]. However, the model did not consider the transmission of information from a transmitter to both the receivers. In contrast to this, the system with two cooperating transmitters and two cooperating receivers is considered here. The data stream from each transmitter is intended for both the receivers.

III. Channel Model and Problem Statement

Consider the nodes 1 and 2 of transmitting cluster, and nodes A and B of the receiving cluster. Such a channel model is depicted in Figure 2. The transmitters send the information cooperatively and similarly receiving nodes decode the transmitted data on cooperation. Each node receives an attenuated and

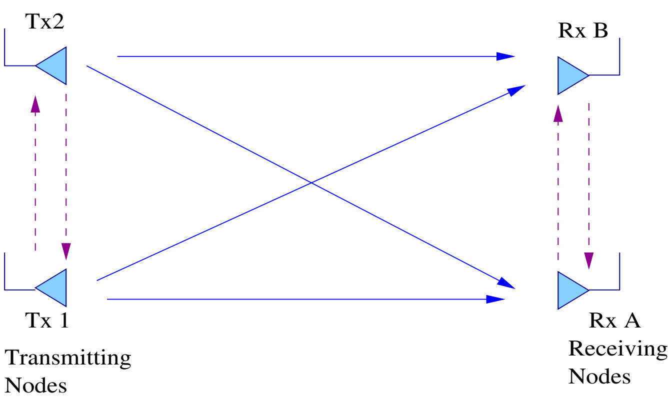

Fig. 2. Channel model for 4-node cooperation

noisy version of the transmitted data. Note that the transmission from receiver to transmitter is not allowed for the obvious reasons of not being practical given the limited energy budget. Let $Y_i(t)$ be the received baseband signals for the nodes $i = 1, 2, A, B$. The channel model can then be mathematically expressed as,

$$
\begin{aligned}
Y_1(t) &= K_{21}X_2(t) + Z_1(t) \\
Y_2(t) &= K_{12}X_1(t) + Z_2(t) \\
Y_A(t) &= K_{1A}X_1(t) + K_{2A}X_2(t) + K_{BA}X_B(t) + Z_A(t) \\
Y_B(t) &= K_{1B}X_1(t) + K_{2B}X_2(t) + K_{AB}X_A(t) + Z_B(t)
\end{aligned}
$$

where $X_i(t)$ is the transmitted signal by node i, $Z_i(t) \sim \mathcal{N}(0, N_i)$ is zero mean additive Gaussian channel noise with variance N_i, and K_{ij} is the path gain from node i to node j corresponding to Rayleigh fading. The nodes are assumed to follow the decode-and-forward [9, 15] form of relaying. Let the power constraint on $X_i(t)$, $\{i = 1, 2, A, B\}$ be P_i and W_j, $j = 1, 2$, be the information content of the transmitters.

Assume a discrete time version of the channel model. Node 1 transmits information $W_{1\to 2}$, $W_{1\to A}$ and $W_{1\to B}$ to nodes 2, A and B respectively. Hence, the rates can be divided as:

$$
\begin{aligned}
R_{1\to A} &= R_{1A} + R_{1BA} + R_{12A} && (1) \\
R_{1\to B} &= R_{1B} + R_{1AB} + R_{12B} && (2) \\
R_{2\to A} &= R_{2A} + R_{2BA} + R_{21A} && (3) \\
R_{2\to B} &= R_{2B} + R_{2AB} + R_{21B} && (4)
\end{aligned}
$$

For simplicity, consider the signal transmitted by node 1:

$$
\begin{aligned}
X_1 = {} & X_{1A} + X_{1B} + X_{12A} + X_{12B} + X_{1BA} + X_{1AB} \\
& + U_{12A} + U_{12B} + U_{1BA} + U_{1AB}
\end{aligned}
$$

where X_{1A}, X_{1B} are direct path signals, X_{12A}, X_{12B}, X_{1BA}, X_{1AB} are the relay path signals, and U_{12A}, U_{12B}, U_{1BA}, U_{1AB} are the coherently combined signals.

The direct path signal conveys the message W_{1A} and W_{1B} with rates R_{1A} and R_{1B} respectively. The relay path signal transmits W_{12A} to node 2 (conveyed by X_{12A}) at rate R_{12A}, W_{12B} to node 2 at rate R_{12B}, W_{1BA} to node B at rate R_{1BA}, and W_{1AB} to node A at rate R_{1AB}. The coherently combining signals involve both transmitter and receiver cooperation. For example, U_{12A} is intended to combine coherently with the signal from node 2 (transmitter cooperation), U_{1BA} is intended to combine with the signals from node B (receiver cooperation) and in both instances the signal is meant for node A. A similar explanation holds for other coherently combining signals.

Expressions similar to that for X_1 can be found for other nodes. Even though the nodes A and B do not have any information of their own to convey they may relay information in the form of coherently combined signals. Based on the composition of X_1, the power is divided into:

$$
\begin{aligned}
P_1 = {} & P_{1A} + P_{1B} + P_{12A} + P_{12B} + P_{1BA} + P_{1AB} \\
& + P_{U12A} + P_{U12B} + P_{U1BA} + P_{U1AB}
\end{aligned}
$$

We assume that the B blocks of length n are transmitted and the block size B and n are sufficiently large for the perfect decoding. The nodes cooperate based on the information stream received in the previous block. The direct path signal depends on the direct path message in the current block and relay path messages transmitted or decoded in the previous block. The relay path signal depends on the relay messages transmitted in the current block and the relay messages transmitted or decoded in the previous block. Similarly, the coherent combining signal from a node to the destination depends on the relay messages transmitted or decoded in the previous block. For example, $X_{1A} = \sqrt{P_{X_{1A}}}\tilde{X}_{1A}[W_{1A}(b), W_{12A}(b-1), W_{21A}(b-1), W_{1BA}(b-1), W_{1AB}(b-1)]$, $X_{12A} = \sqrt{P_{X_{12A}}}\tilde{X}_{12A}[W_{12A}(b), W_{12A}(b-1), W_{21A}(b-1)]$, and $U_{12A} = \sqrt{P_{U_{12A}}}\tilde{U}[W_{12A}(b-1), W_{21A}(b-1)]$.

The other direct-path, relay-path and coherently combined-path signals have analogous expressions. With the above background, the goal is then to carry out an information theoretic analysis of this channel model to derive the upper bound or inner region of the rate region.

IV. Deriving the Rate Region

From the above mentioned channel model, the upper bounds on the achievable rate region can be derived using the technique of forward and backward decoding [18–20] employed in [11, 12]. This technique is the revised version of the one employed in [9]. In contrast to [9], where the relay-path signal is decoded in the forward stage, the revised technique explores the idea of simultaneously decoding as many messages as possible irrespective of their immediate need. Further details on the revised technique are available in [11, 12].

For the sake of brevity and simplicity, consider the rates from the point of view of transmitting node A. We use the forward and backward decoding approach as in [9, 11]. Also, all the direct path signals are intended to be decoded in the backward decoding stage. In the forward decoding stage node A has to decode W_{1AB} and W_{2AB} if these messages are employed. Note that, if node A can decode all the data contained in the signals, the channel is like a multiple access channel [5] from the point of view of node A. Based on this, we obtain a set of rate constraints defining the upper bound on the achievable region with respect to node A. For instance,

$$R_{12} < E\left\{C\left(\frac{K_{1A}^2 P_{12}}{N_A}\right)\right\} \quad (5)$$

$$R_{1A} < E\left\{C\left(\frac{K_{1A}^2 P_{1A}}{N_A}\right)\right\} \quad (6)$$

$$R_{1B} < E\left\{C\left(\frac{K_{1A}^2 P_{1B}}{N_A}\right)\right\} \quad (7)$$

$$R_{BA} < E\left\{C\left(\frac{K_{BA}^2 P_{BA}}{N_A}\right)\right\} \quad (8)$$

$$R_{1AB} < E\left\{C\left(\frac{K_{1A}^2 P_{1AB}}{N_A}\right)\right\} \quad (9)$$

$$R_{2AB} < E\left\{C\left(\frac{K_{2A}^2 P_{2AB}}{N_A}\right)\right\} \quad (10)$$

$$R_{1A} + R_{1B} < E\left\{C\left(\frac{K_{1A}^2 (P_{1A} + P_{1B})}{N_A}\right)\right\} \quad (11)$$

where, $E(\cdot)$ denotes an expectation and $C(x) = (1/2)\log(1 + x)$. Node A has to decode W_{1AB} and W_{2AB} to facilitate a relay in the next block. Thus, constraints on R_{1AB} and R_{2AB} are required. The above rate constraints (5)-(11) are only few examples of a large set of constraints that could be penned. The other rate constraints, such as $\{R_{21}, R_{2A}, R_{2B}, R_{12A}, R_{1BA}, \ldots, R_{21A}, R_{21B}, \ldots, R_{1A}+ R_{12}, \ldots, R_{1A} + R_{1B} + R_{12} + R_{12A} + R_{12B} + R_{1AB} + R_{1BA}, \ldots\}$, could be derived for node A.

Following the same approach as for node A, the rate constraints for nodes B,1 and 2 could also be derived. Hence, in the forward decoding stage node B has to decode W_{1BA} and W_{2BA}, node 1 needs to decode W_{21A} and W_{21B}, and node 2 will need to decode W_{12A} and W_{12B}.

In the backward decoding stage, node A has to decode W_{1A}, W_{2A}, W_{1BA}, W_{2BA}, W_{12A} and W_{21A}. Therefore,

TABLE I
MESSAGES REQUIRED BY NODES $\{1, 2, A, B\}$ TO DECODE IN FORWARD AND BACKWARD DECODING

Decoding Stage	Transmitting Nodes		Receiving Nodes	
	1	2	A	B
Forward	W_{21A}, W_{21B}	W_{12A}, W_{12B}	W_{1AB}, W_{2AB}	W_{1BA}, W_{2BA}
Backward	W_{21}	W_{12}	W_{1A}, W_{2A}, W_{1BA}, W_{2BA}, W_{12A},W_{21A}	W_{1B}, W_{2B}, W_{1AB}, W_{2AB}, W_{12B}, W_{21B}

$$R_{1A} < E\left\{C\left(\frac{K_{1A}^2 P_{1A}}{N_A}\right)\right\} \quad (12)$$

$$R_{2A} < E\left\{C\left(\frac{K_{2A}^2 P_{2A}}{N_A}\right)\right\} \quad (13)$$

But, if say W_{1B} can also be decoded rather than being considered as noise then,

$$R_{1B} < E\left\{C\left(\frac{K_{1A}^2 P_{1B}}{N_A}\right)\right\} \quad (14)$$

Similarly, if W_{2B}, W_{12} and W_{21} can also be decoded instead of being considered as noise, then:

$$R_{1B} < E\left\{C\left(\frac{K_{1A}^2 P_{1B}}{N_A}\right)\right\} \quad (15)$$

$$R_{2B} < E\left\{C\left(\frac{K_{2A}^2 P_{2B}}{N_A}\right)\right\} \quad (16)$$

$$R_{12} < E\left\{C\left(\frac{K_{1A}^2 P_{12}}{N_A}\right)\right\} \quad (17)$$

$$R_{21} < E\left\{C\left(\frac{K_{2A}^2 P_{21}}{N_A}\right)\right\} \quad (18)$$

The above mentioned rate constraints are only a few from the large set of constraints. As in forward decoding, a huge set of rate constraints can be derived for nodes A, B, 1, and 2. The messages that the nodes have to decode in forward and backward decoding have been summarized in Table IV. Note, that some of the rate constraints in the set describing them might be redundant depending on the scenarios.

The rate constraints derived, so far, handle the cooperation implicitly. We shall now explicitly concentrate on transmitter and/or receiver cooperation and examine their consequences. Consider the rate required to transmit from node 1 to A exploiting cooperation, $R_{1\to A}$, as described in (1):

$$R_{1\to A} = R_{1A} + R_{1BA} + R_{12A} \quad (19)$$

The rates R_{1A}, R_{1BA} and R_{12A} were derived during the backward decoding. Now, by employing backward decoding for node A and considering cooperation, the rate, $R_{1A} + R_{12A}$ is given by:

$$R_{1A} + R_{12A} < E\left\{C\left(\frac{K_{1A}^2 P' + K_{2A}^2 P_{U21A} + 2K_{1A}K_{2A}\sqrt{P_{U12A}P_{U21A}}}{N_A}\right)\right\}$$

where $P' = P_{1A} + P_{12A} + P_{U12A}$. Similarly, the other rates in (1) could be evaluated to yield:

$$R_{1\to A} \quad < \quad E\left\{C\left(\frac{Z}{N_A}\right)\right\} \tag{20}$$

where,

$$Z = K_{1A}^2 P_{1\to A} + K_{2A}^2 P_{U'} + K_{BA}^2 P_{UBA} + 2K_{1A}K_{2A}P_{U''}$$

with,

$$\begin{aligned} P_{1\to A} &= P_{1A} + P_{1BA} + P_{12A} + P_{U1BA} + P_{U12A} \\ P_{U'} &= P_{U2BA} + P_{U21A} \\ P_{U''} &= \sqrt{P_{U12A}P_{U21A}} + \sqrt{P_{U1BA}P_{U2BA}} \end{aligned}$$

In (20), U_{BA} represents the receiver cooperative signal coherently combined with the signals transmitted from nodes 1 and 2. Although U_{1BA} and U_{2BA} are coherent signal meant to combine at node B, they do consist of transmitter cooperation as the signals transmitted to node B also results from cooperative coding at the transmitters. If no transmitter cooperation is allowed, then the signals available at B will only be the relay signals from nodes 1 and 2 coherently combining. Hence, clearly the effect of transmitter cooperation is higher than the receiver cooperation. Also, by appropriate selection of power values in (20), transmitter and/or receiver cooperation can be explored. Similarly to (20), the other rate constraints described in (2), (3), and (4) can also be derived. Based on these, define:

$$R_1 = R_{1\to A} + R_{1\to B} \tag{21}$$

$$R_2 = R_{2\to A} + R_{2\to B} \tag{22}$$

Using the above definition for R_1 and R_2, the advantages gained by using cooperation over not using any are summarized in Figure 3. Here, the distance between transmitters (within a cluster) is assumed to be much less than that between the transmitter and receiver. The distance between the receivers is assumed to be same as that between the transmitters. From Figure 3, it is quite evident that there are for sure advantages of using cooperation. However, employing only receiver cooperation does not yield significant gain compared to employing only transmitter cooperation. Using both transmitter and receiver cooperation is superior to using either alone. This result coincides with that in [17], although they had different channel models. Hence, it is sufficient to use only transmitter cooperation if the rate is not a hard constraint. It should be noted that if the distance between the nodes within a cluster is increased, then the gain obtained from cooperation will certainly reduce. If the distance between the nodes is large enough then the rate region will collapse to that without cooperation.

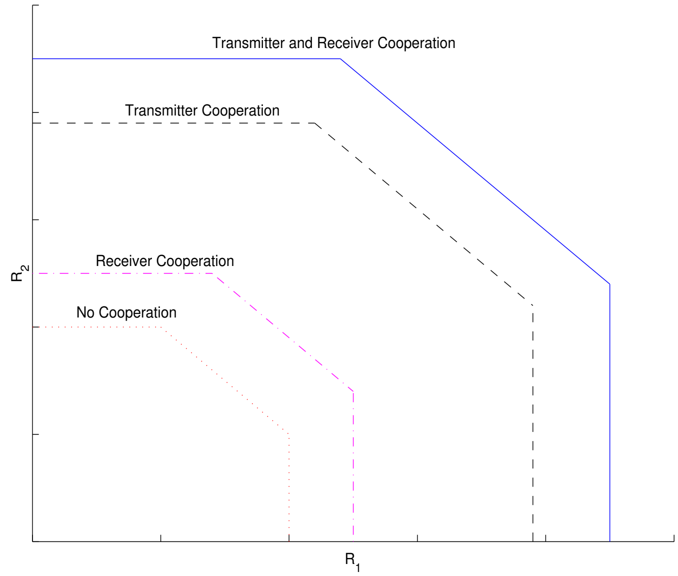

Fig. 3. Rate Region for 4-node channel model with different cooperating scenarios

V. Discussion and Practical Implementation

The above sections discussed the cooperative rate regions for a four nodes channel model. From the derivations, it is evident that there certainly is gain in using cooperation. However, as few good things come for free, there is a high processing cost to pay. Hence, cooperative transmission should be used only when necessary. For instance, if a relay node is available, then it is always advisable to opt for it.

Also note that the cooperative transmission channel is very much different from the known channels such as multiple access, broadcast, relay [5], and multiple input multiple output (MIMO) [21]. However, these channels can be considered as the special cases of our formulation. Another channel model similar to cooperative transmission is distributed MIMO [22] but this involves high cost in node synchronization and also involves only one source and destination.It should be noted that the application of a cooperative strategy is highly dependent on geographical locations. For example, if two transmitters are collocated, then it can essentially behave as one node [11, 12]. Hence, the decision to implement a cooperative scheme also depends on the geographical constraints.

The theoretical analysis of the 4-node channel model suggested that the transmitter cooperation outperforms receiver cooperation significantly. Hence, in practice it is wise to consider only a transmitter cooperative strategy. This will also reduce the processing and implementation cost which otherwise is very high. One of the approaches is to implement the algorithms suggested in [15, 16] depending on the channel conditions. Another approach is superposition of the nodes. For instance, consider the channel model as in Figure 2. The four nodes, $\{1, 2, A, B\}$, can be divided into two sets of three nodes, for example $\{1, 2, A\}$ and $\{1, 2, B\}$. The partitioning of the sets depend on the geographic location. For instance, consider the channel model as in Figure 4.

In this case, the transmitting nodes 1 and 2 cooperatively transmit a message to node B, which in turn relays the infor-

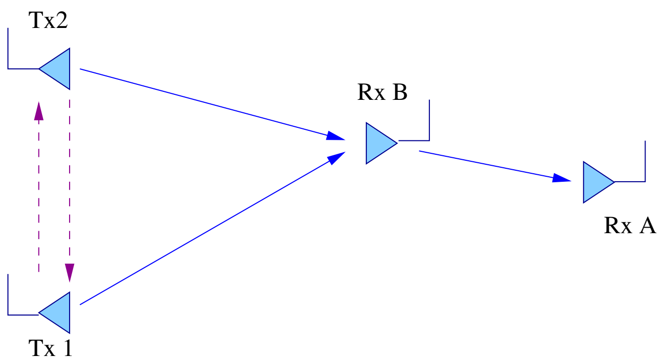

Fig. 4. Channel model for superposition

mation stream to node A. Once the set is partitioned into a 3 node channel model, the algorithm suggested in [9, 10] could be implemented.

VI. Conclusions

This paper considered the problem of communications between two distant clusters of nodes. A cooperative transmission strategy was considered to overcome the gap in the multihop networks. The outer bound on the rates was derived. From the information theoretical analysis, it is evident that the transmitter cooperation is more significant than the receiver cooperation. Some practical implementation techniques were outlined. It may be noted that due to the high cost in implementation involved in cooperative coding, whenever relays are available they should be used.

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