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CONTROL ORIENTED FORMULATION FOR STRUCTRUCES INTERACTING WITH MOVING LOADS*

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ABSTRACT

This paper addresses the control oriented formulation of structures interacting thorough moving contact points. The example used for illustration is the dynamic interactions of moving vehicles and bridges structure. Such systems are time varying due to the moving vehicles. The system representation can be partitioned into time invariant part (bridge and vehicle dynamics) and time varying part (moving contact points). Such system's approach facilitates modularized analytical model development, analysis, simulation, and control system design. Block diagram based simulation with integral causality will be presented to demonstrate the utility of this approach for parametric investigation of the system dynamics. Further, it will be shown that such problem can be put into the linear parameter varying (LPV) standard feedback form to exploit the available theoretical and numerical tools.

1. Introduction

Systems interacting with one another through time varying interfaces occurs in such diverse situations as moving vehicles over bridge or flexible foundations, moving cutters in machining process, moving energy sources in thermal processing, and etc. Such class of problems typically include both lumped and distributed parameter systems interacting through moving contacts. Taking bridge-vehicle systems as an example, the dynamic response of the entire system depends on the dynamic properties of the traversing vehicles and bridge, vehicle operating speeds, surface quality of the roadway, and so forth. To control the magnitude of the bridge vibrations, it is critically important to be able to accurately predict bridge response to the action of crossing vehicles and the resulting responses of the vehicles.

The dynamics of vehicles or loads interacting with highway infrastructures (roads and bridges) has been examined by researchers around the world. The vehicles are commonly modeled by a multiple degrees-of-freedom system incorporating the dynamics of the axles and suspensions, which determine the fundamental frequencies of the vehicles. When these frequencies are close to the fundamental frequencies of the bridges, a "quasi-resonance" of parametric excitation can occur and accelerates the fatigue of bridge structures [1]. The considerations of truck suspension interacting with pavement or bridges have recently been studied [2-6]. In general, three types of fundamental problems are reported in the literature. These are the *moving force*, *moving mass* and *moving oscillator* problems. The modeling of a vehicle traveling along a bridge as a moving force [7] neglects the inertia of the moving subsystem and no dynamic interaction is considered. When the inertia of the moving subsystem is small, the constraint or coupling force may be treated as a moving force. This is called the moving force problem and occurs in high-speed machining processes. When the inertia of the subsystem cannot be neglected, a moving mass model is employed [8, 9]. The inclusion of both the subsystem inertia and interaction forces in the moving oscillator problem is found in the recent study of vehicle-bridge interactions [10-15]

Experimental tests have also been conducted to verify the critical influence of vehicle dynamic loads on pavement wear/damage and bridge structure life [16], which concludes that pavement wear under steel suspensions is at least 15 per cent faster than under air suspensions and that the concentration of dynamic loads for air suspensions is only about half the magnitude of that for steel suspensions. However, the results on the vehicle-bridge interactions were much less conclusive due to the fact that there are more complex factors involved.

The objective of this work is to develop a system's approach to represent the multi-vehicle and bridge dynamics as a multivariable system model and use it to facilitate dynamic analyses and controls.

The rest of the paper is organized as follows. The next section develops the system's model for general combined lumped and distributed parameter system with moving interactions. The system's block diagram and discussions of incorporating external disturbances and control system are given. Section 3 provides the system model for a simplified case that applies to the bridge

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dynamics – proportionally damped non-gyroscopic system. Section 4 presents a simulation example of multiple moving vehicles, tuned with different suspension characteristics, traversing a bridge, and is followed by the conclusions in the last section.

2. System Model.

In this section, we first formulate the system's state equation for a spatially one-dimensional, linear non-self-adjoint distributed continuum interacting point-wise with multiple lumped parameter systems. The main approach uses modal expansion of the distributed parameter system. We will form the system in a feedback form where the interacting force and motion at the contact points are the input and output variables of sub-systems.

Non-self-adjoint Continuum. The equation of motion of the continuum is given by

$$L_M \frac{\partial^2}{\partial t^2} w(x,t) + L_D \frac{\partial}{\partial t} w(x,t) + L_K w(x,t) = f(x,t),$$

$$x \in [0, \ L], \ t \geq 0, \tag{1}$$

where $w(x,t)$ is the transverse displacement of the continuum, L_M, L_D and L_K are spatial operators defined on functions satisfying necessary smoothness requirements and boundary conditions, and $f(x,t)$ is an externally applied force. Both boundary and initial conditions for the continuum ca be assumed to be homogenous without loss of generality. In accordance with the notation of Figure 1, let
the m point forces acting on the continuum be the interaction forces $W_i(t)$ due to the moving vehicles located at $\zeta_i(t)$. Then,

$$f(x,t) = \sum_{i=1}^{m} W_i(t) \ \ \delta(x - \zeta_i(t)), \tag{2}$$

where $\delta()$ is the Dirac delta function and $\zeta_i(t)$ represents the actual position of the vehicles or axles along the continuum axis. For the non-self-adjoint problem described here the continuum response $w(x,t)$ can be expressed as a series in terms of the complex eigenfunctions of the continuum [10-13]:

$$\dot{q}_n(t) = \lambda_n q_n(t) + \frac{1}{\lambda_n} Q_n(t), \ n = 1,2,...,N \tag{3}$$

$$Q_n(t) = \sum_{i=1}^{m} \bar{\psi}_n(\zeta_i(t)) W_i(t) \tag{4}$$

$$w(x,t) = \sum_{n=1}^{\infty} \mathrm{Re}[\phi_n(x)\, q_n(t)] \tag{5}$$

where $q_n(t)$ is the generalized displacement and $Q_n(t)$ is the generalized force corresponding to the n'th mode. $\lambda_n = \alpha_n + j\eta_n$ are the complex eigenvalues of the continuum. $\phi_n(x) = \phi_n^R(x) + i\,\phi_n^I(x)$, and $\psi_n(x) = \psi_n^R(x) + i\,\psi_n^I(x)$ are the right and left hand eigenfunctions of the continuum representing solution of the right and left eigen-problems, respectively, and the overbar denotes complex conjugation.

For the purpose of formulating system's equation in the matrix form, we will use curly bracket to denote real vectors and straight bracket to denote real matrices. We denote by $\{\phi^R\}$, $\{\phi^I\}$, $\{\psi^R\}$, $\{\psi^I\}$, $\{q^R\}$, $\{q^I\}$, $\{Q^R\}$, $\{Q^I\}$ the N-dimensional vectors composed of the components $\phi_n^R, \phi_n^I, \psi_n^R, \psi_n^I, q_n^R, q_n^I, Q_n^R$, and Q_n^I, respectively; and by [α] and [ω] the diagonal matrices of order N:

$$[\alpha] = diag\ [\alpha_1,...,\alpha_n],\ \ [\eta] = diag\ [\eta_1,...,\eta_n]$$

Introducing the 2N-dimensional vectors

$$\{\phi\} = \begin{Bmatrix} \{\phi^R\} \\ \{\phi^I\} \end{Bmatrix}, \{\psi\} = \begin{Bmatrix} \{\psi^R\} \\ \{\psi^I\} \end{Bmatrix}, \{q\} = \begin{Bmatrix} \{q^R\} \\ \{q^I\} \end{Bmatrix}, \{Q\} = \begin{Bmatrix} \{Q^R\} \\ \{Q^I\} \end{Bmatrix}$$

the 2Nx2N block matrix

$$[\Omega] = \begin{bmatrix} [\alpha] & -[\eta] \\ [\eta] & [\alpha] \end{bmatrix},$$

Also introduce the m x 1 displacement vector *{w}* and m x 1 force vector *{W}* at the m interface points

$$\{w\} = \{w(\zeta_1(t),t), \ \ ..., w(\zeta_m(t),t)\}^T,$$

$$\{W\} = \{W_1(t), \ \ ..., W_m(t)\}^T,$$

and 2N x m matrices

$$[\Phi(\zeta(t))] = [\ \{\phi(\zeta_1(t)\},..., \{\phi(\zeta_m(t)\}\ ],$$

$$[\Psi(\zeta(t))] = [\ \{\psi(\zeta_1(t)\},..., \{\psi(\zeta_m(t)\}\ ].$$

The continuum can now be represented as a state equation with m x 1 input force vector *{W(t)}* and m x 1 output displacement vector *{w}:*

$$\{\dot{q}(t)\} = [\Omega]\{q(t)\} + [\Omega]^{-1}\{Q(t)\} \tag{6}$$

$$Q(t) = [\bar{\Psi}(\zeta(t))]\{W(t)\} \tag{7}$$

$$\{w(t)\} = [\Phi(\zeta(t))]^T \{q(t)\} \tag{8}$$

Note that the system is time invariant, but it becomes time varying when the input $\{W(t)\}$ and output $\{w(t)\}$ are included.

Now consider the lumped parameter systems. Let each vehicle dynamics be represented by a state space model with a displacement input vector $\{u_i\}$ and the force output vector $\{y_i\}$:

$$\begin{aligned}\{\dot{x}_i\} &= [A_i]\{x_i\} + [B_i]\{u_i\} \\ \{y_i\} &= [C_i]\{x_i\} + [D_i]\{u_i\}\end{aligned} \tag{9}$$

The lumped parameter systems are also time invariant. They are connected to the distributed parameter system in the following way. The input of one of the lumped parameter systems $u_i(t)$ is the displacement of the continuum at the contact point plus an external disturbance $r(t)$, which represents the road surface profile for the bridge problem

$$u_i(t) = w(\zeta_i(t), t) + r(\zeta_i(t)) . \tag{10}$$

The input of the continuum with respect to one of the lumped parameter systems is the force output of the lumped parameter system $y_i(t)$ plus an external disturbance $d_i(t)$.

$$W_i(t) = y_i(t) + d_i(t) \tag{11}$$

By stacking up the m lumped parameter systems, we have

$$\{x^T(t)\} = \{\{x_1^T(t)\},\ldots,\{x_m^T(t)\}\},$$
$$\{u^T(t)\} = \{u_1^T(t),\ldots,u_m^T(t)\},$$
$$\{y^T(t)\} = \{y_1^T(t),\ldots,y_m^T(t)\},$$
$$\{r^T(t)\} = \{r_1^T(t),\ldots,r_m^T(t)\},$$
$$\{d^T(t)\} = \{d_1^T(t),\ldots,d_m^T(t)\},$$
$$[A_L] = \text{block diag } [A_1,\ldots,A_m],$$
$$[B_L] = \text{block diag } [B_1,\ldots,B_m],$$
$$[C_L] = \text{block diag } [C_1,\ldots,C_m];$$

The lumped system becomes

$$\begin{aligned}\{\dot{x}(t)\} &= [A_L]\{x(t)\} + [B_L]\{u(t)\} \\ \{y(t)\} &= [C_L]\{x(t)\} + [D_L]\{u(t)\}\end{aligned} \tag{12}$$

$$\{u(t)\} = \{w(t)\} + \{r(t)\} \tag{13}$$

$$\{W(t)\} = \{y(t)\} + \{d(t)\} \tag{14}$$

To account for the moving vehicle dynamic behaviors before and after leaving the continuum, the continuum's eigenfunctions can be defined over an extended domain, the function values are the same as the original ones if the location is inside the original continuum domain, and the function values are defined as zeros if the locations are outside of the continuum.

System's Block Diagram. The system of equations developed above can be viewed as a feedback system shown in Figure 2. This system block diagram representation has the following features:

(i) System's component models and their connections follow integral causality, i.e. no differentiation. Thus the system forms a simultaneous first order differential equations (state equations) solved by numerical integration schemes.

(ii) System's component models have clear correspondence to physical components, i.e. vehicle, bridge, road profile, and traffic conditions. This makes it convenient to conduct parametric analysis.

(iii) System theory can be applied directly to the analysis and control of the system. The bridge and vehicle dynamics are time invariant. The only time varying part is the memoryless blocks, which depend on the vehicle locations on the bridge. Thus the system can be classified as a linear parameter varying system.

(iv) It's convenient to add nonlinear effects in the component blocks. For example, the nonlinear spring and damping effects of the suspension, and separation of the vehicle tires from the bridge.

External Disturbances. The external disturbances have been represented in Eqs. (13) and (14) as {r(t)} for motion and {d(t)}for force input respectively. The effects of randomly varying road surface profiles and bumps and potholes at specific locations in the roadway are also important factors as they participate as disturbance inputs in the vehicle-bridge system. The bump/pothole problem can be explicitly accounted for in the formulation through the function $r(\zeta(t))$ in Eq. (13). Various scenarios can be presented in the framework of this examination. For example: What is a "dangerous" pothole for given set of vehicle parameters? How can the effect of a given pothole be minimized by an appropriate vehicle suspension control (active or semi-active) or structure (bridge) control? Finally, The vehicle static gravity can be lumped into the disturbance term as $d_i(t) = M_i g$. Other external disturbances such as the earth quake ground motion and wind gust distributed forces can also be included but are omitted here for brevity.

Control Systems. Active or semi-active vehicle suspension control can be considered as a way to modify the vehicle dynamics to cope with the bridge dynamics and external road surface disturbances. This is shown as transfer function matrix $[K_L]$ in Figure 2. Furthermore, the continuum's structural control can also be included where the point-wise feedback sensor and force actuator locations are represented in the modal transformation matrices $[\Phi(\zeta_{sensor})]$ and $[\psi(\zeta_{actuator})]$ respectively in Figure 2. [Kc] represents the structural controller.

3. Proportionally Damped Non-Gyroscopic Systems.

Bridge structures can usually be considered as proportionally damped system from practical system modeling and identification standpoint. In this case, the general results in Eqn. (6~8) can be further simplified. It has been well established that the eigenfunctions are real and coincide with the eigenfunctions $\varphi_n(x)$ of the self-adjoint undamped problem

$$L_M \frac{\partial^2}{\partial t^2} w(x,t) + L_K w(x,t) = 0 . \tag{15}$$

After manipulations, the continuous system can be put in the following familiar state space form:

$$\begin{Bmatrix} \dot{q}^R(t) \\ \ddot{q}^R(t) \end{Bmatrix} = [\mathrm{A}_c] \begin{Bmatrix} q^R(t) \\ \dot{q}^R(t) \end{Bmatrix} + \begin{Bmatrix} 0 \\ Q^R(t) \end{Bmatrix} \tag{16}$$

$$[\mathrm{A}_c] = \begin{bmatrix} [0] & [I] \\ -[\alpha^2 + \eta^2] & [2\alpha] \end{bmatrix} = \begin{bmatrix} [0] & [I] \\ [-\omega^2] & [-2\zeta\omega] \end{bmatrix} \tag{17}$$

The interaction forces and displacements with the lumped systems are related to the eigenfunctions by

$$\begin{aligned} &\{Q^R(t)\} = [\Gamma(\zeta(t))]\{W(t)\}, \\ &\{w(t)\} = [\Gamma(\zeta(t))]^T \{q^R(t)\}, \\ &[\Gamma(\zeta(t))]_{Nxm} = [\ \{\varphi(\zeta_1(t)\}, ..., \{\varphi(\zeta_m(t)\}\ ]. \end{aligned} \tag{18}$$

The moving vehicle interacts with the continuum through a spring of stiffness *k*. The force of interaction may take both positive and negative values. The possibility of separation of the vehicle from the continuum is not considered herein, but can be included in the simulation. The profile of the road surface (i.e., its deviation from the ideal surface) can be modeled as a general function *r(t)*, which can be stochastic, representing the random profile of the road surface, or deterministic in the case of bumps and potholes at specific locations.

The vehicle dynamics with the interaction motion as the input and the interaction force as the output always have zero relative order. Therefore, to ensure that the entire feedback system have integral causality and is free of algebraic loop, the continuous system must have at least one integrator (relative order 1) between the input and output variables. Indeed, this is true for output to be the displacement $q^R(t)$ or velocity $\dot{q}^R(t)$ in view of Eq. (16) & (17). This is not always true for the more general case in Eq. (6). Indeed if the tire model contains a damper, then the velocity is considered as the input variable to the lumped parameter system. In this case the input is represented as

$$\begin{aligned} \{\dot{w}(t)\} &= [\Gamma(\zeta(t))]^T \{\dot{q}^R(t)\} + [\dot{\Gamma}(\zeta(t))]^T \{q^R(t)\} \\ &= [\Gamma(\zeta(t))]^T \{\dot{q}^R(t)\} + diag[\dot{\zeta}_i \frac{\partial(.)}{\partial \zeta_i}][\Gamma(\zeta(t))]^T \{q^R(t)\} \end{aligned} \tag{19}$$

The input to the lumped system in Eq. (13) should be changed accordingly

$$\{u(t)\} = \{\dot{w}(t)\} + \{\dot{r}(t)\} = \{\dot{w}(t)\} + \{\dot{\zeta}\}\{\frac{\partial(.)}{\partial \zeta}\}^T r(\zeta(t)) \tag{20}$$

However, for the general case of non-self-adjoint continuous system in Eq. (6), we have

$$\begin{aligned} \{\dot{w}(t)\} &= [\Phi(t)]^T \{\dot{q}(t)\} + [\dot{\Phi}(t)]^T \{q(t)\} \\ &= ([\Phi(t)]^T [\Omega] + diag[\dot{\zeta}_i \frac{\partial(.)}{\partial \zeta_i}][\Phi(t)]^T)\{q(t)\} + \\ &[\Phi(t)]^T [\Omega]^{-1} [\overline{\Psi}(t)]\{W(t)\} \end{aligned}$$

The system would avoid algebraic loop if and only if the feed through term, i.e. the last term of the right hand side of the equation, is zero.

4. Simulation Example.

Based on the above formulation, block diagram based simulation was constructed by using commercially available software (Simulink) to simulate the case of multiple vehicles traversing through a bridge represented by a simply supported beam. Here we consider the bridge as a 20 m long simply supported beam with first natural frequency at about 2 Hz. All the modes are assumed to have 5% damping ratio. We consider simple vehicle dynamics as single mass-spring-damper-system as shown in Figure 1 with a mass of 30000 kg and damping ratio of 10%. Two spring stiffness, a soft spring that gives 1.5 Hz. vehicle natural frequency and a hard spring (or higher order mode) that gives 15 Hz. natural frequency, are considered. A stream of five vehicles traversing at 30 m/sec with a uniform spacing of 15 m was simulated. These different suspension characteristics can be considered as actively tuned suspension control to account for interactions with the bridge. The bridge displacement at various locations and the tire force for each vehicle are respectively shown in the plots. Figure 3 shows the results for soft spring and figure 4 shows that of hard spring. Clearly, the 2 Hz. Vehicle passing frequency matches the bridge's natural frequency and causes quasi resonance. The dynamic force due to the stiff spring is much larger than that of the soft spring. For the case of stiff spring, the dynamic tire force of the fifth car is much larger than that of the leading car. If we had considered this system as a moving (gravity) force problem, there would not have been the dynamic tire force components, i.e. $\{y(t)\}=\{0\}$, as illustrated in the simulation. While this is an oversimplified simulation to represent the real vehicle-bridge system dynamics, it demonstrates the possible detrimental effect of a heavy vehicle platoon.

5. Conclusions

We proposed a system model representation for general multiple moving lumped parameter systems interacting with a distributed parameter systems and applied it to the heavy vehicle-bridge interaction problem. The system model is modularized that complex and realistic vehicle and bridge dynamic model, road surface disturbances, and various traffic conditions can be included conveniently and separately into the different sub-systems in the system model. An importance aspect of this approach, where we differ from the work of others, is in the recognition of not only vehicle-bridge interactions but also interactions between vehicles while on the bridge. Another important aspect is that the form of the linear parameter varying system developed herein allows us to consider the analysis and control design using the theoretical results in that field and also employ them to other applications. Both aspects are our ongoing research effort.

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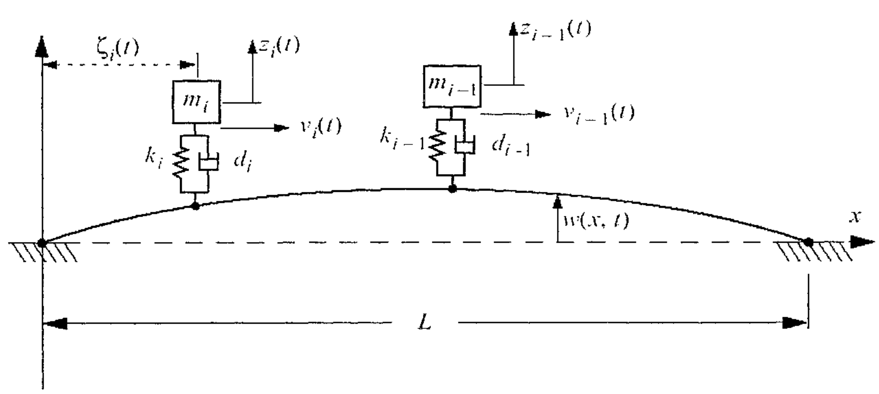

Figure 1 Continuous-lumped system

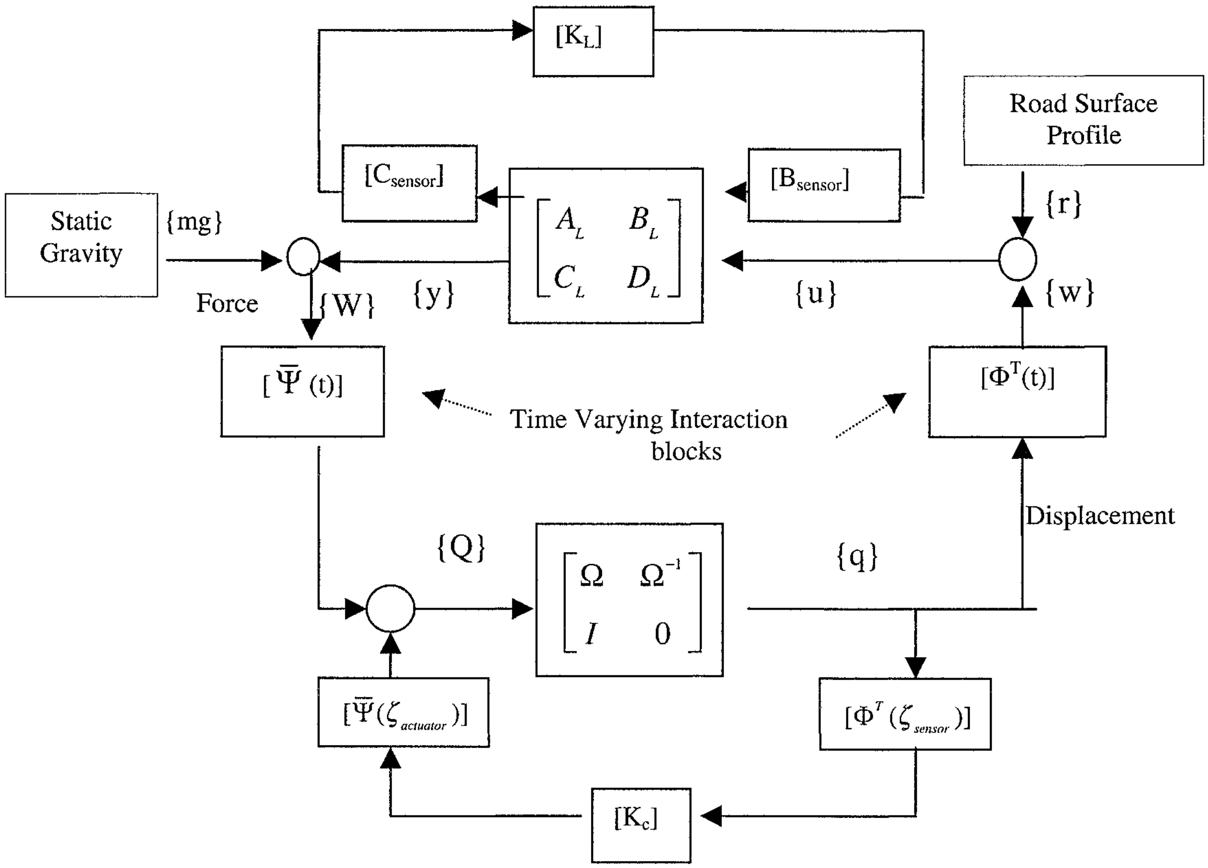

Figure 2 System's block diagram

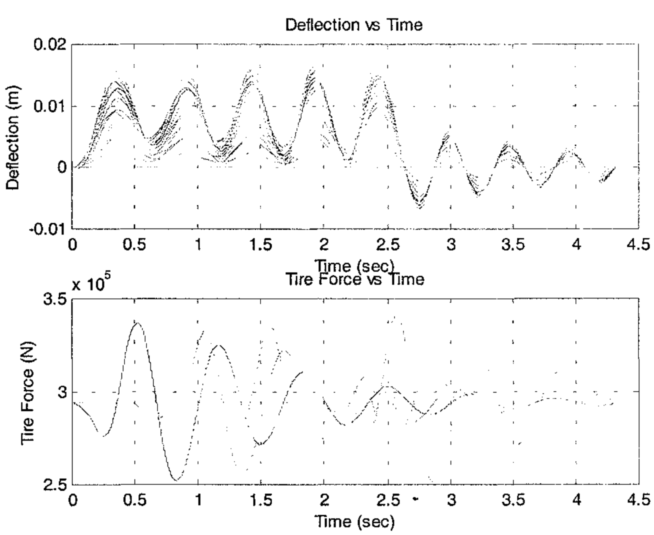

Figure 3 Platoon of vehicles with soft suspension. upper traces: displacements of the bridge at various locations. lower traces: tire forces.

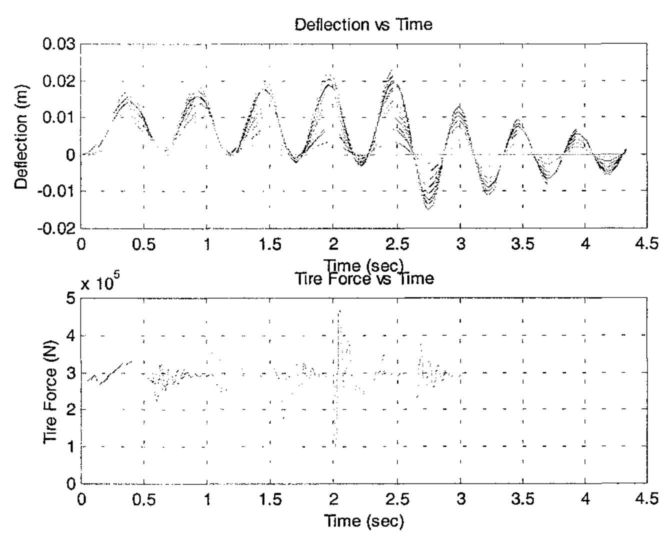

Figure 4 Platoon of vehicles with hard suspension. upper traces: displacements of the bridge at various locations. lower traces: tire forces.