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Nonlinear Backstepping Control of an Electrohydraulic Material Testing System

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Abstract

While the theory of backstepping control design for nonlinear systems has existed for about a decade, there are still very few successful implementation results of this scheme. This paper presents the design and experiment of nonlinear backstepping control for an electrohydraulic material testing fixture. Dynamic displacement tracking control for a nonlinear elastomer specimen is considered. It is shown by experimental comparisons that superior tracking control performance can be achieved by the backstepping design approach when the specimen nonlinearity is accurately modeled. Additionally, the difficulties of implementing the backstepping controller, such as design parameter tuning, transient response, and input saturation issues, are discussed. Heuristic experiences to these issues are presented as well.

1 Introduction

Mechanical properties are identified by material testing operations such as the impact test, creep test, tensile/compression test and fatigue test, etc. Among others, the tensile/compression test is the most frequently performed test. The tensile/compression test measures the resistance of a material by statically or dynamically applied force. A testing material is placed between the upper grip and the lower grip. A force, called load, is then applied by an actuator. Sensors are used to measure the force, strain, and net displacement of the specimen, respectively.

Electrohydraulic actuators are widely used in material testing operations because of its ability to generate a large force with wide bandwidths. In most cases, they are operated under a simple P or PI closed loop control to establish stability. Manual tuning of the controller gains are usually exercised for a specific test, depending on the specimen stiffness, the testing profile and ranges. Typical modes of material testing control include the load (force or differential pressure) control, strain control, and displacement control of the specimen. In dynamic testing where the specified profile is dynamic, it is desirable to closely track the desired profile to fulfill the testing design specification. The nonlinearities presented in the system prevent the simple linear feedback control to achieve good tracking performance.

Several papers on the control application of material testing machines have been published by Lee and Srinivasan. Their first work includes a discussion of the development of the linear model of a hydraulic actuator for a low frequency test and the online identification of plant parameters [1]. They also combined the online identification scheme with a self tuning controller based upon the pole placement method [2]. Their linear model based controller performs well only in low frequency, and low velocity range testing, where the system is assumed to be close to linear.

An adaptive PID control scheme was applied to the material testing machine by Hinton and Clarke [3]. Their adaptive PID controller is able to correct variations caused by the change in specimen stiffness. They used a recursive least squares estimator with a variable for estimating the stiffness of the specimen. The PID gains were then adjusted in real time by the estimates of the stiffness of the specimen. Even though this adaptive PID controller was implemented successfully, most tests were performed on metal specimens, which have linear stiffness. No tests were performed on specimens that had nonlinear stiffness, such as an elastomer.

Backstepping control theory was introduced in early 1990's. However, the published results on applications and implementations of backstepping control have been scarce. Nonlinear backstepping design for the control of active suspension systems was conducted by Lin and Kanellakopoulos [4]. Simulation was performed to show improvement in the performance when compared with conventional approaches. However, no experiment was performed.

Experimental investigation to compare the backstepping approach with passivity based controllers was studied by Bupp, Bernstein and Coppola [5]. The main challenge they encountered was the large control input occurred during the transient stage, which could not be effectively tamed by posing input saturation limits. More recently Alleyne and Lui

[7] experimentally compared the performance of the backstepping controller and their synthetic input nonlinear controller, which is based on passivity formulation for the stability analysis, for the actuator differential pressure control of an electrohydraulic system. Their experimental result showed that the tracking performance was better in backstepping controller than their controller.
This paper presents the displacement control of an electrohydraulic material testing fixture. Displacement control mode is chosen among the three material testing modes because its sensor signal is easier to obtain. Also, the displacement control dynamics have larger relative degree than the load control, implying more complexity in the backstepping control design since more derivatives must be sequentially applied till the control input appears for control synthesis. The modeling, design, and implementation of the backstepping control will be presented in the paper.

2 The Experimental Setup and System Model

A double acting equal area hydraulic actuator (Moog Inc. Model No: 853-038) was mounted on the top of a specially made material testing frame, which is designed for elastomer testing application. The servovalve capacity was 7.5 gpm and the actuator force area was 0.56 in^2. Thus, the maximum force was 1,680 lbf and the maximum velocity of the actuator was approximately 2.34 m/sec.
The nonlinear model of the electrohydraulic system has been studied by several researchers. The linearization of the nonlinear model of the electrohydraulic system was performed in [6], which derived the 8^{th} order linearized model for the electrohydraulic system. The linearized 8^{th} order model consists of a cascaded system of 3^{rd} order actuator model and 5^{th}order servovalve model. Even though the linearized 8^{th} model fits the system's frequency response closely up to 1000 Hz, model reduction was performed to simplify the plant model for the controller design. First, the response of the servovalve was much faster than that of the actuator. It was shown in [6] that first-order servo valve model fits the actual frequency response well up to about 100 Hz. Second, from the standpoint of the backstepping controller, the high order servovalve dynamics would imply power explosion of differentiated terms from the nonlinear terms. Figure 1 shows the overall hydraulic system setup. The 8^{th} order system was reduced to 4^{th} order system as shown in Eq. (1)

$$\begin{aligned}
\dot{x}_1 &= x_2 \\
\dot{x}_2 &= \frac{1}{M}(-bx_2 + Ax_3) \\
\dot{x}_3 &= \frac{4\beta_e}{V_t}(-Ax_2 + K'x_4) \\
\dot{x}_4 &= -\frac{1}{\tau}x_4 + \frac{K}{\tau}u
\end{aligned} \tag{1}$$

where

x_1 : displacement of actuator
x_2 : velocity of actuator
x_3 : difference of pressure
x_4 : displacement of spool

M : mass of double ended actuator and clevis
b : damping coefficient of the actuator
A : actuator ram area
V_t : total actuator volume
β_e : effective bulk modulus
K' : flow coefficient caused by pressure
τ : time constant
K : amplifier gain

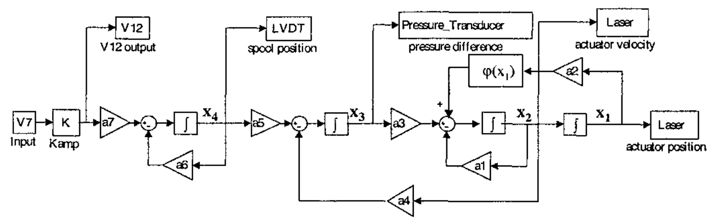

Figure 1: Hydraulic Actuator Model

Four sensors were attached to the material testing frame. These sensors were a laser transducer, a load cell, a Linear Variable Differential Transformer (LVDT) and a pressure transducer. The laser transducer measures the displacement of the hydraulic actuator. The LVDT measures the servovalve spool position. The two pressure transducers with low-pass filters and summation circuit measure the differential pressure of the actuator. The load cell was attached at the bottom of the material testing frame to measure the force applied on the specimen during the operations.
The elastomer, a number of natural and synthetic linear polymers, displays a large amount of deformation when a force is applied and presents substantial nonlinearity between force and displacement.
The elastomer's force versus displacement data was first obtained experimentally and then curve fitted with parameterized functions. We tried three different cases to fit the curve. In the first case, we fit the nonlinear curve with a 4^{th} order polynomial as shown in Figure 2. The polynomial equation is written in Eq. (2). In the second case we fit the nonlinear curve with three straight lines as shown in Figure 3. The three lines are listed in Eq. (3). In the third case, we simply fit the nonlinear curve with a straight line as shown in Figure 4. The equation is shown in Eq. (4).
The primary reason we tried three different cases for the specimen model was to show the importance of the nonlinear terms presented by specimen. The backstepping control performance using these models will be compared

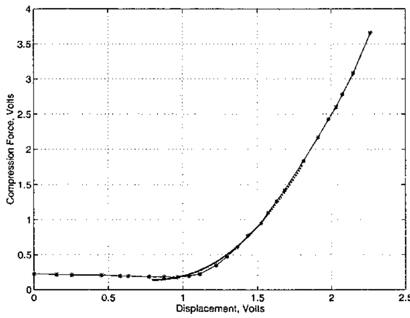

Figure 2: Case 1 : Stiffness Curve Fitting

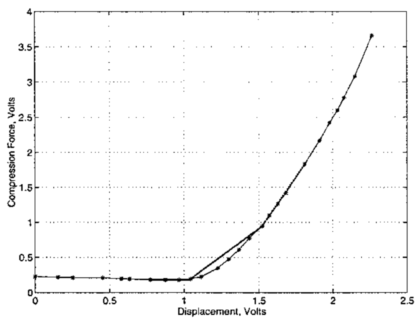

Figure 3: Case 2 : Stiffness Curve Fitting

to signify the important role of the specimen nonlinear model in the overall system.

$$\text{Case } 1: f(x) = -0.3548x^4 + 2.0256x^3 - 2.4435x^2 + 0.7805x + 0.1876 \quad (2)$$

$$\text{Case } 2: f(x) = 0.182 \quad (0.8 < x \leq 1.0)$$
$$f(x) = 1.5688x - 1.4413 \quad (1.0 < x \leq 1.5)$$
$$f(x) = 3.076x - 3.7366 \quad (1.5 < x \leq 1.8) \quad (3)$$

$$\text{Case } 3: f(x) = 1.5892x - 1.3440 \quad (4)$$

where
x: displacement of actuator
$f(x)$: force applied on the specimen

3 Nonlinear Backstepping Controller

The plant, which was controlled by the backstepping controller, was a cascaded system of hydraulic system and the specimen. The specimen used had a nonlinear stiffness. Thus, the specimen term was modeled as a nonlinear term as shown in Eq. (5).

$$\begin{aligned} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= -a_1 x_2 - f(x_1) + a_3 x_3 \\ \dot{x}_3 &= -a_4 x_2 + a_5 x_4 \\ \dot{x}_4 &= -a_6 x_4 + a_7 u \end{aligned} \quad (5)$$

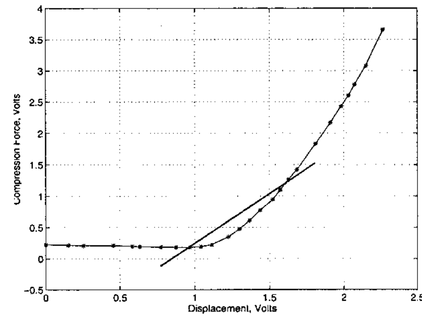

Figure 4: Case 3 : Stiffness Curve Fitting

where

$$a_1 = \frac{b}{M} \qquad a_2 = \frac{1}{M}$$
$$a_3 = \frac{A}{M} \qquad a_4 = \frac{4\beta_e A}{V_t}$$
$$a_5 = \frac{4\beta_e K'}{V_t} \qquad a_6 = \frac{1}{\tau}$$
$$a_7 = \frac{K}{\tau} \qquad f(x_1) = a_2 \varphi(x_1)$$

$\varphi(x_1)$: stiffness of specimen

Among the three material testing control modes, we chose the position control mode in order to demonstrate how effectively the backstepping controller can compensate for nonlinear terms. It was not convenient to mount the sensor on the elastomer for strain control. The displacement control is more challenging than the load control or pressure control from backstepping design standpoint because the displacement output has higher order relative degree than the other two variables. The control objectives were to stabilize the plant and to track the given reference signal asymptotically. The detailed derivations of finding backstepping control input are covered in next 5 steps.
Step 1. Define the first error term, z_1.

$$\begin{aligned} z_1 &= x_1 - y_r \\ \dot{z}_1 &= \dot{x}_1 - \dot{y}_r \\ &= x_2 - \dot{y}_r \end{aligned} \quad (6)$$

where y_r : reference input
Take x_2 as the virtual control. The first stabilizing function α_1 would be chosen as shown in Eq. (7).

$$\alpha_1 = -C_1 z_1 + \dot{y}_r \quad (7)$$

where C_1 : the first design parameter
The partial differentiations of α_1 with respect the terms x_1, y_r and $\dot{y}_r$ are performed for use in the next steps.

$$\alpha_{11}: \quad \frac{\partial \alpha_1}{\partial x_1} = -C_1 \frac{\partial z_1}{\partial x_1} = -C_1 \quad (8)$$

$$\alpha_{12}: \quad \frac{\partial \alpha_1}{\partial y_r} = -C_1 \frac{\partial z_1}{\partial y_r} = C_1 \quad (9)$$

$$\alpha_{13}: \quad \frac{\partial \alpha_1}{\partial \dot{y}_r} = 1 \quad (10)$$

Step 2. Define the second error term, z_2.

$$\begin{aligned}
z_2 &= x_2 - \alpha_1 \\
\dot{z}_2 &= \dot{x}_2 - \dot{\alpha}_1 \\
&= -a_1 x_2 - a_2 \varphi(x_1) + a_3 x_3 - \frac{\partial \alpha_1}{\partial x_1}\dot{x}_1 - \frac{\partial \alpha_1}{\partial y_r}\dot{y}_r - \frac{\partial \alpha_1}{\partial \dot{y}_r}\ddot{y}_r \\
&= -a_1 x_2 - a_2 \varphi(x_1) + a_3 x_3 - \alpha_{11} x_2 - \alpha_{12}\dot{y}_r - \alpha_{13}\ddot{y}_r
\end{aligned} \tag{11}$$

Take $a_3 x_3$ as the second virtual control. The second stabilizing function α_2 would be chosen as shown in Eq. (12).

$$\alpha_2 = a_1 x_2 + a_2 \varphi(x_1) + \alpha_{11} x_2 + \alpha_{12}\dot{y}_r + \alpha_{13}\ddot{y}_r - z_1 - C_2 z_2 \tag{12}$$

where C_2 : the second design parameter

Again, the following partial differentiations of α_2 with respect to the terms x_1, x_2, y_r, $\dot{y}_r$ and $\ddot{y}_r$ are performed below.

$$\alpha_{21}: \quad \frac{\partial \alpha_2}{\partial x_1} = a_2 \frac{\partial \varphi(x_1)}{\partial x_1} - \frac{\partial z_1}{\partial x_1} - C_2 \frac{\partial z_2}{\partial x_1} = a_2 \frac{\partial \varphi(x_1)}{\partial x_1} - 1 - C_1 C_2 \tag{13}$$

$$\alpha_{22}: \quad \frac{\partial \alpha_2}{\partial x_2} = a_1 + \alpha_{11} - C_2 \frac{\partial z_2}{\partial x_2} = a_1 - C_1 - C_2 \tag{14}$$

$$\alpha_{23}: \quad \frac{\partial \alpha_2}{\partial y_r} = -\frac{\partial z_1}{\partial y_r} - C_2 \frac{\partial z_2}{\partial y_r} = 1 + C_1 C_2 \tag{15}$$

$$\alpha_{24}: \quad \frac{\partial \alpha_2}{\partial \dot{y}_r} = \alpha_{12} - C_2 \frac{\partial z_2}{\partial \dot{y}_r} = C_1 + C_2 \tag{16}$$

$$\alpha_{25}: \quad \frac{\partial \alpha_2}{\partial \ddot{y}_r} = \alpha_{13} = 1 \tag{17}$$

Step 3. Define the third error term, z_3.

$$\begin{aligned}
z_3 &= a_3 x_3 - \alpha_2 \\
\dot{z}_3 &= a_3 \dot{x}_3 - \dot{\alpha}_2 \\
&= a_3(-a_4 x_2 + a_5 x_4) - \frac{\partial \alpha_2}{\partial x_1}\dot{x}_1 - \frac{\partial \alpha_2}{\partial x_2}\dot{x}_2 \\
&\quad - \frac{\partial \alpha_2}{\partial y_r}\dot{y}_r - \frac{\partial \alpha_2}{\partial \dot{y}_r}\ddot{y}_r - \frac{\partial \alpha_2}{\partial \ddot{y}_r}\dddot{y}_r \\
&= -a_3 a_4 x_2 + a_3 a_5 x_4 - \alpha_{21} x_2 - \alpha_{22}(-a_1 x_2 - a_2 \varphi(x_1) \\
&\quad + a_3 x_3) - \alpha_{23}\dot{y}_r - \alpha_{24}\ddot{y}_r - \alpha_{25}\dddot{y}_r
\end{aligned} \tag{18}$$

Take $a_3 a_5 x_4$ as the third virtual control. The third stabilizing function α_3 would be chosen as shown in Eq. (19).

$$\begin{aligned}
\alpha_3 &= a_3 a_4 x_2 + \alpha_{21} x_2 + \alpha_{22}(-a_1 x_2 - a_2 \varphi(x_1) + a_3 x_3) \\
&\quad + \alpha_{23}\dot{y}_r + \alpha_{24}\ddot{y}_r + \alpha_{25}\dddot{y}_r - C_3 z_3 - z_2
\end{aligned} \tag{19}$$

where C_3 : the third design parameter

Differentiate α_3 with respect to the state variables and the reference terms, x_1,x_2, x_3, y_r, $\dot{y}_r$, $\ddot{y}_r$ and $\dddot{y}_r$.

$$\begin{aligned}
\alpha_{31}: \quad \frac{\partial \alpha_3}{\partial x_1} &= \frac{\partial \alpha_{21}}{\partial x_1} x_2 - a_2 \alpha_{22} \frac{\partial \varphi(x_1)}{\partial x_1} - \frac{\partial z_2}{\partial x_1} - C_3 \frac{\partial z_3}{\partial x_1} \\
&= a_2 \frac{\partial^2 \varphi(x_1)}{\partial x_1^2} x_2 - a_2 \alpha_{22} \frac{\partial \varphi(x_1)}{\partial x_1} - C_1 + C_3 \alpha_{21}
\end{aligned} \tag{20}$$

$$\alpha_{32}: \quad \frac{\partial \alpha_3}{\partial x_2} = a_3 a_4 + \alpha_{21} - a_1 \alpha_{22} - \frac{\partial z_2}{\partial x_2} - C_3 \frac{\partial z_3}{\partial x_2} = a_3 a_4 + \alpha_{21} - a_1 \alpha_{22} - 1 + C_3 \alpha_{22} \tag{21}$$

$$\alpha_{33}: \quad \frac{\partial \alpha_3}{\partial x_3} = a_3 \alpha_{22} - C_3 \frac{\partial z_3}{\partial x_3} = a_3 \alpha_{22} - a_3 C_3 \tag{22}$$

$$\alpha_{34}: \quad \frac{\partial \alpha_3}{\partial y_r} = -\frac{\partial z_2}{\partial y_r} - C_3 \frac{\partial z_3}{\partial y_r} = C_1 + C_3 \alpha_{23} \tag{23}$$

$$\alpha_{35}: \quad \frac{\partial \alpha_3}{\partial \dot{y}_r} = \alpha_{23} - C_3 \frac{\partial z_3}{\partial \dot{y}_r} - \frac{\partial z_2}{\partial \dot{y}_r} = \alpha_{23} + C_3 \alpha_{24} + 1 \tag{24}$$

$$\alpha_{36}: \quad \frac{\partial \alpha_3}{\partial \ddot{y}_r} = \alpha_{24} - C_3 \frac{\partial z_3}{\partial \ddot{y}_r} = \alpha_{24} + C_3 \tag{25}$$

$$\alpha_{37}: \quad \frac{\partial \alpha_3}{\partial \dddot{y}_r} = \alpha_{25} = 1 \tag{26}$$

Step 4. Define the fourth error term, z_4.

$$\begin{aligned}
z_4 &= a_3 a_5 x_4 - \alpha_3 \\
\dot{z}_4 &= a_3 a_5 \dot{x}_4 - \dot{\alpha}_3 \\
&= a_3 a_5(-a_6 x_4 + a_7 u) - \frac{\partial \alpha_3}{\partial x_1}\dot{x}_1 - \frac{\partial \alpha_3}{\partial x_2}\dot{x}_2 - \frac{\partial \alpha_3}{\partial x_2}\dot{x}_3 \\
&\quad - \frac{\partial \alpha_3}{\partial y_r}\dot{y}_r - \frac{\partial \alpha_3}{\partial \dot{y}_r}\ddot{y}_r - \frac{\partial \alpha_3}{\partial \ddot{y}_r}\dddot{y}_r - \frac{\partial \alpha_3}{\partial \dddot{y}_r}\ddddot{y}_r \\
&= -a_3 a_5 a_6 x_4 + a_3 a_5 a_7 u - \alpha_{31} x_2 \\
&\quad - \alpha_{32}(-a_1 x_2 - a_2 \varphi(x_1) + a_3 x_3) - \alpha_{33}(-a_4 x_2 + a_5 x_4) \\
&\quad - \alpha_{34}\dot{y}_r - \alpha_{35}\ddot{y}_r - \alpha_{36}\dddot{y}_r - \alpha_{37}\ddddot{y}_r - C_4 z_4 - z_3
\end{aligned} \tag{27}$$

The control input, u, is appeared in the Eq. (27). Thus, control input, u, will be selected to make the closed loop system stable. It is shown in Eq. (28).

$$\begin{aligned}
u = \frac{1}{a_3 a_5 a_7}[&a_3 a_5 a_6 x_4 + \alpha_{31} x_2 - \alpha_{32}(-a_1 x_2 - a_2 \varphi(x_1) \\
&+ a_3 x_3) + \alpha_{33}(-a_4 x_2 + a_5 x_4) + \alpha_{34}\dot{y}_r + \alpha_{35}\ddot{y}_r \\
&+ \alpha_{36}\dddot{y}_r + \alpha_{37}\ddddot{y}_r - C_4 z_4 - z_3]
\end{aligned} \tag{28}$$

Eq. (28) is in fact in the form of nonlinear state feedback and linear feedforward from the reference and its derivatives up to the plant's relative order.

Step 5. Stability check.

Closed loop error equations are shown below.

$$\begin{bmatrix} \dot{z}_1 \\ \dot{z}_2 \\ \dot{z}_3 \\ \dot{z}_4 \end{bmatrix} = \begin{bmatrix} -C_1 & 1 & 0 & 0 \\ -1 & -C_2 & 1 & 0 \\ 0 & -1 & -C_3 & 1 \\ 0 & 0 & -1 & -C_4 \end{bmatrix} \begin{bmatrix} z_1 \\ z_2 \\ z_3 \\ z_4 \end{bmatrix} \quad (29)$$

Lyapunov stability check can be performed.

$$\begin{aligned} V &= \frac{1}{2}z_1^2 + \frac{1}{2}z_2^2 + \frac{1}{2}z_3^2 + \frac{1}{2}z_4^2 \\ \dot{V} &= z_1\dot{z}_1 + z_2\dot{z}_2 + z_3\dot{z}_3 + z_4\dot{z}_4 \\ &= z_1(-C_1 z_1 + z_2) + z_2(-C_2 z_2 + z_3 - z_1) \\ &\quad + z_3(-C_3 z_3 + z_4 - z_2) + z_4(-C_4 z_4 - z_3) \\ &= -C_1 z_1^2 - C_2 z_2^2 - C_3 z_3^2 - C_4 z_4^2 \leq 0 \\ &\text{Thus, } \dot{V} \leq 0 \end{aligned} \quad (30)$$

Since $\dot{V} \leq 0$, the closed loop system is GAS. The equilibrium points are $z_1 = z_2 = z_3 = z_4 = 0$. $z_1 = 0$ implies asymptotic tracking of the reference signal.

In fact, if the system was linear, the above backstepping control design would result in a linear controller where the closed loop dynamics are exactly Eq. (29). The eigenvalues of the system matrix in Eq. (29) would be close to (-C1, -C2, -C3, -C4) when the values of these parameters are much larger than 1.

4 Implementation of Backstepping Controller

While the backstepping control input was designed in continuous time domain, the controller was implemented by a digital signal processor at a sampling frequency of 5000 Hz. Two major difficulties were encountered in implementing the backstepping controller. First was the selection of the parameters for the closed loop error dynamics in Eq. (29), and the other difficulty was the very large and chattering control input signals during the transition state. It easily went over the maximum input limits set to safeguard the plant.

To choose the design parameters, let's inspect the closed loop equation, Eq. (29). Note that C_1 is the pole of the first error equation. Thus, the convergence rate of the z_1 is dependent upon the magnitude of C_1. For example, C_1 should be set set at a high value when the rapid convergence of z_1 is desired and should be set at a low value when a lower rate of convergence is required. The same observations can be made in the second, third and forth error equations. Thus, the role of these design parameters is to control the rate of the convergence of error equations in the closed loop.

The second issue was that the control input was too large in the transition state. This was also detected in the simulation. The primary reason for this occurrence is that the backstepping controller input is found sequentially and the error terms are defined as the difference between the states and the desired values of these states. Thus, the gap between each state and its desired value is initially quite large but gets smaller and smaller in the steady state. Moreover, each of these error terms are embedded in the next error term. Thus, the last error term, z_4, has the largest magnitude in the transition state. Since the backstepping control input contains these error terms, it becomes large enough to go beyond the saturation limit. In order to force these error terms to converge to zero more quickly, it seems that the design parameters should be set at a larger value. In practice, however, choosing large design parameters could lead to a large control input, which in turn may lead to input saturation. Let's take a look at Eq. (28). The α terms shown in the formula are explicit variables of combinations of the design parameters, C_1 through C_4. Thus, the control input will be large when the design parameters are set at a large value. Consequently, there is a trade off between choosing parameters for the fast convergence and avoiding input saturation. It should be noticed that the feedforward terms from the reference signal and its derivatives terms should not generate the input saturation problem at all. Otherwise, it simply indicates the inability of the plant to track that particular reference trajectory.

We grouped the backstepping input Eq. (28) into two categories as shown in Eq. (31).

$$\begin{aligned} u = {} & f(C_1, C_2, C_3, C_4, x_1, x_2, x_3, x_4) \\ & + \beta_1 y_r + \beta_2 \dot{y}_r + \beta_3 \ddot{y}_r + \beta_4 \dddot{y}_r + \beta_5 \ddddot{y}_r \end{aligned} \quad (31)$$

Where

$$\begin{aligned} \beta_1 &= 1 + C_1C_2 + C_1C_4 + C_3C_4 + C_1C_2C_3C_4 \\ \beta_2 &= 2C_1 + C_2 + C_3 + 2C_4 + C_1C_2C_3 + C_2C_3C_4 + C_1C_3C_4 \\ &\quad + C_1C_2C_4 + C_1C_3C_4 + C_1C_2C_4 \\ \beta_3 &= 3 + C_1C_2 + C_1C_3 + C_1C_4 + C_2C_3 + C_2C_4 + C_3C_4 \\ \beta_4 &= C_1 + C_2 + C_3 + C_4 \\ \beta_5 &= 1 \end{aligned}$$

Note that the coefficients of the feedforward part correspond to those of the characteristic polynomial of Eq. (29). To avoid input saturation and chattering, reduction on the design parameters C_i's were implemented during the transient stage, which in effect corresponds to reducing gains on the state's feedback terms. The feedforward part remains unchanged to force fast transient tracking errors convergence without causing saturation. As soon as the control input reaches near steady state, the reduction gains on the feedback part are removed. Thus any disturbance coming in after this will be rejected at the designed rate. This method ensures that applying the reduction factors would not have any affect upon the tracking performance in the steady state since the feedforward part is supposed to take care of the major part of the tracking and is not altered during the transient stage.

In the experiment, the displacement reference signal to be tracked was a 30 Hz sine wave between 0.8 and 1.8 volts. Figure 5 shows the experimental result using the most accurate nonlinear specimen model in Case 1. The design parameters were C_1 = 900, C_2 = 1000, C_3 = 1000 and C_4 = 1000. The tracking error was little less than $\pm$ 0.05 (V).

The significance of the nonlinear term and its derivatives can be demonstrated by using the piecewise linear and linear models in the Case 2 and 3 respectively. Figure 6 shows the experimental tracking error for the piecewise linear model in Case 2 function. The best tracking performance obtained was ± 0.1 (V). While the piecewise linear function appears to agree with the nonlinear curve well as shown in Figure 3, its derivatives do not match that well. These high order derivative terms cause the tracking performance degradation and illustrate their significance in the backstepping control. The experimental results for the simple linearization model, Case 3, is shown in Figure 7. It was difficult to stabilize the plant with this controller, but, by trial and error, we were able to reduce the tracking error down to ± 0.24 (V).

Since a load cell had been instrumented in the system, we also attempted to replace the nonlinear term, $\varphi(x_1)$, by the measured force feedback signal in the implementation of Case 1. The derivative terms from the nonlinear model were maintained in the control implementation. Since the force feedback represents a direct measurement of the nonlinear force, a better tracking performance is expected. Indeed, the tracking error was further reduced to ± 0.008 (V), as shown in Figure 8. This result again suggests the importance of an accurate nonlinear model in the backstepping control.

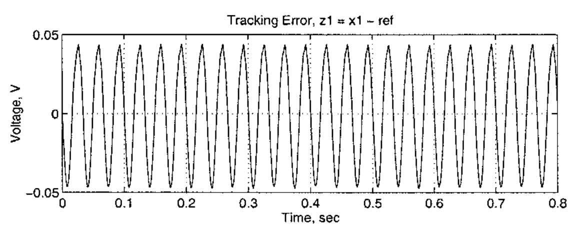

Figure 5: Backstepping Control Tracking Error of Case 1 Model

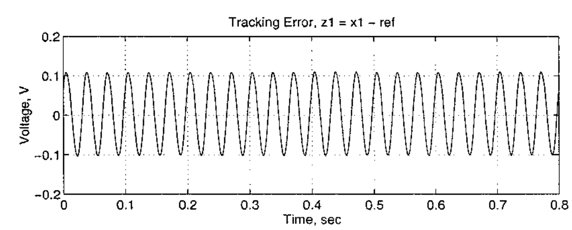

Figure 6: Backstepping Control Tracking Error of Case 2 Model

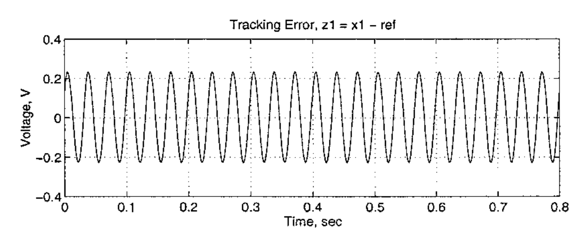

Figure 7: Backstepping Control Tracking Error of Case 3 Model

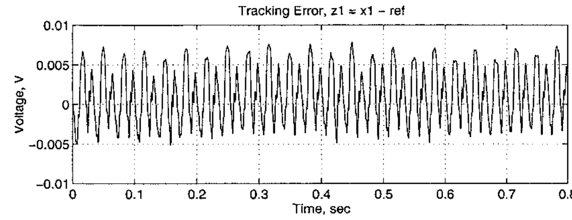

Figure 8: Backstepping Control Tracking Error of Case 1 Model and Force Feedback

5 Conclusions

The backstepping controller was designed and implemented to control a nonlinear electrohydraulic actuated material testing machine. Commonly occurred large control input and chattering in the transition state are avoided by applying reduction factors in the feedback terms during the initial transient stage. Since the nonlinear terms and derivatives propagate in the backstepping control, it is important to have an accurate nonlinear model for achieving superior control performance.

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