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REJECTION OF REPEATABLE AND NON-REPEATABLE DISTURBANCES FOR DISK DRIVE ACTUATORS

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ABSTRACT
This paper presents an application of two-parameter robust repetitive control (TPRRC) in disk drive servo control, where the repeatable runout (RRO) and non-repeatable runout (NRRO) are rejected simultaneously. The repeatable disturbances are rejected by the internal model structure of TPRRC, while the non-repeatable disturbances are attenuated by the robust performance specification. The design procedure of TPRRC is given. A simple structure of the robust performance weighting function $W_p(j\varpi)$ is proposed and the relationship between the coefficients of $W_p(j\varpi)$ and the performance specification over the non-repeatable disturbance is discussed too. The simulation results using the measured position error signals (PES) of a disk drive as output disturbance are presented to show the effectiveness of the disturbance rejection of disk drive servo control and demonstrate that it is a cost effective way to improve the track density of disk drives.

Keyword: Disk Drive Servo Control, Repetitive Control and Disturbance Rejection.

1. INTRODUCTION

The capacity of magnetic hard disk drive has increased significantly in recent years, while the size of the hard disk drive is keeping smaller and smaller. This trend is probably to continue in the near future. It is estimated that a high track density of 25000 tracks per inch (TPI) will be expected by the end of this century [2]. This will require better performance of disk drive servo control.

The read/write operation of a disk drive is usually categorized into the track-seeking mode and the track-following mode. In track-seeking mode, the head positioning servo control loop moves the head to the vicinity of a target track. In track-following mode, the head positioning servo control loop precisely positions the head on the desired track for reading or writing of data. As the track density increases, the disk drive servo control becomes more difficult since various disturbances caused by manufacturing processes will limit the performance improvement of the servo control. The disturbances can be divided into repeatable and non-repeatable disturbances. For example, in track following, the eccentricity of the spindle will cause the repeatable runout (RRO). The disk vibration and bearing defects will cause the non-repeatable runout (NRRO).

Much research has been done to deal with the repeatable disturbances. The internal model based repetitive control is one of the well-known methods and has been demonstrated to be effective in rejecting repeatable disturbances in disk drives [3] [6]. However, this repetitive control using zero phase error tracking controller (ZPETC) can reject only the repeatable disturbance of a fundamental frequency and its harmonics. In order to achieve high track density, we have to deal with those NRRO. One way is to reduce the NRRO during the disk drive manufacturing processes. The disadvantage is that it will increase the cost of disk drive significantly. The other way is to design a better control algorithm with the capability to reject both RRO and NRRO simultaneously.

Guo(1997) proposed a disturbance rejection controller based on the repetitive controller using ZPETC, where two polynomials of R(z) and S(z) are added to do frequency shaping of sensitivity function over the frequency range of NRRO [4]. However, considering the uncertainty of the model, which is inevitable in practice, the structures of R(z) and S(z) are constrained in order to achieve robustness. Therefore the frequency shaping of sensitivity function based on R(z) and S(z) is limited and the tradeoff between robustness and performance is not elucidated.

Li and Tsao (1998) proposed a framework for the synthesis and analysis of robust repetitive controller using structured singular values [8]. It provided a systematic way to tradeoff between robustness and performance by converting the repetitive control design into a standard problem in linear fractional transformation (LFT) form. The robust performance can be achieved not only to the repeatable signal of fundamental frequency and its harmonics, but also to the non-repeatable signals whose frequency range is determined by the performance weighting $W_p(j\omega)$. To make the controller realizable in practice, a fictitious uncertainty was introduced to substitute the big delay term of repetitive control, such that the designed controller is low order and can be implemented efficiently.

To further push the robust performance, Li and Tsao (1998) also presented a two-parameter robust repetitive controller (TPRRC) [9]. An additional design freedom was added to improve the robust performance over the non-repeatable signals. Since TPRRC can deal with both repeatable and non-repeatable signals, it is a good alternative for disk drive servo control. The challenge is that the non-repeatable disturbances in disk drive servo control (NRRO) usually locate in the high frequency, around 400 Hz as reported in [4] [7] [10].

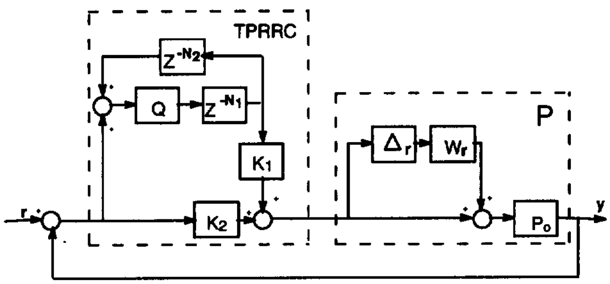

Figure 1. The Block Diagram of TPRRC

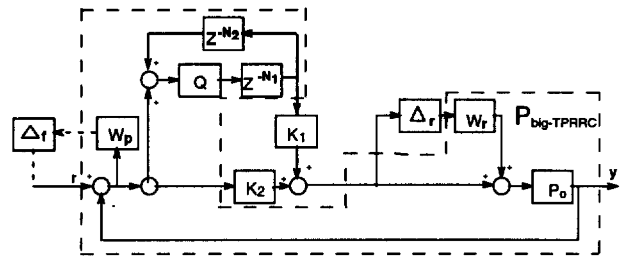

Figure 2. The Design Framework of TPRRC

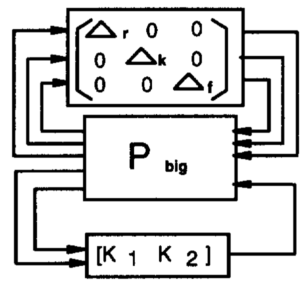

Figure 3. The LFT Form of TPRRC

This paper presents the repeatable and non-repeatable disturbances rejection in disk drive servo control using TPRRC. It demonstrates that it is a cost-effective way to improve the track density of disk drives by improving the performance of the servo control. In section 2, a brief review of TPRRC and design procedure is given. In section 3, the issues of selection of control model and performance weighting for disk drive servo control are discussed. The relationship between the coefficients of $W_p(j\varpi)$ and the performance specification over the non-repeatable disturbance is discussed too. The simulation results using the measured position error signal (PES) of disk drives as output disturbance are presented in section 4 to show the effectiveness of repeatable and non-repeatable disturbances rejection in disk drive servo control. Finally, conclusions are given in section 5.

2. TWO PARAMETER ROBUST REPETITIVE CONTROL (TPRRC)

There are two objectives in TPRRC:

(a). Perfectly reject the repeatable disturbance and its harmonics. In discrete time domain, the repeatable signals are governed by

$$f(k+N) = f(k) \quad \text{for } \forall k \tag{1}$$

where k stands for time and N is the period of the repeatable signal.

(b). Achieve robust performance over non-repeatable signals, which is specified by following inequality

$$\left|W_p(j\varpi)S_{TPRRC}(j\varpi)\right| < 1 \qquad \text{for } \forall\varpi \tag{2}$$

where S_{TPRRC} is the sensitivity function of the closed loop system of TPRRC. $W_p(j\varpi)$ is the performance weighting.

The block diagram of TPRRC is shown in figure 1, where P_o is the nominal plant, W_r is the weighting of unmodeled dynamics. The unmodeled dynamics Δ_r is assumed stable and the norm of Δ_r is less than or equal to 1, i.e. $\Delta_r \in \Re H_\infty$ and $\|\Delta_r\| \le 1$. The actual plant is $P_o\,(I + W_r\,\Delta_r)$. Where $N_1 + N_2 = N$, N is the period of repeatable signal and $N_1 >> N_2$. Q is a low pass filter, which is necessary for robustness of the closed loop system.

The task is to design controller K_1 and K_2, such that the closed loop system is stable and inequality (2) is satisfied. It is easy to show that the closed loop system can reject the repeatable signals with period of N and its harmonics by internal model principle, since there is a periodic signal generator in the structure of TPRRC. In order to design a low order controller which can be realized efficiently, the proposed design framework is shown in figure 2, where the big delay term of repetitive control is substituted by a fictitious uncertainty Δ_k, where Δ_k is assumed stable and the norm of Δ_k is less than or equal to 1, i.e. $\Delta_k \in \Re H_\infty$ and $\|\Delta_k\| \le 1$.

The design procedure of TPRRC is following:

(1). Design the controller K_1 and K_2 under the design framework of Figure 2, which is converted into a standard problem in LFT form as shown in figure 3, such that

$$\sup_{\varpi} \mu_{\Delta_p}\left(F_\ell(P_{big}(j\varpi),\ [K_1(j\varpi)\ \ K_2(j\varpi)])\right) < 1 \tag{3}$$

where $\Delta_p := \{\mathrm{diag}\,(\Delta_r, \Delta_k, \Delta_f),\ \Delta_r \in C,\ \Delta_k \in C,\ \Delta_f \in C\}$. This is a standard μ synthesis problem and can be solved systematically in Matlab.

(2). Implement the TPRRC in figure 1, using the controller K_1 and K_2 designed in step (1).

(3). It is guaranteed that the TPRRC in figure 1 will achieve the two objectives, i.e. perfectly reject the repeatable signal with period of N and its harmonics, and the sensitivity function of the closed loop system of TPRRC is shaped by inequality (2). Please refer to [9] for proof.

3. SYSTEM MODEL AND PERFORMANCE WEIGHTING

3.1 System Model. A lot of researchers have tried to get an accurate mathematical model of disk drive [1], [10]. These models in complex mathematical form describe the dynamics of each component of disk drive, such as the multiple flexible disks, spindle, bearing and etc.. These models help us to have more

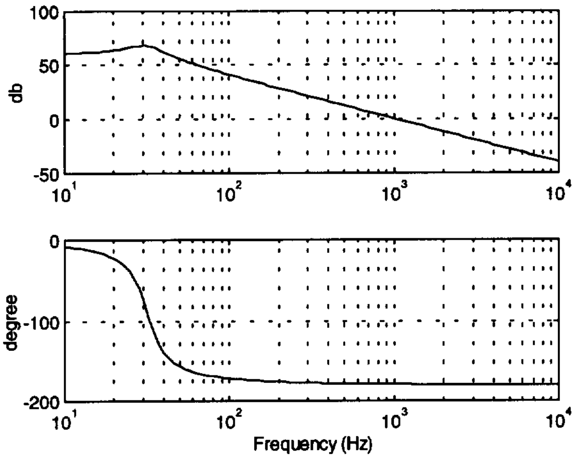

Figure 4. The Frequency Response of System Model

insight of the dynamics of disk drive and can be used to predict the resonance of the unbalanced mode and forced response analytically.

However, for a control engineer, a simplified model is preferred because of the complexity of controller, speed limit of DSP and other implementation issues. A conventional disk drive actuator has ball bearings and a voice coil motor (VCM). A simplified form of the actuator's transfer function is proportional to a double integrators. However, the nonlinear dynamics due to the actuator's ball bearings and pivot friction affects the form of the transfer function. A second order model with damp is popular in the design of disk drive servo control. In this paper, the following second order model is used.

$$P(s) = \frac{K_p \varpi_p^{\,2}}{S^2 + 2\xi_p \varpi_p S + \varpi_p^{\,2}} \tag{4}$$

The frequency response of the model is shown in figure 4.

3.2 Performance Weighting $W_p(j\varpi)$. As we know, the repeatable signals are taken care of by the internal model structure of periodic signal. However, the performance over non-repeatable signals is described by inequality (2). So, the selection of performance weighting $W_p(j\varpi)$ depends on the performance specification over the non-repeatable signals. Figure 7 shows the measured track-following position error signal (PES) of a disk drive running at 5400 rpm. The power spectrum of the PES is shown in Figure 8, where there is a strong non-repeatable disturbance at 420 Hz as predicted by mathematical analysis. A weighting function as simple as possible is preferred, such that the control system is low order and can be implemented efficiently. In this paper, a third order weighting function is used.

$$Wp(\,j\varpi\,) = \frac{\varpi_N^{\,2}}{(\,j\varpi\,)^2 + 2\xi\varpi_N(\,j\varpi\,) + \varpi_N^{\,2}} * \frac{K\varpi_L}{j\varpi + \varpi_L} \tag{5}$$

The reason choosing the third order function of (5) as the performance weighting is that we can easily specify the performance specification over non-repeatable signals by

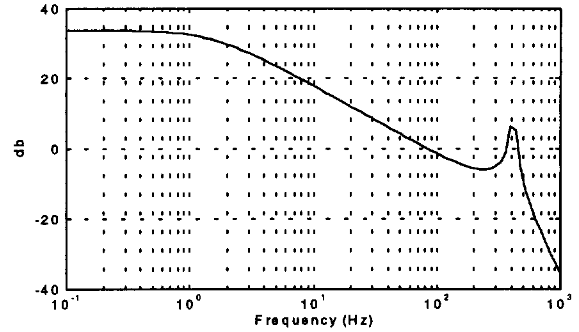

Figure 5. The Frequency Response of Performance Weighting $W_p(j\varpi)$

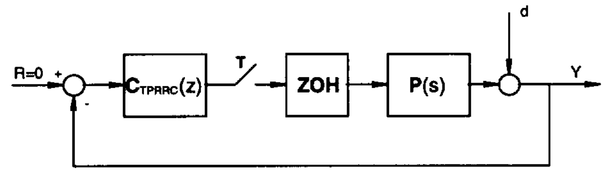

Figure 6. The Block Diagram of a Disk Drive Control

assigning appropriate values of the parameters ϖ_N, ξ, K and ϖ_L of (5). The relationship between the parameters and the performance specification over non-repeatable disturbances are described as following:

> ϖ_N: ϖ_N roughly approximates the frequency of non-repeatable disturbances, when it is the middle value of the frequency range where the non-repeatable disturbances locate.
>
> ξ: ξ corresponds to the performance weight placed on the rejection of the non-repeatable disturbances. ξ is less than 1. The smaller the ξ, the heavier weight we place on the rejection of non-repeatable disturbances.
>
> K: K specifies the performance weight over the low frequency. If a small steady state error at low frequency is desired, a high gain of K is used.
>
> ϖ_L: ϖ_L specifies the low frequency below which small steady state error is desired.

The frequency response of the weighting function $W_p(j\varpi)$ is shown in figure 5.

4. SIMULATION RESULTS

The simulation is conducted in Simulink of Matlab. To simulate the real disk drive servo system as accurate as possible, a sampled-data control system is constructed. The block diagram of the sampled-data control system is shown in figure 6, where P(s) is the transfer function of the disk drive servo loop in continuous time domain. The $C_{TPRRC}(z)$ is the discrete time controller of TPRRC, designed in discrete time domain. A zero-order hold is used to connect discrete and continuous signals. The disturbance d used in the simulation is a measured and normalized track-following position error signal (PES) of a disk drive running at 5400 rpm. The PES in time domain is shown in figure 7. The

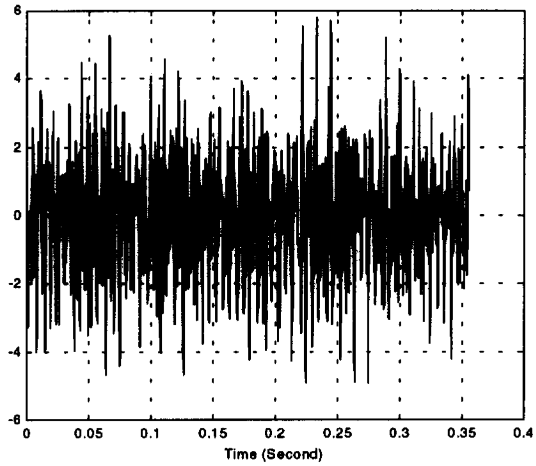

Figure 7. Time Trace of PES (rms=1.7808)

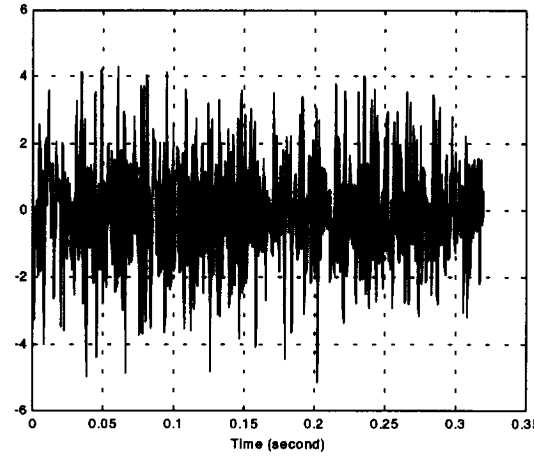

Figure 9. The Time Trace of Output of ZPETC (rms=1.6083)

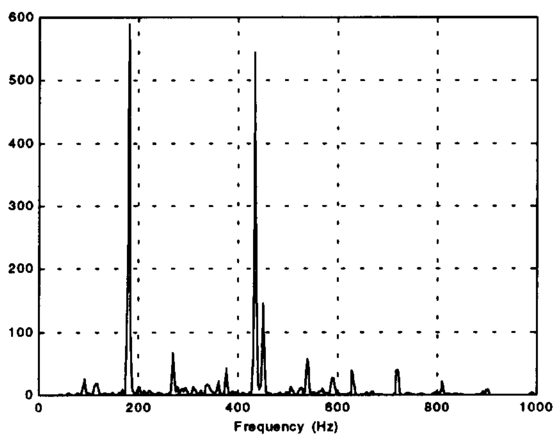

Figure 8. The Power Spectrum of PES

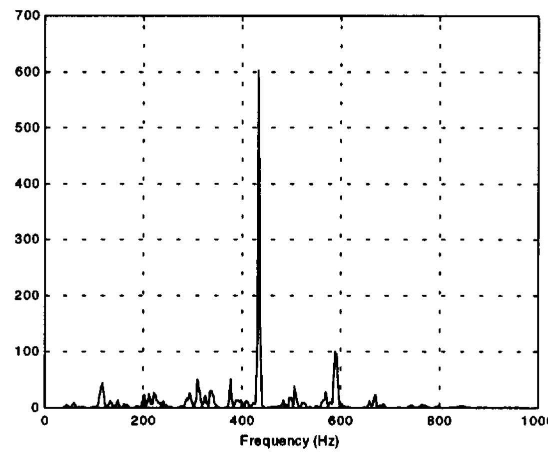

Figure 10. The Power Spectrum of Output of ZPETC

RMS of PES is 1.78. To understand the characteristics of the PES, analysis in frequency domain is preferred. The power spectrum of the PES is shown in figure 8. It is obvious that there is a strong non-repeatable runout around 420 Hz, which is not a harmonic of fundamental frequency 90 Hz.

For easy comparison, a repetitive controller based on ZPETC is designed for the same system. Figure 9 shows the track-following PES in time domain. We can see that the RMS of PES decreases a little bit from original 1.78 to 1.61. The power spectrum of the PES of ZPETC is shown in figure 10. It is clear that the repeatable disturbance and its harmonics are rejected. However, the non-repeatable disturbance at 420 Hz and other non-harmonics are
amplified, which explains why the RMS of PES is only decreased by 10 percent.

A TPRRC is designed for the disk drive servo system to reject both repeatable and non-repeatable disturbances. The performance weighting $W_p(j\varpi)$ of figure 5 is used and the frequency response of the sensitivity function (i.e. the transfer function from disturbance d to system output y) is shown in figure 11. The sensitivity function keeps the shape of repetitive control at the fundamental frequency 90 Hz and its harmonics and is also shaped to attenuate the non-harmonics around 420 Hz. From the frequency response of sensitivity function, we can see that the non-repeatable disturbances in the frequency range from 300 Hz to 500 Hz will be attenuated. The PES in time domain is shown in figure 12. The RMS of PES is reduced from original 1.78 to 0.655, which is a 63 percent improvement of RMS. The power spectrum of the output is shown in figure 13. It is obvious that TPRRC reject not only the repeatable disturbance of 90 Hz an its harmonics, but also the non harmonics. In this application, the non-repeatable disturbances around 420 Hz are attenuated dramatically.

5. CONCLUSIONS

The two-parameter robust repetitive control (TPRRC) is successfully applied to disk drive servo control, such that both repeatable runout (RRO) and non-repeatable runout (NRRO) can be rejected simultaneously. The systematic design procedure of TPRRC and a simple structure of performance weighting function $W_p(j\varpi)$ are presented. The relationship between the coefficients of $W_p(j\varpi)$ and the performance specification over the non-repeatable disturbances is discussed. The simulation results using the measured position error signal (PES) of a disk drive are presented to show the effectiveness of the disturbance rejection in disk drive

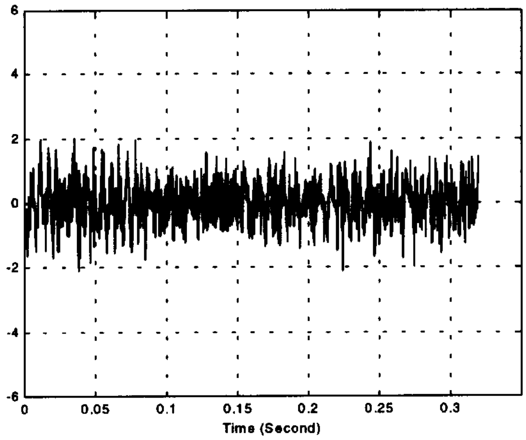

Figure 12. Time trace of output (rms=0.6554)

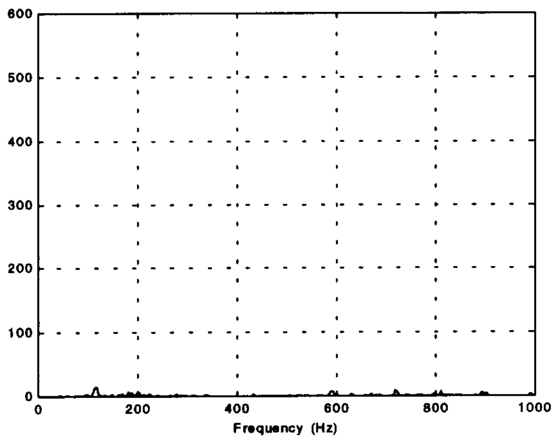

Figure 13. The Power Spectrum of PES of TPRRC

servo control and demonstrate that it is a cost-effective way to improve the track density of disk drives.

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