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REPETITIVE CONTROL FOR LINEAR TIME VARYING SYSTEMS

Zongxuan Sun

Research and Development Center
General Motors Corporation
Warren, MI 48090

Tsu-Chin Tsao

Department of Mechanical and Aerospace Engineering
University of California, Los Angeles
Los Angeles, CA 90095

ABSTRACT

Repetitive control that asymptotically tracks or rejects periodic signals has been widely used in many applications. For linear time invariant system, this problem has been thoroughly studied and solved. This paper presents the analysis and synthesis of repetitive control algorithms to track or reject periodic signals for linear time varying systems. Both continuous and discrete time domain results will be presented. A time varying internal model is embedded in the feedback loop to ensure asymptotic performance. It is shown that asymptotic performance can't be achieved with a finite dimensional controller in the continuous time domain, while it is possible in the discrete time domain. Simulation results demonstrate the effectiveness of the proposed algorithms.

1. INTRODUCTION

Repetitive control that asymptotically tracks or rejects periodic signals has been widely used in many applications. To name a few, Non-Circular Turning Process (NCTP) for piston and camshaft machining, computer hard disk drive track following, optical turning, steel casting etc.. Early work on repetitive control was initiated by Inoue et. al. (1981). Hara et. al. (1988) presented the stability analysis of the infinite dimensional repetitive control system. Tomizuka et. al. (1989) presented the analysis and synthesis of the discrete time repetitive controllers. Zero Phase Error Tracking algorithm (Tomizuka, 1987) was used in the repetitive control synthesis. Tsao and Tomizuka (1994) presented a robust repetitive control algorithm by using Q filter, which will turn off the leaning scheme at high frequencies where unmodeled dynamics present. This is the tradeoff between tracking performance and system robustness.

To achieve asymptotic tracking performance for control plants with unknown parameters, Tsao and Tomizuka (1987) presented an adaptive feedforward zero phase error tracking algorithm. Sun and Tsao (2000) presented the adaptive repetitive control algorithm and its application to an Electro-hydraulic Actuator.

Omata et. al. (1985) presented the repetitive control for linear periodic system. Hanson and Tsao (1996) addressed the discrete time repetitive control for LTI system sampled at a periodic rate. Less conservative criteria have been achieved comparing with Omata's results. Tsakalis and Ioannou (1993) discussed the internal mode principle based tracking control design for linear time varying systems.

Inspired by the repetitive control structure, Sun and Tsao (1999, 2001) presented the nonlinear internal model principle control and predictive internal model control for linear systems with nonlinear disturbance dynamics especially chaotic disturbances in the discrete and continuous time domain respectively. Further more Sun and Tsao (2002) presented the nonlinear internal model principle control for nonlinear systems with nonlinear disturbance dynamics.

This paper presents the analysis and synthesis of repetitive control algorithms to track or reject periodic signals for linear time varying systems. Both continuous and discrete time domain results will be presented. Necessary conditions to achieve asymptotic performance are first derived based on the proposed control structure. Inspired by the unique structure of the necessary conditions, a set of sufficient conditions is then proposed. The controller contains a time varying internal model in the feedback loop to ensure asymptotic performance. Similar to the LTI repetitive control design, it is shown that asymptotic performance can't be achieved with a finite dimensional controller in the continuous time domain, while it is possible in the discrete time domain. Analytical results on the achievable system performance with finite dimensional controllers in the continuous time domain are also presented. Simulations have been conducted to demonstrate the effectiveness of the proposed algorithms.

The rest of this paper is organized as follows. Section 2 describes the plant and disturbance models; Section 3 presents the feedback control design; Section 4 shows the simulation results and Section 5 is the conclusion.

2. PROBLEM DESCRIPTION

Before proceeding to the problem description, we would like to introduce the following definitions (Tsakalis and Ioannou, 1993) on polynomial differential operator (PDO) and polynomial integral operator (PIO) that will be used to represent the system model and controllers.

Definition 1: Let σ represent the differential operator $\frac{d}{dt}(\cdot)$. The left polynomial differential operator (PDO) $P(\sigma,t)$ is defined as:

$$P(\sigma,t) = a_n(t)\sigma^n + a_{n-1}(t)\sigma^{n-1} + \cdots + a_0(t)$$

Similarly, the right PDO is defined as:

$$P(\sigma,t) = \sigma^n a_n(t) + \sigma^{n-1} a_{n-1}(t) + \cdots + a_0(t)$$

Definition 2: A left (right) polynomial integral operator (PIO) $P^{-1}(\sigma,t)$ is defined as the operator that maps the input $u(t)$ to the zero state response of the differential equation $P(\sigma,t)[y] = u$ where $P(\sigma,t)$ is the left (right) monic polynomial differential operator (PDO). More specifically,

$$P^{-1}(\sigma,t)[u](t) = C(t)\int_{t_0}^{t} \Phi(t,\tau)B(\tau)u(\tau)d\tau$$

where $\Phi(t,\tau)$, $B(t)$, $C(t)$ are the state transition matrix, the input and output matrix, respectively, corresponding to the observer (controller) realization of the differential equation.

Consider the following single input single output linear time varying (LTV) plant:

$$y = A_p^{-1}(\sigma,t)B_p(\sigma,t)[u] + d \tag{1}$$

where $u(t)$ and $y(t)$ are the input and output signals respectively. $d(t)$ is the disturbance and it satisfies the following linear time invariant (LTI) dynamic model:

$$\Lambda(\sigma)[d] = 0 \tag{2}$$

We start with the general form of disturbance dynamics as shown above and later we will apply it to the periodic signals. The following assumptions are made on the plant and the disturbance:

Assumption 1: There exists a unique control signal $u^* \in L_{pe}$ such that $A_p^{-1}(\sigma,t)B(\sigma,t)[u^*] = -d$.

Assumption 2: The disturbance dynamic model $\Lambda(\sigma)$ doesn't have zeros in the right half plane.

Assumption 3: The disturbance is unmeasurable, but bounded and smooth. To be specific, $d(t)$ and its derivatives up to the r th order ($r \geq 1$) are bounded.

The control laws we are considering are output feedback. Detailed control structures will be presented in the following section. The control objective is to achieve $\lim_{t\to\infty} y(t) = 0$ for any initial conditions of the plant and disturbance.

3. OUTPUT FEEDBACK CONTROL DESIGN

As shown in Figure 1, the output feedback control law is as follows:

$$\begin{aligned} u &= P(\sigma,t)Q^{-1}(\sigma)[u_1 - u_2] \\ u_1 &= N(\sigma,t)M^{-1}(\sigma,t)[-y] \\ u_2 &= G^{-1}(\sigma,t)F(\sigma,t)[u] \end{aligned} \tag{3}$$

The motivation behind control structure (3) is that we need a self-excitation mechanism in the feedback loop so that it will drive the system to cancel out the persistent but bounded disturbance once the output becomes zero.

It is well known from the internal model principle that the disturbance model need to be included in the feedback loop to ensure asymptotic performance. To insert the internal model into the feedback structure as shown in Figure 1, we have the following condition:

$$d = -G^{-1}(\sigma,t)F(\sigma,t)P(\sigma,t)Q^{-1}(\sigma)[d] \tag{4}$$

As we know, if the plant model (1) is linear time invariant (LTI), condition (4) is both necessary and sufficient to achieve asymptotic performance provided that the closed loop system is asymptotically stable. Obviously this claim is not true anymore for the LTV systems, it requires more conditions to achieve asymptotic performance.

Our approach to solve the above problem is to analyze the closed loop system first, find out what is the extra necessary condition needed to achieve asymptotic performance, and then synthesize the controller based on solutions of the necessary conditions.

3.1 Necessary Conditions For Asymptotic Performance

In this section, we derive the necessary conditions to achieve asymptotic performance for plant (1) with control laws (3) and (4). For notation convenience, we will drop the notions σ and t for the PDO's and PIO's used in the following derivation.

Theorem 1: Consider the linear time varying plant (1) and the control laws (3) and (4), it is necessary to satisfy the following condition to achieve asymptotic performance:

$$A_p^{-1}B_pPQ^{-1}G^{-1}F[u^*] = G^{-1}FPQ^{-1}A_p^{-1}B_p[u^*] \tag{5}$$

Proof: As shown in Figure 2, assume asymptotic disturbance rejection has been achieved, i.e. $y(t) \equiv 0$, along with assumption 1, we have:

$$A_p^{-1}B_p[u^*] = -d \tag{6}$$

So $d = A_p^{-1}B_pPQ^{-1}G^{-1}F[u^*]$ (7)

Combining (4) and (6), we have

$$d = G^{-1}FPQ^{-1}A_p^{-1}B_p[u^*] \tag{8}$$

Comparing (7) and (8), we conclude:

$$A_p^{-1}B_pPQ^{-1}G^{-1}F[u^*] = G^{-1}FPQ^{-1}A_p^{-1}B_p[u^*].$$

3.2 Sufficient Conditions for Asymptotic Performance

Theorem 1 shows that the *extra* necessary condition required for the LTV system is condition (5). A closer look at condition (5) reveals an interesting fact: this condition is nothing but swapping between the operators. Obviously this will always be true for linear time invariant operators, but not for the general linear time varying operators.

Inspired by the unique structure of condition (5), we choose

$$G(\sigma,t) = A_p(\sigma,t) \text{ and } F(\sigma,t) = B_p(\sigma,t) \tag{9}$$

It is easy to verify then condition (5) is satisfied. And now the control law (3) becomes:

$$u = P(\sigma,t)Q^{-1}(\sigma)[u_1 - u_2] \tag{10}$$
$$u_1 = N(\sigma,t)M^{-1}(\sigma,t)[-y]$$
$$u_2 = A_p^{-1}(\sigma,t)B_p(\sigma,t)[u]$$

The condition (4) becomes:

$$d = -A_p^{-1}(\sigma,t)B_p(\sigma,t)P(\sigma,t)Q^{-1}(\sigma)[d]$$

i.e. $\{1 + A_p^{-1}(\sigma,t)B_p(\sigma,t)P(\sigma,t)Q^{-1}(\sigma)\}[d] = 0$

$$A_p^{-1}(\sigma,t)\{A_p(\sigma,t)Q(\sigma) + B_p(\sigma,t)P(\sigma,t)\}Q^{-1}(\sigma)[d] = 0$$

If we can design $P(\sigma,t)Q^{-1}(\sigma)$ such that

$$A_p(\sigma,t)Q(\sigma) + B_p(\sigma,t)P(\sigma,t) = X(\sigma,t)\Lambda(\sigma)$$

for some $X(\sigma,t)$ with bounded coefficients, the above equation becomes:

$$A_p^{-1}(\sigma,t)X(\sigma,t)\Lambda(\sigma)Q^{-1}(\sigma)[d]$$
$$= A_p^{-1}(\sigma,t)X(\sigma,t)Q^{-1}(\sigma)\Lambda(\sigma)[d] = 0$$

Based on the disturbance model (2), the above condition is obviously true.

With this, we propose the following theorem as sufficient conditions to achieve asymptotic performance.

Theorem 2: Consider the plant and disturbance models (1) and (2), together with the control law (10), the following conditions are sufficient to achieve asymptotic disturbance rejection:

$$A_p(\sigma,t)Q(\sigma) + B_p(\sigma,t)P(\sigma,t) = X(\sigma,t)\Lambda(\sigma) \tag{11}$$

$$X(\sigma,t)\Lambda(\sigma)\tilde{M}(\sigma,t) + B_p(\sigma,t)P(\sigma,t)\tilde{N}(\sigma,t) = A_*(\sigma,t) \tag{12}$$

is asymptotically stable.

where $\tilde{M} = Q^{-1}M$ and $\tilde{N} = Q^{-1}N$.

Proof: As shown in Figure 3, the output of the system is:

$$y = A_p^{-1}B_p[u] + d$$

From (10), we have

$$(1 + PQ^{-1}A_p^{-1}B_p)[u] = PQ^{-1}[u_1]$$

So $y = d - A_p^{-1}B_p(1 + PQ^{-1}A_p^{-1}B_p)^{-1}PQ^{-1}NM^{-1}[y]$

$$y = [1 + (B_p^{-1}A_p + PQ^{-1})^{-1}PQ^{-1}NM^{-1}]^{-1}[d]$$

$$y = M[M + Q(A_p Q + B_p P)^{-1} B_p P\tilde{N}]^{-1}[d]$$

By condition (11), we get

$$y = M[M + Q(X\Lambda)^{-1} B_p P\tilde{N}]^{-1}[d]$$

$$y = M[X\Lambda Q^{-1}M + B_p P\tilde{N}]^{-1} X\Lambda Q^{-1}[d]$$

$$y = M[X\Lambda \tilde{M} + B_p P\tilde{N}]^{-1} XQ^{-1}\Lambda[d]$$

Since $\Lambda[d] = 0$, together with condition (12) we have $\lim_{t\to\infty} y(t) = 0$.

Remark: Condition (11) is the time varying internal model, while condition (12) specifies the stabilizing controller.

Remark: Another important observation is that a special case of the proposed control design is Internal Model Control (IMC) by letting $N(\sigma,t) = -1$ and $M(\sigma,t) = 1$.

Remark: Tsypkin and Holmberg (1995) presented the internal model principle and internal model control together (IMPACT) for the discrete time linear time invariant (LTI) system. The object was to provide a simplified design method for disturbance rejection under the internal model control structure. Two LTI polynomial equations need to be solved, although they are not necessary conditions to achieve asymptotic performance. Tsakalis and Ioannou (1993) presented a control design for linear time varying systems to track a class of measurable signals. Two LTV polynomial equations need to be solved. Conditions (11) and (12) are similar to the LTV polynomial equations derived by Tsakalis and Ioannou (1993). However they are designed to asymptotically reject unmeasurable disturbances and as we have shown they are the only obvious solutions of the necessary condition (5). So although we call them sufficient conditions, they are actually close to necessary conditions.

Using similar approaches, we can extend theorem 2 to the case of linear time varying plant with linear time varying disturbance dynamics. Then the plant and disturbance models (1) and (2) become:

$$y = A_p^{-1}(\sigma,t)B_p(\sigma,t)[u] + d \qquad (13)$$

$$\Lambda(\sigma,t)[d] = 0 \qquad (14)$$

The control law (10) becomes:

$$u = P(\sigma,t)Q^{-1}(\sigma,t)[u_1 - u_2] \qquad (15)$$

$$u_1 = N(\sigma,t)M^{-1}(\sigma,t)[-y]$$

$$u_2 = A_p^{-1}(\sigma,t)B_p(\sigma,t)[u]$$

Theorem 3: Consider plant and disturbance models (13) and (14), together with the control law (15), the following conditions are sufficient to achieve asymptotic disturbance rejection:

$$A_p(\sigma,t)Q(\sigma,t) + B_p(\sigma,t)P(\sigma,t) = X(\sigma,t)\Lambda(\sigma,t)Q(\sigma,t) \qquad (16)$$

$$X(\sigma,t)\Lambda(\sigma,t)M(\sigma,t) + B_p(\sigma,t)P(\sigma,t)\tilde{N}(\sigma,t) = A_*(\sigma,t) \qquad (17)$$

is asymptotically stable.

where $\tilde{N} = Q^{-1}N$.

Proof: As shown in Figure 3, following similar derivations in theorem 2, we have:

$$y = M[M + Q(A_p Q + B_p P)^{-1} B_p P\tilde{N}]^{-1}[d]$$

By condition (16), we get

$$y = M[M + (X\Lambda)^{-1} B_p P\tilde{N}]^{-1}[d]$$

$$y = M[X\Lambda M + B_p P\tilde{N}]^{-1} X\Lambda[d]$$

Since $\Lambda[d] = 0$, together with condition (17) we have $\lim_{t\to\infty} y(t) = 0$.

Remark: The main constraint of condition (16) is that the order of the disturbance model $\Lambda(\sigma,t)$ can't be higher than the plant order, i.e. the order of $A_p(\sigma,t)$.

3.3 Repetitive Control Design

In this section, we will apply theorem 2 to both continuous and discrete time domain repetitive control designs. The objective of repetitive control is to track or reject unmeasurable periodic signals, where the period is known, but the amplitude and phase are unknown. The Laplace domain representation of the periodic disturbance model is:

$$\Lambda(s) = 1 - e^{-sT} \qquad (18)$$

where T is the period.

Since the above disturbance model is infinite dimensional, there is no finite dimensional controller PQ^{-1} which can solve condition (11) associated with the disturbance model (18).

Define

$\Lambda_0(\sigma) = \sigma$, $\Lambda_1(\sigma) = \sigma(\sigma \pm jw)$, and

$$\Lambda_k(\sigma) = \sigma(\sigma \pm jw)\cdots(\sigma \pm jkw) \qquad (19)$$

where $w = \frac{2\pi}{T}$

Substitute (19) into (11), we get

$$A_p(\sigma,t)Q(\sigma) + B_p(\sigma,t)P(\sigma,t) = X(\sigma,t)\Lambda_k(\sigma) \quad (20)$$

Theorem 4: Consider the plant model (1), disturbance model (18) and the control law (10), if PQ^{-1} and NM^{-1} satisfy the following conditions:

$$A_p(\sigma,t)Q(\sigma) + B_p(t)P(\sigma,t) = X(\sigma,t)\Lambda_k(\sigma) \quad (21)$$

$$X(\sigma,t)\Lambda_k(\sigma)\tilde{M}(\sigma,t) + B_p(\sigma,t)P(\sigma,t)\tilde{N}(\sigma,t) = A_k(\sigma,t) \quad (22)$$

is asymptotically stable.

The closed loop system will be asymptotically stable and the steady state output is:

(a) $\| y \|_\infty \le \| MA_k^{-1}XQ^{-1} \|_1 \cdot \| \Lambda_k \|_1 \sum_{|n|>k} | c_n |$ (23)

where $c_n = \frac{1}{T}\int_0^T d(t)e^{-jnwt}\, dt$.

(b) $y(t) \to 0$, as $k \to \infty$

Proof:

(a) By (22), we know the closed loop system is asymptotically stable. Similarly to theorem 2, we can get:

$$y = M[X\Lambda_k\tilde{M} + B_pP\tilde{N}]^{-1}XQ^{-1}\Lambda_k[d]$$

So $\| y \|_\infty \le \| M[X\Lambda_k\tilde{M} + B_pP\tilde{N}]^{-1}XQ^{-1} \|_1 \cdot \| \Lambda_k[d] \|_\infty$

$$\| \Lambda_k[d] \|_\infty = \| \Lambda_k[\sum_n c_n e^{-jnwt}] \|_\infty$$

$$= \| \Lambda_k[\sum_{|n|>k} c_n e^{-jnwt}] \|_\infty \le \| \Lambda_k \|_1 \sum_{|n|>k} |c_n|$$

where $c_n = \frac{1}{T}\int_0^T d(t)e^{-jnwt}\, dt$.

By assumption 2, $d(t)$ and its derivatives up to the r th order are bounded. Then

$$c_n \sim O(\frac{1}{n^{r+1}}),\ r \ge 1 .$$

Obviously, $\sum_{|n|>k} |c_n|$ is bounded, and thus well defined.

Then we conclude $\| y \|_\infty \le \| MA_k^{-1}XQ^{-1} \|_1 \cdot \| \Lambda_k \|_1 \sum_{|n|>k} | c_n |$, which proves condition (23).

(b) Since A_k^{-1} and Q^{-1} are strictly stable, both $\| MA_k^{-1}XQ^{-1} \|_1$ and $\| \Lambda_k \|_1$ are bounded. From (a), we have:

$\| y \|_\infty \le \| MA_k^{-1}XQ^{-1} \|_1 \cdot \| \Lambda_k \|_1 \sum_{|n|>k} | c_n | \le C \sum_{|n|>k} | c_n |$, where C is a bounded positive constant.

Since $c_n \sim O(\frac{1}{n^{r+1}})$, $r \ge 1$, $\sum_{n=-\infty}^{\infty} | c_n |$ is bounded and $\lim_{k\to\infty} \sum_{|n|>k} | c_n | = 0$.

So we have $\lim_{k\to\infty} \| y \|_\infty \le C \lim_{k\to\infty} \sum_{|n|>k} | c_n | = 0$

We then conclude $y(t) \to 0$, as $k \to \infty$.

Remark: Obviously, c_n is the coefficient of the Fourier series of the periodic disturbance. To minimize the right hand side of inequality (23), one can always choose $\tilde{\Lambda}_k(\sigma)$ such that $\sum | c_n |$ is minimized for the remaining harmonics.

Now let's apply the above results to the discrete time domain repetitive control design. In this case, we can replace the differential operator σ with the time advance operator q. The periodic disturbance model becomes:

$$\Lambda(q) = 1 - q^{-N} \quad (24)$$

where N is the period.

Obviously we can solve the problem with a finite dimensional controller since the disturbance model (24) is only finite dimensional.

Theorem 5: Consider the discrete form of plant (1) and the control law (10), together with the disturbance model (24), the following conditions are sufficient to achieve asymptotic disturbance rejection:

$$A_p(q,k)Q(q) + B_p(q,k)P(q,k) = X(q,k)\Lambda(q) \quad (25)$$

$$X(q,k)\Lambda(q)\tilde{M}(q,k) + B_p(q,k)P(q,k)\tilde{N}(q,k) = A_*(q,k) \quad (26)$$

is asymptotically stable.

Proof: Similar to theorem 2. Omitted.

4. SIMULATION RESULTS

Consider the following continuous time single input single output linear time varying system:

$$y = A_p^{-1}(\sigma,t)B_p(\sigma,t)[u] + d \tag{27}$$

where $A_p(\sigma,t) = \sigma + a(t)$, $a(t) > 0$ and $B_p(\sigma,t) = b$, $b \neq 0$.

The unmeasurable disturbance has the following form:

$$d(t) = A\sin(\omega t + \alpha) \tag{28}$$

where ω is known, A and α are unknown.

Based on (28), we choose $\Lambda(\sigma)$ as:

$$\Lambda(\sigma) = (\sigma + j\omega)(\sigma - j\omega) = \sigma^2 + \omega^2$$

Now we are ready to design the feedback controllers described in (10) to satisfy the conditions (21) and (22). They are designed in the following two steps:

Step 1: Design $P(\sigma,t)Q(\sigma)^{-1}$ to satisfy condition (21).

Choose $Q(\sigma) = \sigma + 1$, $P(\sigma,t) = \sigma p_1(t) + p_0(t)$, $X(\sigma,t) = 1$ and $\omega = 1$, plug them into equation (21):

$$(\sigma + a(t))(\sigma + 1) + b(\sigma p_1(t) + p_0(t)) = \sigma^2 + 1 \tag{29}$$

Solve the above equation, we get:

$$p_0(t) = \frac{1 - a(t) + \dot{a}(t)}{b} \tag{30}$$

$$p_1(t) = \frac{-1 - a(t)}{b} \tag{31}$$

Obviously, $P(\sigma,t)Q(\sigma)^{-1}$ is a strictly stable controller.

Step 2: Design $\tilde{N}(\sigma,t)\tilde{M}(\sigma,t)^{-1}$ to stabilize the closed loop system according to condition (22).

Since $X(\sigma,t)\Lambda(\sigma)$ is second order, we choose $\tilde{N}\tilde{M}^{-1}$ as the following form:

$$\tilde{N} = \sigma n_1(t) + n_2(t)$$

$$\tilde{M} = \sigma + n_3(t)$$

Then the closed loop polynomial differential operator becomes:

$$X\Lambda\tilde{M} + B_p P\tilde{N} \tag{32}$$

$$= (\sigma^2 + 1)(\sigma + n_3(t)) + b(\sigma p_1(t) + p_0(t))(\sigma n_1(t) + n_2(t)) = A_*(\sigma,t)$$

Let $A_*(\sigma,t) = (\sigma + 1)^3$ and solve the above equation, we get:

$$\begin{bmatrix} -1 - a(t) & 0 & 1 \\ 1 - a(t) + 2\dot{a}(t) & -1 - a(t) & 0 \\ \dot{a}(t) - \ddot{a}(t) & 1 - a(t) + \dot{a}(t) & 1 \end{bmatrix} \begin{bmatrix} n_1(t) \\ n_2(t) \\ n_3(t) \end{bmatrix} = \begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix} \tag{33}$$

Choose $a(t) = 1 + 0.5\sin t$ and $b = 1$, then (30), (31) and (33) become:

$$p_0(t) = 0.5\cos t - 0.5\sin t \tag{34}$$

$$p_1(t) = -2 - 0.5\sin t \tag{35}$$

$$\begin{bmatrix} n_1(t) \\ n_2(t) \\ n_3(t) \end{bmatrix} = \begin{bmatrix} \dfrac{-16 - 8\sin t + 4\cos t}{18 + 12\sin t + 4\cos t - 2\sin t\cos t + \sin^2 t} \\ \dfrac{-16 - 4\sin t - 12\cos t}{18 + 12\sin t + 4\cos t - 2\sin t\cos t + \sin^2 t} \\ \dfrac{22 + 12\sin t + 2\cos t - 4\sin t\cos t - \sin^2 t}{18 + 12\sin t + 4\cos t - 2\sin t\cos t + \sin^2 t} \end{bmatrix} \tag{36}$$

It is easy to verify that

$18 + 12\sin t + 4\cos t - 2\sin t\cos t + \sin^2 t > 0$ for all t .

So (36) is well defined. Also we can verify that

$n_3(t) = \dfrac{22 + 12\sin t + 2\cos t - 4\sin t\cos t - \sin^2 t}{18 + 12\sin t + 4\cos t - 2\sin t\cos t + \sin^2 t} > 0$ for all t .

So $\tilde{N}\tilde{M}^{-1}$ is a strictly stable controller.

Now we have obtained all the parameters for the linear time varying controllers. The closed loop system used for the simulation is shown in Figure 3. During the simulation, we choose $A = 10$ and $\alpha = \frac{\pi}{6}$ for the disturbance model (28) and Figure 4 shows the periodic disturbance. Figure 5 shows the plant output and control signal. As predicted by theorem 2 and 4, asymptotic performance has been achieved. Figure 6 and 7 show the time varying control parameters for $P(\sigma,t)$, $\tilde{N}(\sigma,t)$ and $\tilde{M}(\sigma,t)$ respectively.

5. CONCLUSIONS

This paper presents the analysis and synthesis of repetitive control algorithms to asymptotically track or reject periodic signals for linear time varying systems. Both continuous and discrete time domain results have been provided. Necessary conditions to achieve asymptotic performance are first derived based on the proposed control structure. Sufficient conditions are then proposed in the form of two Diophantine equations. It is shown that asymptotic tracking can't be achieved with a finite dimensional controller in the continuous time domain, while it is possible in the discrete time domain.

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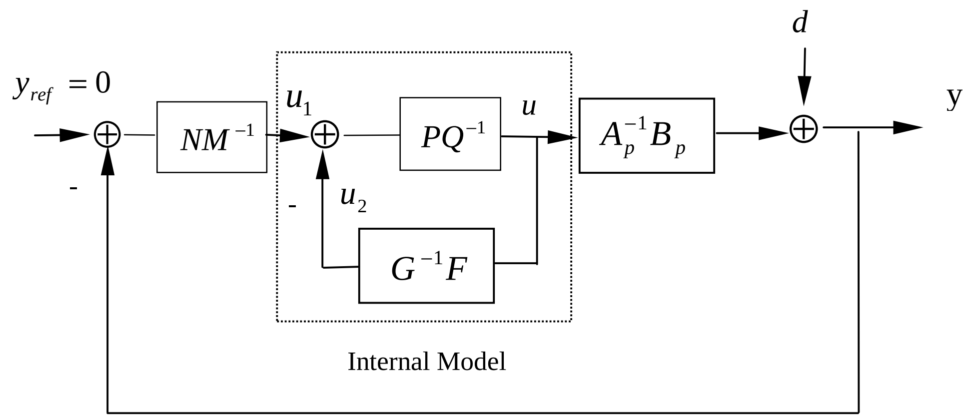

Figure 1. Block Diagram for the Output Feedback Control

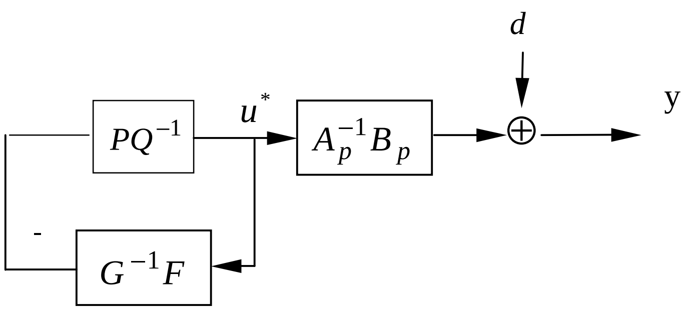

Figure 2. Block Diagram for Asymptotic Disturbance Rejection

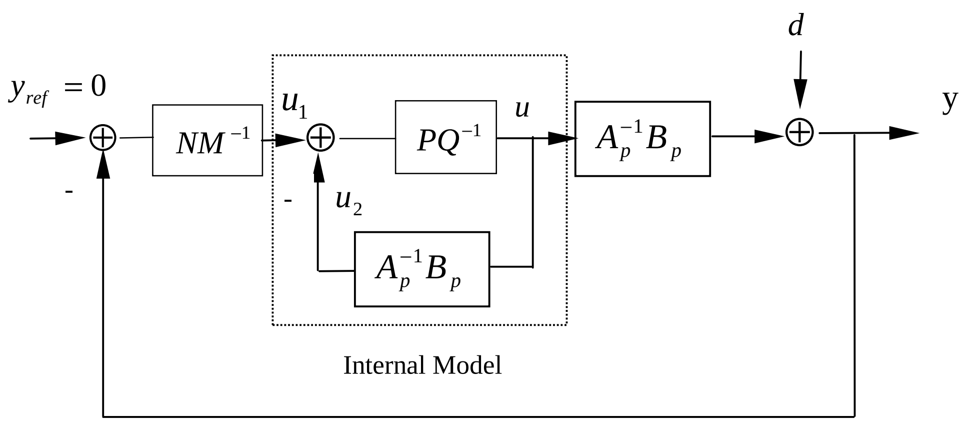

Figure 3. Repetitive Control Block Diagram

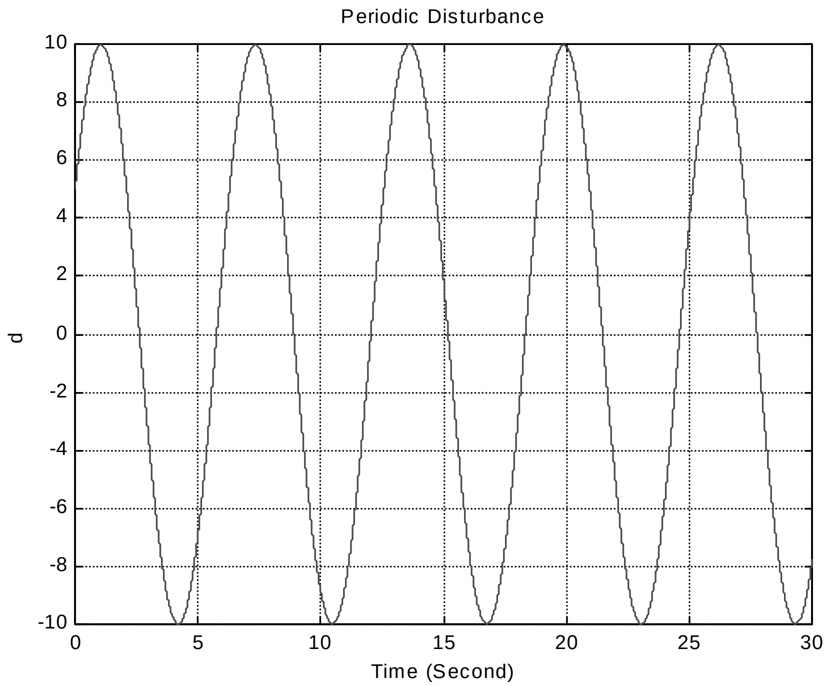

Figure 4. The periodic disturbance

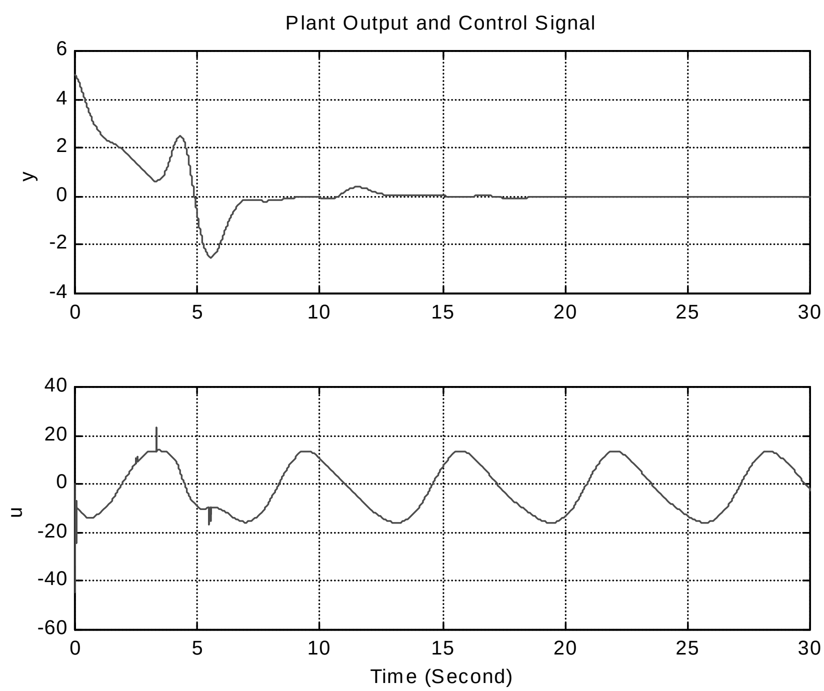

Figure 5. Plant output and control signal

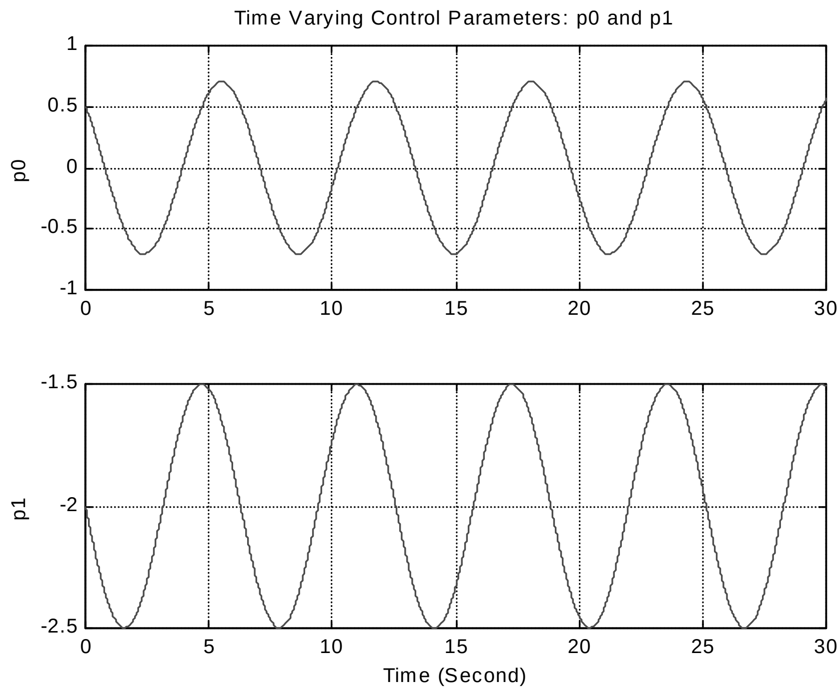

Figure 6. Time varying control parameters: p0 and p1

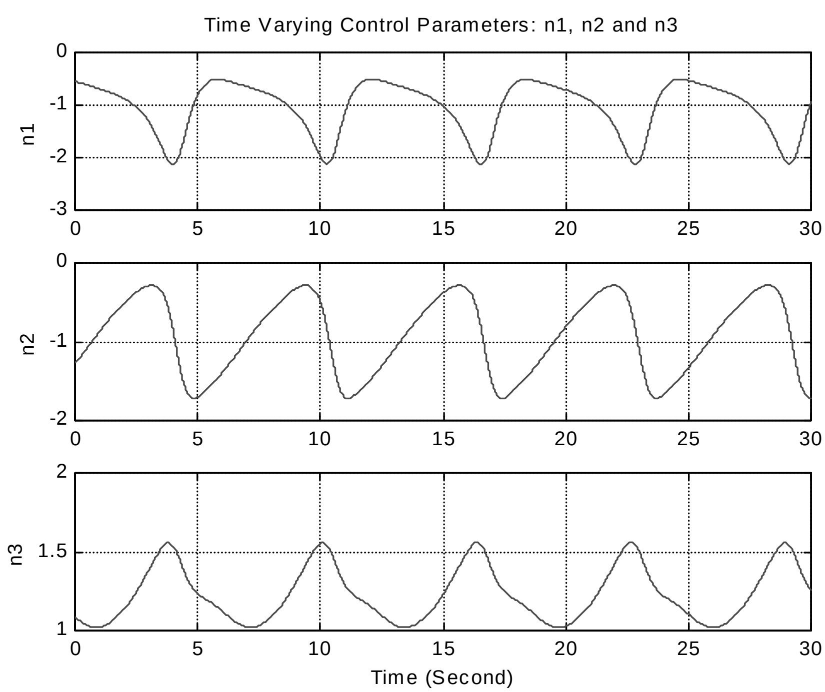

Figure 7. Time varying control parameters: n1, n2 and n3