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Repetitive Control for Asymptotic Tracking of Periodic Signals With an Unknown Period

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Repetitive control schemes for asymptotic tracking and disturbance rejection of periodic signals with an unknown period are presented. A sampled data recursive scheme for identifying the period of a periodic signal with a resolution finer than the sampling interval is presented. Discrete-time self-tuning repetitive controllers, which adapt both the periodic signal period and sampling interval, are proposed based on the period identification scheme. The fine adaptation of the controller sampling interval makes the identified signal period an exact integer multiples of the controller sampling interval and renders a superior tracking performance than that of the conventional fixed sampling interval repetitive controllers. Experimental results on a linear motion system are presented to demonstrate the effectiveness of the proposed control schemes. [S0022-0434(00)01402-7]

1 Problem Statement

Repetitive control, which is based on the internal model principle, has proven to be a useful technique for asymptotic tracking and rejection of exogenous periodic signals (Hara et al. [1], Tomizuka et al. [2]). It has been successfully applied to areas such as non-circular turning (Tsao and Tomizuka [3]), mechanical manipulators (Omata et al. [4], Tsai et al. [5]), computer disk drives (Chew and Tomizuka [6]), magnetic bearings (Higuchi et al. [7]), spindle runout compensation (Tsao et al. [8], Tsao, and Pong [9]), spindle speed regulation under periodic disturbances (Kobayashi et al. [10], Tsao and Pong [11]), and etc. In some situations, the period of the repetitive signal is uncertain or slowly changing. For example, the periodic disturbances caused by inevitable dynamic imbalance in rotational machinery may have uncertain period due to variations of machine rotational speed. If the angular position of the rotating axis is available, the repetitive controller may be designed and implemented by using the angular position as the "time" variable so that the disturbance period is always exactly one rotation regardless of speed variations (Tsao and Pong [11]). However, the angular position measurement may not be accessible or cost effective. Periodic motion generation is another example found in numerous manufacturing processes. The motion trajectory period may vary according to the changes of the trajectory's traversal speed, which is usually controlled by a higher level supervisory control or an upper stream process operation. Such situations require a repetitive controller which can self-tune the repetitive signal generator period to match the external signal period closely.

The performance of the repetitive control system relies on the matching of the repetitive controller's signal generator period with the exogenous periodical signal period. The repetitive controller generates a comb-filter shaped (in the frequency domain) closed-loop sensitivity function. The notches of the comb-filter provide substantial sensitivity reduction but they are sharply narrow. The sensitivity function (i.e., the error magnitude) at the Fourier harmonic frequencies (i.e., $\omega=2\pi n/T_p$) of the periodic signal is proportional to the following:

$$|1-e^{-j2\pi n\hat{T}_p/T_p}|=|1-e^{-j2\pi n\varepsilon}| \quad \left(\varepsilon=\frac{\hat{T}_p-T_p}{T_p}\right), \qquad (1)$$

where T_p is the signal period and $\hat{T}_p$ is the period assumed by the repetitive controller and ε is the period mismatch ratio. In the case of the exact matching, i.e., $\hat{T}_p=T_p$, the error magnitude is zero. However, a slight mismatch of $\varepsilon=1\%$ increases the error magnitude ratio to about 0.1 at the first harmonic frequency $(n=1)$. The error is even larger for larger n values, i.e., higher Fourier harmonic frequencies.

Self-tuning of the digital repetitive control period has been proposed to adjust the repetitive controller's signal generator period (Tsao and Nemani [12]; Hu [13]) with respect to the exogenous signal. Tsao and Nemani [12] proposed a recursive single parameter period identification scheme. The digital implementation of the scheme estimates the period as integer multiples of the signal sampling interval. Hu [13] suggested the use of an ARX model, whose order is as large as the upper bound of the period, and least squares parameter adaptation algorithm for period estimation. The period is determined by inspecting the coefficients of the identified ARX. Of course, the signal period could also be estimated by inspecting the signal's power spectrum using Discrete Fourier Transform. All these methods, barring the computation issue, have a finite resolution on the estimated period limited by the sampling interval, and hence pose a limitation on the period matching precision. This paper presents a recursive period estimation scheme, which significantly reduces the finite resolution problem mentioned above and is computationally efficient. This estimated period is then used to adjust the discrete-time repetitive controller's integer period length. Furthermore, by a fine adjustment of the controller sampling period, the controller signal generator's period is matched precisely with the identified signal period. Finally, experimental results for the implementation on a motor driven linear slide are presented to demonstrate the effectiveness of the proposed control schemes.

2 Recursive Identification of Signal Period

Consider a continuous-time domain linear time invariant system in input-output model form:

$$A_c(D)e(t)+B_c(D)u(t)=C_c(D)w(t), \qquad (2)$$

where $A_c(D)$, $B_c(D)$, and $C_c(D)$ are the time domain rational transfer functions of the time derivative operator D. Both the control $u(t)$ and the error $e(t)$ are measurable and the disturbance $w(t)$ is a periodic signal that is in general unmeasurable. Tracking control of a signal $y_d(t)$ can be considered as the disturbance rejection problem by defining (Tomizuka et al. [2])

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$$e(t)=yd(t)-y(t),\quad w(t)=yd(t),\quad C_c(D)=A_c(D) \tag{3}$$

Since only the knowledge of the signal period is needed in the repetitive control system, it is not necessary to know the entire exact disturbance signal waveform. Therefore, even if the plant model is not accurately known, one can use the left-hand side of Eq. (2) or the steady-state waveforms of $u(t)$ and $y(t)$ to estimate the period since these signals are periodic with the same period as $w(t)$.

The period identification algorithm is based on the gradient minimization of a quadratic energy function as described below:

Theorem 1: Let $f(t)$ be a periodic function with the basic (smallest) period τ^*, and $f(t)$ is not identically zero.

$$\text{Let } J(\tau)=\frac{1}{2}\int_{t-T_d}^{t}[f(s)-f(s-\tau)]^2 ds,\quad \text{where } T_d>\tau^*. \tag{4}$$

Consider the following gradient adaptation algorithms:

$$\text{Continuous iteration: } \frac{d\tau}{dt}=-\gamma\frac{\partial J(\tau(t))}{\partial\tau},\quad \tau(0)=\tau_0. \tag{5}$$

$$\text{Discrete iteration: } \tau(t+1)=\tau(t)-h\frac{\partial J(\tau(t))}{\partial\tau},\quad \tau(0)=\tau_0. \tag{6}$$

If the initial condition τ_0 is close enough to τ^*, that is, there is $\sigma>0$ and $|\tau_0-\tau^*|<\sigma$, then there exist adaptation gains γ, $h>0$ such that the adaptations in (5) and (6) converges, i.e., $\tau\to\tau^*$.

Proof: First, we note that τ^* is a stationary point of the continuously differentiable function $J(\tau)$ since the first derivative as shown in Eq. (7) is zero at $\tau=n\tau^*$, $n=0,\pm1,\pm2,\ldots$.

Further, these points are local mimima since the second derivatives at these points are positive, as shown in Eq. (8). Therefore, with small enough adaptation gains γ, h, the gradient schemes in (5) and (6) converge to the local mimima depending on the initial condition. The adaptation converges to the basic period τ^* when the initial condition is close enough.

$$\frac{\partial J(\tau(t))}{\partial\tau}=\int_{t-T_d}^{t}[f(s)-f(s-\tau)]\dot{f}(s-\tau)ds \tag{7}$$

$$\frac{\partial^2 J(\tau(t))}{\partial\tau^2}=\int_{t-T_d}^{t}-[f(s)-f(s-\tau)]\ddot{f}(s-\tau)+\dot{f}^2(s-\tau)ds$$

$$\frac{\partial^2 J(n\tau^*)}{\partial\tau^2}=\int_{t-T_d}^{t}\dot{f}^2(s)ds>0. \tag{8}$$

The cost function J is periodic and has local minima at integer multiples of the base period. Any integer multiple, except for zero, of the base period may be used for repetitive control period. Therefore, the initial condition of the period estimation is not critical.

Since every periodic signal has a Fourier series representation, it can be easily verified that for the nth harmonic, the cost function is periodic with the same period. As the harmonic number n increases the convex region gets narrower for each local minimum. Therefore, to ensure large enough convex region, low-pass filtering of the signal entering the estimation algorithm is necessary. Further, the signal time derivative is required in the adaptation. This noncausal differentiation is not a problem here since the differentiation may be combined with the low-pass filter to render causal filtering.

Discrete Implementation of Period Identification. For practical implementation purpose, this algorithm is more conveniently implemented using sampled data. Let the sampling interval of the data be T and the estimated period $\hat{T}=\eta T$. Denote $\lfloor\eta\rfloor$ as the integer part of η. The integration in Eq. (4) is carried over a period of length T_d greater than the upper bound of the signal period $\bar{\tau}=\bar{\eta}T$ and the first derivative in Eq. (7) can be evaluated at the following two integer points:

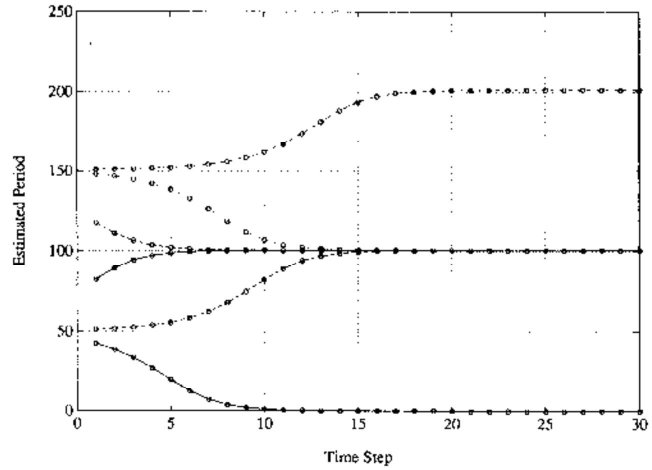

Fig. 1 Simulated results of period estimations with various initial conditions

$$\frac{\partial J(\lfloor\eta\rfloor T)}{\partial\tau}\cong\sum_{i=\bar{\eta}+1}^{L+\bar{\eta}}(f_i-f_{i-\lfloor\eta\rfloor})\frac{f_{i-\lfloor\eta\rfloor+1}-f_{i-\lfloor\eta\rfloor-1}}{2} \tag{9}$$

and

$$\frac{\partial J((\lfloor\eta\rfloor+1)T)}{\partial\tau}\cong\sum_{i=\bar{\eta}+1}^{L+\bar{\eta}}(f_i-f_{i-\lfloor\eta\rfloor-1})\frac{f_{i-\lfloor\eta\rfloor}-f_{i-\lfloor\eta\rfloor-2}}{2} \tag{10}$$

Central difference is used to approximate the signal partial derivatives to avoid biased estimation when random white noise exists in the signal.

The partial derivative of J at the point η as needed in Eq. (6) is obtained by the linear interpolation at the two adjacent integer points obtained in (9) and (10):

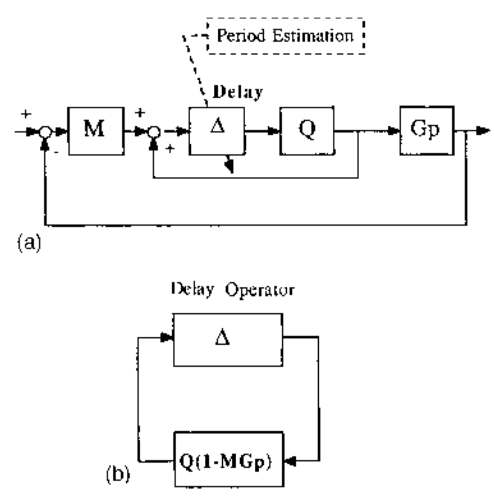

Fig. 2 (Adaptive) repetitive control system, 2(*a*) is the block diagram, 2(*b*) is the equivalent block diagram

$$\frac{\partial J(\eta T)}{\partial \tau} \cong \frac{\partial J(\lfloor \eta \rfloor T)}{\partial \tau}(\lfloor \eta \rfloor + 1 - \eta) + \frac{\partial J((\lfloor \eta \rfloor + 1)T)}{\partial \tau}(\eta - \lfloor \eta \rfloor) \tag{11}$$

The discrete recursive period identification Eq. (7) is then

$$\eta(t+1) = \eta(t) - h' \frac{\partial J(\eta T)}{\partial \tau} \quad \left(h' = \frac{h}{T} \right) \tag{12}$$

Figure 1 shows the numerical simulation results of the period estimation of a pure sine wave of 0.503 second period. The signal was sampled with 0.005 second sampling period. The period estimate η converged to 100.60014, which is very close to the exact value 100.60000. The estimates converged to within (100.599, 100.601) in about 25 iterations. Triangular wave was also used for period estimation. The result had no significant difference from that shown in Fig. 2. When a strong high frequency harmonic was added to the triangular wave, the estimation could not converge, but a low-pass filtering of the signal solved the convergence problem as expected. As illustrated in Fig. 1, the iterations converge to different local minima depending on the initial conditions.

3 Self-Tuning Repetitive Control

The period identification scheme using sampled data may be applied to both the continuous-time and discrete-time repetitive control systems. Herein, we consider the discrete-time repetitive controller (Tomizuka et al. [2]), whose block diagram is shown in Fig. 2(*a*), of the following form:

$$u(t) = h_Q(t) * (u(t-N) + h_M(t) * e(t-N)), \tag{13}$$

where t is the time index, $h_M(t)$ and $h_Q(t)$ are the impulse responses of stable linear filters $M(z^{-1})$ and $Q(z^{-1})$, which are designed to ensure closed loop robust stability. Particularly, if the open loop plant is stable, $M(z^{-1})$ may be designed as the stable inverse (or stable approximate inverse for nonminimum phase system) of the plant and $Q(z^{-1})$ may be a low-pass filter, which scarifies high frequency tracking/disturbance rejection performance for robust stability. It is well established (Hara et al. [1], Tomizuka et al. [2]) that the above repetitive control system is stable for any fixed signal period N provided that the following sufficient condition is met:

$$\|Q(z^{-1})(1 - M(z^{-1})G_p(z^{-1}))\|_\infty < 1. \tag{14}$$

Let the discrete transfer function of the plant in Eq. (2) with a zero-order hold and sampler be $G_p(z^{-1}) = B(z^{-1})/A(z^{-1})$. The prototype discrete-time repetitive controller is (Tsao and Tomizuka [3], Tomizuka et al. [2])

$$M(z^{-1}) = \frac{K_r A(z^{-1}) B^-(z)}{B^+(z^{-1}) b}, \quad 0 < K_r < 2, \quad b \geq \max |B^-(e^{-j\omega})|^2$$

$$Q(z^{-1}) = F(z^{-1})F(z), \quad |F(e^{-j\omega})| \leq 1, \quad 0 \leq \omega < \pi, \tag{15}$$

where $B(z^{-1}) = B^+(z^{-1})B^-(z^{-1})$ and $B^-(z^{-1})$ contains all the unstable plant zeros. For the self-tuning repetitive control, certainty equivalence principle is applied to adjust the repetitive controller period accordingly and frozen-time stability is established by (14) for any estimated N.

The period N must be an integer in the discrete-time repetitive control implementation. The estimated signal period η_t, however, is in general not an integer. Without changing the digital control sampling period, one can at best set the repetitive controller signal period to the nearest integer of the estimated signal period:

$$N_{k+1} = \lfloor \eta_k + 0.5 \rfloor, \tag{16}$$

where k represents the iteration index. With this, the difference between the identified signal period and the integer controller period can be represented as

$$\varepsilon = \left| \frac{N_{k+1} - \eta_k}{\eta_k} \right| \leq \frac{0.5}{\eta_k} \tag{17}$$

Equation (1) can then be used to determine whether the repetitive control performance is acceptable with such period mismatching. To further reduce the difference between the repetitive controller period and the identified signal period, the digital control sampling interval can be fine tuned around the nominal sampling interval T. The integer delay N_{k+1} is updated according to (16) and the sampling time T_{k+1} is updated according to the following equation such that their product equals to the estimated period ($\eta_k T_k$) and that the new sampling time T_{k+1} is minimally deviated from the nominal sampling period T:

$$T_{k+1} = \frac{\eta_k T_k}{N_{k+1}} \tag{18}$$

The deviation of the new sampling interval from the nominal sampling interval T, denoted as δ, is

$$\delta = \left| \frac{T_{k+1} - T}{T} \right| \leq \frac{0.5}{N_{k+1}} \tag{19}$$

Since the change of sampling time δ is typically small enough, for example $\delta = 0.5\%$ for $N_{k+1} = 100$, the closed-loop system stability can be maintained without changing the repetitive controller parameters that are designed based on the nominal sampling interval T. This is because the discrete-time system matrix eigenvalues are continuous functions of the matrix elements, which in turn are continuous functions of the sampling period.

Slow Adaptation Scheme. When the plant is not accurately known and the signal $w(t)$ in Eq. (2) is not available, the steady-state signal $u(t)$ or $y(t)$ can be used for period identification since it has the same period as $w(t)$. To ensure that the signal sampled for computation is close to periodic, the sampling time must be fixed in each cycle of data collection. Furthermore, the controller period and sampling time must be updated infrequent enough so that the closed-loop system can practically reach the steady-state response. Therefore, we assume that the period and sampling time updating rate is much slower than the control system transient response. With this assumption, the identifier gets near periodic signals from the near steady state control system and the slowly time-varying control system stability can be determined by its frozen-time model. Frozen-time stability is thus assured if Eq. (14) is satisfied for the small range of sampling interval variations. This means that the H_∞ norm of the transfer function in (14) should be sufficiently less than one so that Eq. (14) can be satisfied under the variations. We call this slow adaptation scheme.

The slow adaptation scheme can be implemented in the following fashion. Let the period identification and repetitive control have the same sampling time although, in general, they need not be identical. Let each tuning cycle be P steps. The controller period N_k and sampling interval T_k are updated at beginning of each tuning cycle and they remain fixed for the entire cycle. Within cycle k, first P_1 steps are for the signal to settle to steady state, the subsequent P_2 steps are for collecting data for period identification, and the last P_3 steps are for iterative period identification computation.

Fast Adaptation Scheme. In the case that a periodic signal is available at all time disregarding the control system transient responses, the period identification and the adaptive controller may be updated every time step to catch up with the change of signal period more quickly. This is the case when the signal $w(t)$ in Eq. (2) is available for measurement or the plant model is accurate enough so that $w(t)$ can be computed from the input and output signals by using Eq. (2). In this case, the period identifier and the repetitive control period are updated every controller sampling interval. We call this fast adaptation scheme. A sufficient condition for the l_∞ BIBO stability of such linear time varying system is that the l_1 norm of the transfer function in Eq. (14) be less than one. Referring to the equivalent block diagram, Fig. 2(*b*), the time delay perturbation is now a linear time varying perturbation operator, whose l_∞ induced norm is one. Therefore, applying the small gain theorem [14] gives the above condition. The BIBO stability for variable time delay repetitive control can also be proven based on state space formulation (Hu [13]).

When applying variable sampling time adaptive scheme to this fast adaptation version, two independent sampling intervals, one for the period identification and the other for the repetitive controller should be used, with the former fixed while the latter varying. To establish stability, the l_1 norm of the transfer function in (14) should be sufficiently less than one so that even under a small range of sampling interval variations, the induced norm is always less than one.

4 Application to Mechanical Motion Tracking Control

The proposed self-tuning repetitive control algorithms have been applied to a mechanical motion tracking control problem. The hardware system consists of a air bearing supported XY stage driven by DC servo motors through ball screws, PWM servo amplifiers, linear encoders with 1 micron resolution, computer interfaces, and a floating point digital signal processor (TMS320C30) for control implementation. Both analog-to-digital and digital-to-analog converters are of 12 bit size over −10 to 10 volts range. The servo amplifier has an internal velocity feedback and therefore the voltage input to the amplifier represents a velocity command. Only the X axis motion is considered herein for testing the control algorithms.

The transfer function from the voltage input of the amplifier to the encoder position output for the X-axis has been identified:

$$G_x(s)=\frac{0.004(s+1934.7)(s+4749.3)}{s[(s+953.7)^2+841.0]} \quad (20)$$

The zero order hold and sampler discrete transfer function for a 5 millisecond sampling interval is

$$G(z^{-1})=\frac{1.0e^{-3}(0.1034z^{-1}+0.0111z^{-2}+0.0001z^{-3})}{1-0.9917z^{-1}-0.0082z^{-2}-0.0001z^{-3}} \quad (21)$$

A digital PD controller of the following form was introduced to stabilize the system:

$$C(z^{-1})=k_p+k_d\frac{1-z^{-1}}{T} \quad (22)$$

The gains $k_p=4000$, $k_d=20$ were used and the stabilized closed loop transfer function is

$$G_p(z^{-1})=\frac{0.8273z^{-1}-0.3252z^{-2}-0.0432z^{-3}-0.0005z^{-4}}{1-0.1644z^{-1}-0.3333z^{-2}-0.0433z^{-3}-0.0005z^{-4}}, \quad (23)$$

which is the plant model used for repetitive controller design. Since all the zeros of the plant is inside the unit circle, the repetitive controller by applying Eq. (15) for $K_r=1$ and $b=1$ is

$$M(z^{-1})=\frac{1}{G_p(z^{-1})}$$

$$Q(z^{-1})=\frac{z^2+4z+6+4z^{-1}+z^{-2}}{16}$$

Both the H_∞ and l_1 norms of the transfer function in Eq. (14) was verified to be less than one.

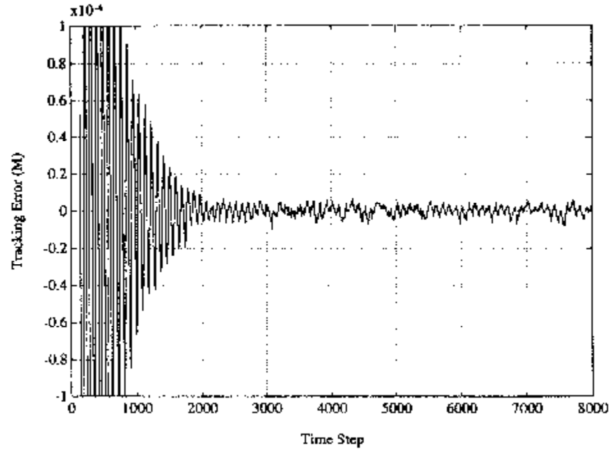

Fig. 3 Experimental results of the nonadaptive repetitive control scheme with exact match of the signal and the controller period (*N*=100)

Slow Adaptation Experimental Results. Experiment was first conducted for the case of exact signal and controller period synchronization. A sine wave, of which the period is 100 times the sampling period (5 ms) and the amplitude is 0.6 mm, was generated within the control software. Figure 3 shows the tracking

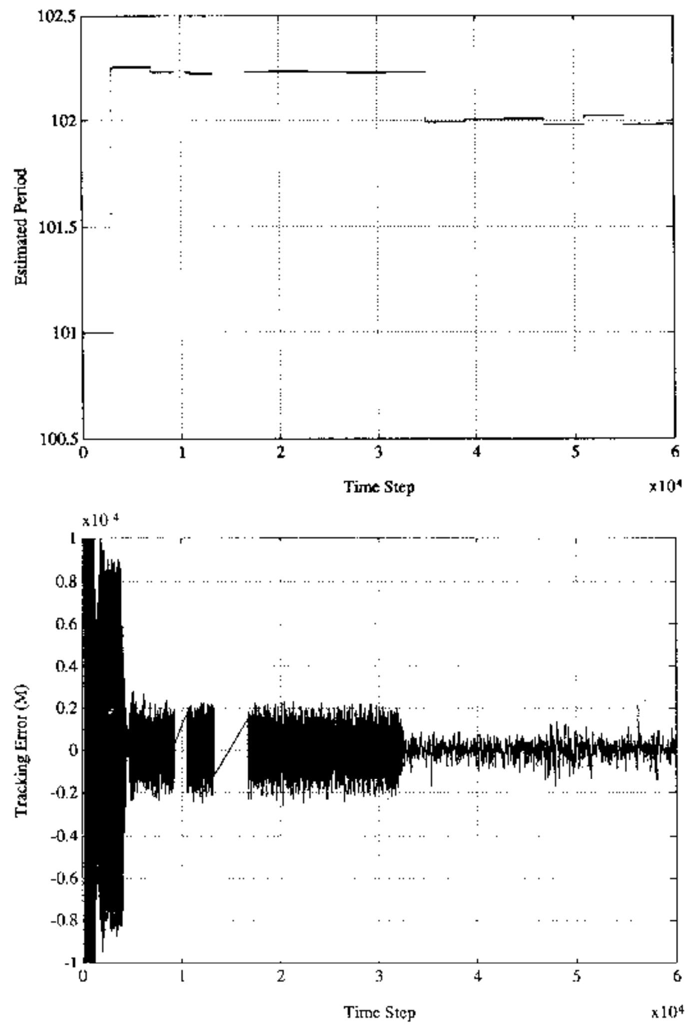

Fig. 4 Experimental results of the slow adaptation scheme with fixed and variable sampling intervals

error response. The r.m.s. value of the steady-state error response was 2.6 microns. This value represents the hardware system performance limit for exactly matched case and will be used for comparison with the adaptive schemes.

An analog sine wave was generated external to the control software by a function generator and was sampled by the control software as the tracking reference signal. The signal used for period identification was taken from the voltage input to the servo amplifier, assuming the periodic reference signal was not available. The signal period was around 500 milliseconds, but was uncertain. Each tuning cycle has $P=4000$ steps with $P_1=2100$, $P_2=900$, $P_3=1000$. The experimental results are shown in Fig. 4. The fixed sampling time adaptive scheme was implemented for the first 32,000 time steps and the variable sampling time adaptive scheme was activated at step 32,000. In the first cycle, the controller period had initial condition at 101 and the steady state r.m.s. error magnitude was 62.2 microns. The period identifier gave $\eta=102.22$ and hence the adaptive repetitive controller applied $N=102$ in the next few cycles. This reduced the r.m.s. error magnitude to 13.3 microns. When switched to variable sampling interval adaptive scheme, the sampling time was adjusted such that both signal and controller periods were 102. The identified period corresponding signal sampling interval is shown in the upper trace of Fig. 4. The r.m.s. error magnitude reduced further to 3.3 microns, close to the limiting value 2.6 microns. The 0.7 micron difference is probably due to combined effect from the controller hardware timer resolution and the noises in the analog signal transmission and conversion. The missing data points in Fig. 4 is due to the fact that host computer processor was not available for data acquisition during those periods.

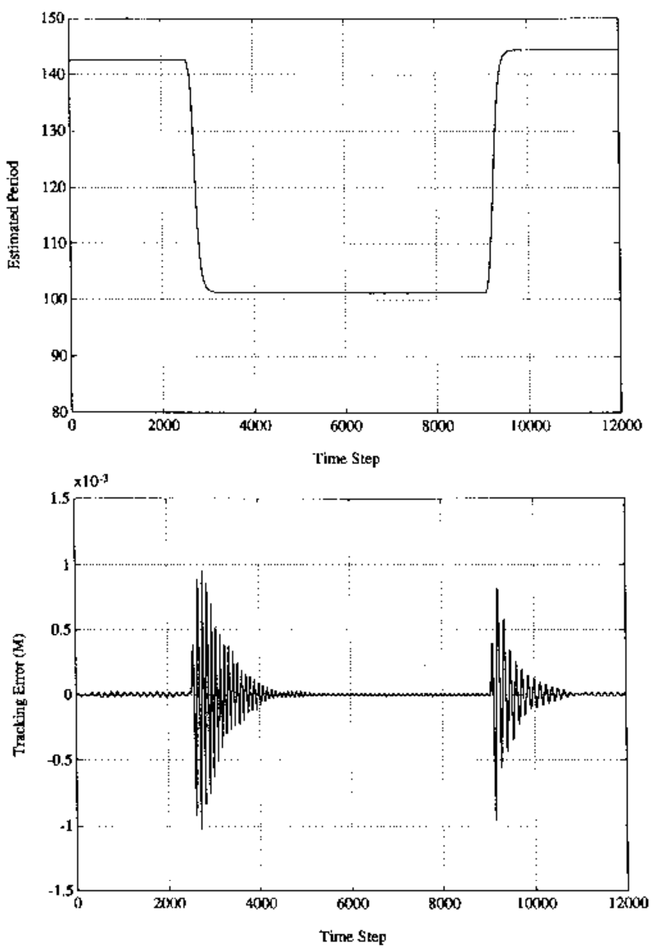

Fig. 5 Experimental results of fast adaptation scheme using reference input signal for period identification

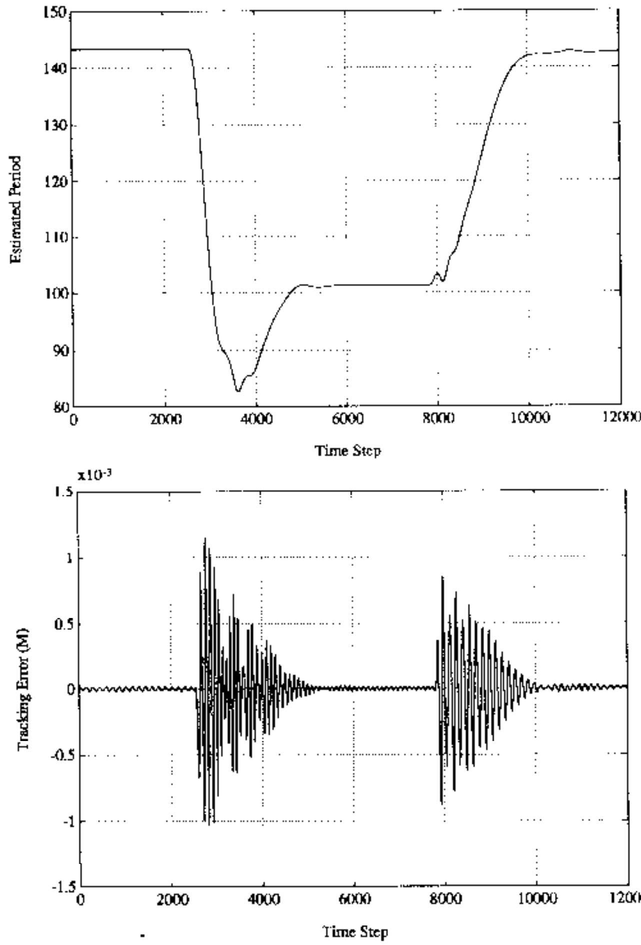

Fig. 6 Experimental results of fast adaptation scheme using the plant model to calculate the periodic input signal

Fast Adaptation Experimental Results. The fast adaptation scheme was implemented with a fixed sampling interval. In the experiment, the function generator signal frequency was manually switched from about 1.4 Hz to about 2 Hz and switched again back to 1.4 Hz. The moving data window for the period identification was 150 steps. The first test used the reference signal from the function generator for period identification and the second test used the error and control signals to calculate the exogenous signal $w(t)$ by Eq. (2). The results are shown in Figs. 5 and 6, respectively. Both cases were able to keep up with the change of periods quickly while maintaining system stability. It is interesting to observe that the period estimation has smooth transition when the signal frequency is changed although in the transition stage there are times at which the 150 data points used for period identification contain both frequencies. As expected, the transient performance of the first test appears better than that of the second test because the signal used for the second test contains "noise" from the unmodeled dynamics.

Finally, it is remarked that the real-time software for implementing the adaptive schemes presented herein only needs to be slightly modified from a fixed sampling interval repetitive control software. In addition to the period estimation software, changing controller period is only a matter of changing the pointer index of a data array, and changing sampling interval is accomplished by changing a timer counter value. Both simulation and experimental results showed that no substantial adverse transient behaviors were observed when changing the repetitive controller period and/or sampling intervals.

5 Concluding Remarks

Repetitive control schemes for tracking periodic signals with unknown or slowly varying period have been presented. A discrete-time domain recursive scheme for identifying continuous-time signal period with better than sampling interval resolution has been proposed. The period adaption scheme was incorporated in the self tuning repetitive control with a slow adaptation scheme and a fast one, each with stability conditions given. A novel way to match the noninteger estimated signal period and the integer repetitive controller period was proposed by changing the control sampling interval. Both the fixed sampling and the variable sampling interval adaptive schemes have been verified by simulation/experiment with the latter having superior performance than the former. The fast adaptation schemes have been able to keep up with the change of periods quickly while maintaining system stability. The slow adaptation scheme with a fixed sampling interval has also been successfully applied to removing unknown periodic disturbance load for the mold level regulation in continuous steel casting process (Manayathara et al. [15]).

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An Anti-Windup Design for Linear System With Asymptotic Tracking Subjected to Actuator Saturation

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This paper deals with asymptotic tracking for linear systems with actuator saturation in the presence of disturbances. Both reference inputs and disturbances are assumed to belong to a class which may be regarded as the zero-input responses of linear systems. The controller includes an anti-windup term which reduces the degradation in the system performance due to saturation. The stability of the overall system is established based on the Lyapunov stability theory. Both state and output feedback solutions are given. The proposed scheme is evaluated for a two axis motion control system by simulation. [S0022-0434(00)01002-9]

1 Introduction

Nearly all physical systems are subjected to some type of control input saturation. Actuator saturation is such a case, and it is often encountered in mechanical control systems. It is known that actuator saturation may have adverse effects on performance and stability of a closed-loop system, if the controller of which is designed without accounting for it. Consequently, there have been a number of studies made for linear systems with input saturation. One research topic is the closed-loop stability, and several significant results have emerged. State and output feedback controllers have been synthesized to achieve global or semi-global stabilization of stabilizable systems with no unstable open-loop eigenvalues [1,2]. Another topic of interest has been the trajectory planning considering the actuator constraints in the framework of off-line optimization [3,4]. A third topic has been the so-called windup. Windup problems were originally encountered in PI/PID controllers. However, it was recognized later that the integrator windup is only a special case of a more general problem. As pointed out by Doyle et al. [5], any controller with relatively slow or unstable modes will experience windup if there are actuator constraints. Substantial research has been done to incorporate modifications into controllers, which have been designed without accounting for constraints, such that the closed-loop behavior is satisfactory even in the presence of constraints. These modifications are usually called anti-windup schemes. Most of the existing anti-windup schemes modify the control law only when the actuator is saturated. Many of them are ad-hoc in nature and do not guarantee asymptotic closed-loop stability even for open-loop stable systems.

This paper is based on work by Kapoor, Teel, and Daoutidis [6,7]. In [6], only regulation was considered and measurement

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