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EE 215C Winter 2009 — Homework 2
Eudean Sun — January 20, 2009

1. (a) i. First, let's find $V_{n,out}^2$. The noise contributions from R_S, R_D, and M_1 all appear as uncorrelated current sources flowing through $R_S \parallel R_D \parallel R_{in}$ (where R_{in} is the input resistance excluding R_S and R_D), which then experience a gain of 1 to the output (since M_2 is biased by an ideal current source). The noise current of M_2 flows to the output directly and gets converted to a voltage via R_{out}.

$$\begin{aligned} V_{n,out}^2 &= \left(\overline{I_{RS}^2} + \overline{I_{RD}^2} + \overline{I_{D1}^2}\right)(R_S \parallel R_D \parallel R_{in})^2 + \overline{I_{D2}^2} R_{out}^2 \\ &= (4kT/R_S + 4kT/R_D + 4kT\gamma g_{m1})(R_S \parallel R_D \parallel R_{in})^2 + 4kT\gamma g_{m2} R_{out}^2 \end{aligned}$$

We can see that $R_{in} = 1/g_{m1}$, since $v_{gs2} = 0$ in the small-signal model means M_1 is effectively diode-connected for small-signal purposes.
Fig. 1 shows the test setup we'll use to find R_{out}.

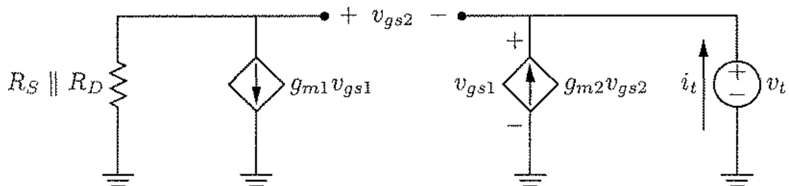

Figure 1: Test setup for finding R_{out}

$$\begin{aligned} i_t &= -g_{m2}v_{gs2} \\ v_{gs2} &= -g_{m1}v_{gs1}(R_S \parallel R_D) - v_t \\ v_{gs1} &= v_t \\ v_{gs2} &= -v_t\left[g_{m1}(R_S \parallel R_D) + 1\right] \\ i_t &= g_{m2}v_t\left[g_{m1}(R_S \parallel R_D) + 1\right] \\ R_{out} &= \frac{v_t}{i_t} = \frac{1}{g_{m2}\left[g_{m1}(R_S \parallel R_D) + 1\right]} \end{aligned}$$

Thus, we have:

$$\begin{aligned} V_{n,out}^2 &= 4kT\left[(1/R_S + 1/R_D + \gamma g_{m1})\left(R_S \parallel R_D \parallel \frac{1}{g_{m1}}\right)^2 + \gamma g_{m2}\left(\frac{1}{g_{m2}\left[g_{m1}(R_S \parallel R_D) + 1\right]}\right)^2\right] \\ &= 4kT\left[\left(\frac{1}{R_S \parallel R_D} + \gamma g_{m1}\right)\left(R_S \parallel R_D \parallel \frac{1}{g_{m1}}\right)^2 + \frac{\gamma}{g_{m2}\left[g_{m1}(R_S \parallel R_D) + 1\right]^2}\right] \\ NF &= \frac{V_{n,out}^2}{A^2}\frac{1}{4kTR_S} \\ A &= \frac{R_D \parallel \frac{1}{g_{m1}}}{\left(R_D \parallel \frac{1}{g_{m1}}\right) + R_S} \end{aligned}$$

$$\begin{aligned} NF &= \frac{1}{R_S}\left(1 + \frac{R_S}{R_D \parallel \frac{1}{g_{m1}}}\right)^2\left[\left(\frac{1}{R_S \parallel R_D} + \gamma g_{m1}\right)\left(R_S \parallel R_D \parallel \frac{1}{g_{m1}}\right)^2 + \frac{\gamma}{g_{m2}\left[g_{m1}(R_S \parallel R_D) + 1\right]^2}\right] \\ &= R_S\left(\frac{1}{R_S \parallel R_D \parallel \frac{1}{g_{m1}}}\right)^2\left[\left(\frac{1}{R_S \parallel R_D} + \gamma g_{m1}\right)\left(R_S \parallel R_D \parallel \frac{1}{g_{m1}}\right)^2 + \frac{\gamma}{g_{m2}\left[g_{m1}(R_S \parallel R_D) + 1\right]^2}\right] \end{aligned}$$

$$
\begin{aligned}
&= 1 + \frac{R_S}{R_D} + \gamma g_{m1} R_S + \frac{\gamma R_S \left(1/R_S + 1/R_D + g_{m1}\right)^2}{g_{m2}\left[g_{m1}\left(R_S \parallel R_D\right) + 1\right]^2} \\
&= \boxed{1 + \frac{R_S}{R_D} + \gamma g_{m1} R_S + \frac{\gamma R_S}{g_{m2}\left(R_S \parallel R_D\right)^2}} \quad \checkmark\ 3/5 \\
\lim_{R_D \to \infty} NF &= \boxed{1 + \gamma g_{m1} R_S + \frac{\gamma}{g_{m2} R_S}} \quad \checkmark\ 3/5
\end{aligned}
$$

As we let $R_D \to \infty$, the contribution to the noise figure from R_D goes to zero, and the contribution to the noise figure from M_2 decreases. Intuitively, we know that the noise current squared from R_D goes as $1/R_D$ while the input resistance (which the noise flows through) is limited by $(R_S \parallel 1/g_{m1})$. Thus, as R_D increases, the noise current drops out while the input resistance stays finite, causing the noise contribution to drop out.
Similar intuition applies to the decrease in noise from M_2. We see that R_{out} decreases as R_D increases. Since the noise current of M_2 flows through R_{out}, we can see that as R_D increases, the noise contribution from M_2 will decrease.

ii. Let R_{in} be the input resistance excluding R_S, R_{out} the output resistance excluding R_D, and A_v the gain excluding R_S (i.e., from the source to the drain of M_1).

$$
\begin{aligned}
V_{n,out}^2 &= \left(\overline{I_{RS}^2} + \overline{I_{D2}^2}\right)\left(R_S \parallel R_{in}\right)^2 A_v^2 + \overline{I_{RD}^2}\left(R_D \parallel R_{out}\right)^2 + \overline{\left\{I_{D1}\left[\left(R_S \parallel R_{in}\right)A_v - \left(R_D \parallel R_{out}\right)\right]\right\}^2} \\
&= \left(\overline{I_{RS}^2} + \overline{I_{D2}^2}\right)\left(R_S \parallel R_{in}\right)^2 A_v^2 + \overline{I_{RD}^2}\left(R_D \parallel R_{out}\right)^2 + \overline{I_{D1}^2}\left[\left(R_S \parallel R_{in}\right)A_v - \left(R_D \parallel R_{out}\right)\right]^2
\end{aligned}
$$

We need to find R_{in}, R_{out}, and A_v. Let's use Fig. 2 to find R_{in}.

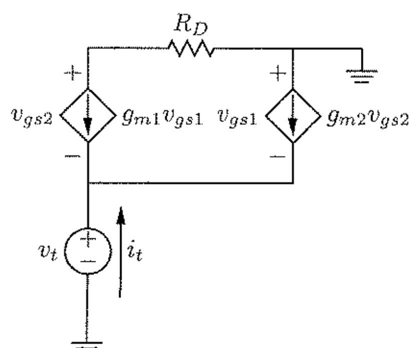

Figure 2: Test setup for finding R_{in}

$$
\begin{aligned}
i_t &= -g_{m1} v_{gs1} - g_{m2} v_{gs2} \\
v_{gs1} &= -v_t \\
v_{gs2} &= -g_{m1} v_{gs1} R_D - v_t \\
&= g_{m1} R_D v_t - v_t \\
&= v_t \left(g_{m1} R_D - 1\right) \\
i_t &= g_{m1} v_t - g_{m2} v_t \left(g_{m1} R_D - 1\right) \\
&= v_t \left[g_{m1} + g_{m2}\left(1 - g_{m1} R_D\right)\right] \\
R_{in} &= \frac{v_t}{i_t} = \frac{1}{g_{m1} + g_{m2}\left(1 - g_{m1} R_D\right)}
\end{aligned}
$$

Let's use Fig. 3 to find R_{out}.

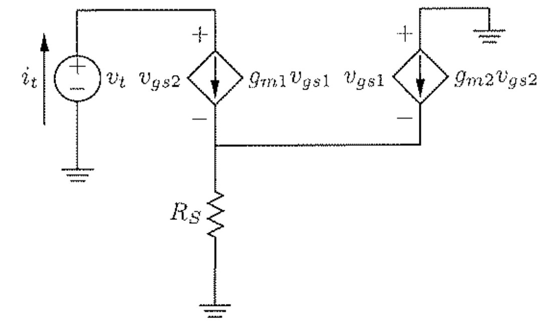

Figure 3: Test setup for finding R_{out}

$$\begin{aligned}
i_t &= g_{m1}v_{gs1} \\
-v_{gs1} &= (i_t + g_{m2}v_{gs2})\,R_S \\
v_{gs2} &= v_t + v_{gs1} \\
-v_{gs1} &= (i_t + g_{m2}v_t + g_{m2}v_{gs1})\,R_S \\
-v_{gs1}\,(1 + g_{m2}R_S) &= (i_t + g_{m2}v_t)\,R_S \\
v_{gs1} &= -\frac{(i_t + g_{m2}v_t)\,R_S}{1 + g_{m2}R_S} \\
i_t &= -g_{m1}\frac{(i_t + g_{m2}v_t)\,R_S}{1 + g_{m2}R_S} \\
i_t\left(1 + \frac{g_{m1}R_S}{1 + g_{m2}R_S}\right) &= -\left(\frac{g_{m1}g_{m2}R_S}{1 + g_{m2}R_S}\right)v_t \\
i_t\left(\frac{1 + (g_{m1} + g_{m2})\,R_S}{1 + g_{m2}R_S}\right) &= -\left(\frac{g_{m1}g_{m2}R_S}{1 + g_{m2}R_S}\right)v_t \\
R_{out} = \frac{v_t}{i_t} &= -\frac{1 + (g_{m1} + g_{m2})\,R_S}{g_{m1}g_{m2}R_S}
\end{aligned}$$

Let's use Fig. 4 to find A_v.

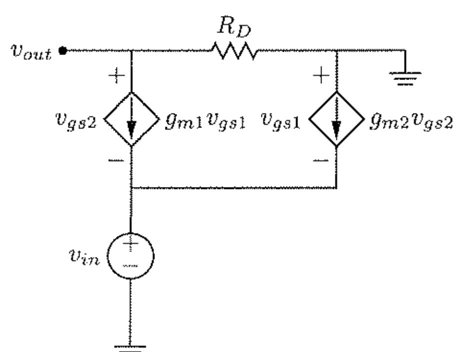

Figure 4: Test setup for finding A_v

$$\begin{aligned}
v_{out} &= -g_{m1}v_{gs1}R_D \\
v_{gs1} &= -v_{in} \\
v_{out} &= g_{m1}R_Dv_{in} \\
A_v &= \frac{v_{out}}{v_{in}} = g_{m1}R_D
\end{aligned}$$

Now we can write $V_{n,out}^2$:

$$\begin{aligned}
R_S \parallel R_{in} &= \frac{\frac{R_S}{g_{m1}+g_{m2}(1-g_{m1}R_D)}}{R_S + \frac{1}{g_{m1}+g_{m2}(1-g_{m1}R_D)}} \\
&= \frac{R_S}{1+(g_{m1}+g_{m2})\,R_S - g_{m1}g_{m2}R_SR_D} \\
R_D \parallel R_{out} &= -\frac{\frac{R_D+(g_{m1}+g_{m2})R_SR_D}{g_{m1}g_{m2}R_S}}{R_D - \frac{1+(g_{m1}+g_{m2})R_S}{g_{m1}g_{m2}R_S}} \\
&= \frac{R_D+(g_{m1}+g_{m2})\,R_SR_D}{1+(g_{m1}+g_{m2})\,R_S - g_{m1}g_{m2}R_SR_D}
\end{aligned}$$

$$\begin{aligned}
V_{n,out}^2 = \; & 4kT\left\{\left(\frac{1}{R_S}+\gamma g_{m2}\right)\left[\frac{g_{m1}R_SR_D}{1+(g_{m1}+g_{m2})\,R_S-g_{m1}g_{m2}R_SR_D}\right]^2\right. \\
& + \frac{1}{R_D}\left[\frac{R_D+(g_{m1}+g_{m2})\,R_SR_D}{1+(g_{m1}+g_{m2})\,R_S-g_{m1}g_{m2}R_SR_D}\right]^2 \\
& \left. + \;\gamma g_{m1}\left[\frac{g_{m1}R_SR_D-R_D-(g_{m1}+g_{m2})\,R_SR_D}{1+(g_{m1}+g_{m2})\,R_S-g_{m1}g_{m2}R_SR_D}\right]^2\right\}
\end{aligned}$$

$$\begin{aligned}
= \; & \frac{4kT}{\left[1+(g_{m1}+g_{m2})\,R_S-g_{m1}g_{m2}R_SR_D\right]^2}\left\{\left(\frac{1}{R_S}+\gamma g_{m2}\right)(g_{m1}R_SR_D)^2\right. \\
& \left. + R_D\left[1+(g_{m1}+g_{m2})\,R_S\right]^2+\gamma g_{m1}R_D^2\left[1+g_{m2}R_S\right]^2\right\}
\end{aligned}$$

$$\begin{aligned}
NF &= \frac{V_{n,out}^2}{A^2}\frac{1}{4kTR_S} \\
A &= \frac{R_{in}}{R_S+R_{in}}A_v \\
&= \frac{g_{m1}R_D}{1+(g_{m1}+g_{m2})\,R_S-g_{m1}g_{m2}R_SR_D}
\end{aligned}$$

$$\begin{aligned}
NF &= \frac{1}{(g_{m1}R_D)^2\,R_S}\left\{\left(\frac{1}{R_S}+\gamma g_{m2}\right)(g_{m1}R_SR_D)^2+R_D\left[1+(g_{m1}+g_{m2})\,R_S\right]^2+\gamma g_{m1}R_D^2\,(1+g_{m2}R_S)^2\right\} \\
&= \boxed{1+\gamma g_{m2}R_S+\frac{\gamma}{g_{m1}R_S}(1+g_{m2}R_S)^2+\frac{1}{g_{m1}^2R_SR_D}\left[1+(g_{m1}+g_{m2})\,R_S\right]^2} \quad \checkmark\; 5/5
\end{aligned}$$

$$\lim_{R_D\to\infty} NF = \boxed{1+\gamma g_{m2}R_S+\frac{\gamma}{g_{m1}R_S}(1+g_{m2}R_S)^2} \quad \checkmark\; 5/5$$

As we let $R_D \to \infty$, the contribution to the noise figure from R_D goes to zero, while all other terms remain the same. Intuitively, the noise current squared of R_D goes as $1/R_D$, while the output resistance is limited by R_{out}, which is independent of R_D (note again that R_{out} refers to the output resistance neglecting R_D, so the total output resistance would be $R_D \parallel R_{out}$). Thus, as R_D goes to infinity, its noise current goes to zero while the resistance it flows through approaches a finite value, meaning its noise contribution goes to zero.

(b) i.

$$\lim_{g_{m2} \to 0} NF = \boxed{\infty}$$ 5/5

As we let $g_{m2} \to 0$, the noise figure goes to infinity. Intuitively, the noise current squared of M_2 goes as g_{m2}, but the output resistance goes at $1/g_{m2}$. Since the noise contribution of M_2 is obtained by multiplying the square of the noise current by the square of the output resistance, the noise contribution of M_2 goes as $1/g_{m2}$. Thus, as g_{m2} goes to zero, the noise contribution of M_2 goes to infinity, causing the noise figure to go to infinity.

ii.

$$\lim_{g_{m2} \to 0} NF = \boxed{1 + \frac{\gamma}{g_{m1} R_S} + \frac{(1 + g_{m1} R_S)^2}{g_{m1}^2 R_S R_D}}$$ 5/5

As we let $g_{m2} \to 0$, the contribution to the noise figure from M_2 goes to zero and the contributions from M_1 and R_D decrease. Intuitively, the noise current from M_2 flows through a finite input resistance, and since the noise current from M_2 goes to zero as $g_{m2} \to 0$, the noise contribution from M_2 goes to zero.

We also see that the output resistance will decrease to R_D as $g_{m2} \to \infty$ (note that it actually is a decrease, not an increase, since $R_{out} < 0$, where again R_{out} is the output resistance neglecting R_D). Since the noise currents of M_1 and R_D flow through this output resistance, their noise contributions will decrease. 5/5

2. (a) Using $\boxed{W_2 = 13.7\ \mu\text{m}}$ causes the drain of M_1 to resonate at 5.2 GHz. The AC response of the circuit is shown in Fig. 5.

(b) Let R_{out1} be the output resistance of the first stage at 5.2 GHz, R_{in1} be the resistance seen at the source of M_1 at 5.2 GHz, and A_{v1} be the gain from the source of M_1 to the drain of M_1.

$$V_{n,out}^2 = \overline{I_{RS}^2} R_{in1}^2 A_{v1}^2 + \overline{I_{RP}^2} R_{out1}^2 + \overline{I_{D1}^2} \left(R_{in1} A_v - R_{out1}\right)^2$$

We can use SPICE to measure R_{in1}, R_{out1}, A_{v1}, and g_{m1}. We can estimate γ using SPICE as well by taking the noise measured from M_1 and dividing it by $4kTg_{m1}$.

$$\begin{aligned}
R_{in1} &= 48.870\ \Omega \\
R_{out1} &= 2.5872\ \text{k}\Omega \\
A_{v1} &= 27.5236 \\
g_{m1} &= 18.1324\ \text{mS} \\
\gamma &= 0.81819 \\
R_{P1} &= 1.3069\ \text{k}\Omega \\
T &= 298.15\ \text{K} \\
k &= 1.38065 \times 10^{-13}\ \text{J/K} \\
V_{n,out}^2 &= 1.0570 \times 10^{-15}\ \text{V}^2/\text{Hz} \\
NF &= \frac{V_{n,out}^2}{A^2} \frac{1}{4kTR_S} \\
A &= 27.708 \\
NF &= \boxed{1.67235}
\end{aligned}$$

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Using SPICE to find the noise figure (by dividing the total output noise by the output noise contributed by the source resistance), we get $NF = \boxed{1.78516}$. The calculated and simulated values differ by about 6.75 %, which is relatively good agreement.

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(c) The noise figure of the overall amplifier is $\boxed{NF = 1.8087}$. The second stage increases the noise figure by only about 1.32 %. This makes sense intuitively because the gain of the first stage is relatively large ($A = 27.708$). This reduces the contribution of the second stage to the noise figure, as can be seen from the Friis equation.

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This is the SPICE netlist used for this problem:

```
* EE215C HW2 Problem 2

.inc ‘215a.sp’

.param W2=13.7u
.param PI=3.14159265358979323846
.param Q=5
.param FREQ=5.2e9
.param L1=8n
.param RP1=‘2*PI*L1*FREQ*Q’
.param L2=6n
.param RP2=‘2*PI*L2*FREQ*Q’

vdd  vdd   gnd   1.8V
vin  vin   gnd   AC 1
rs   vin   1     50
c1   1     vs1   1u
i1   vs1   gnd   2.5m
m1   vd1   vdd   vs1   gnd   CMOSN W=50u L=0.18u
+PS=‘2*5e-5+1.2e-6’ PD=‘2*5e-5+1.2e-6’ AS=‘5e-5*0.6e-6’ AD=‘5e-5*0.6e-6’
l1   vdd   vd1   L1
rp1  vdd   vd1   RP1

m2   vout  vd1   vs2   gnd   CMOSN W=W2 L=0.18u
+PS=‘2*W2+1.2e-6’ PD=‘2*W2+1.2e-6’ AS=‘W2*0.6e-6’ AD=‘W2*0.6e-6’
i2   vs2   gnd   2m
c2   vs2   gnd   750f
c3   vout  gnd   60f
l2   vdd   vout  L2
rp2  vdd   vout  RP2

* Used for finding the resonance frequency.
.ac  lin   1000  5G   5.4G

* Used for finding the noise figure.
** .noise   v(vout)   vin 1
.option post nomod accurate
.end
```

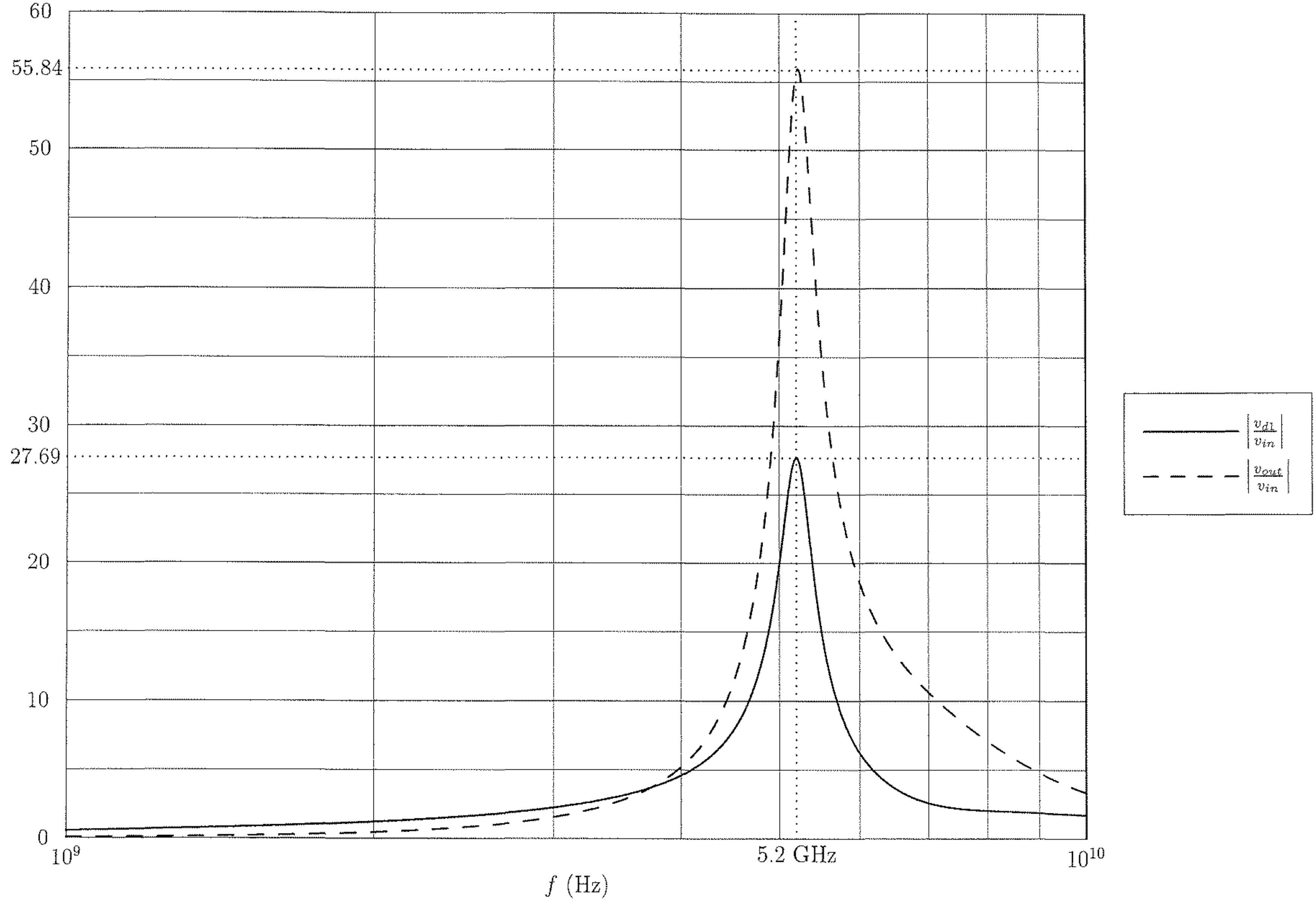

Figure 5: AC response of the LNA