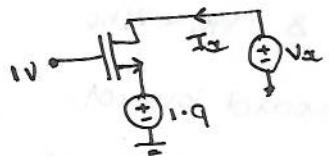


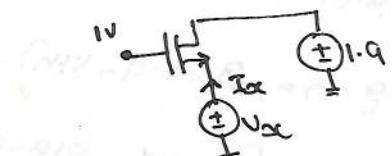
2.5c QN: Design as a function of $V_{OC} \in [0, VDD]$



$$\beta = HnCoSw/C$$

In all questions.

i) Starting at $V_{OC} = 0$ & showing Source at lowest voltage

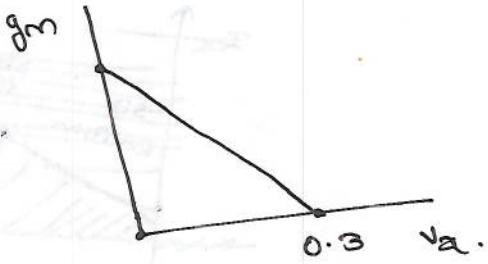
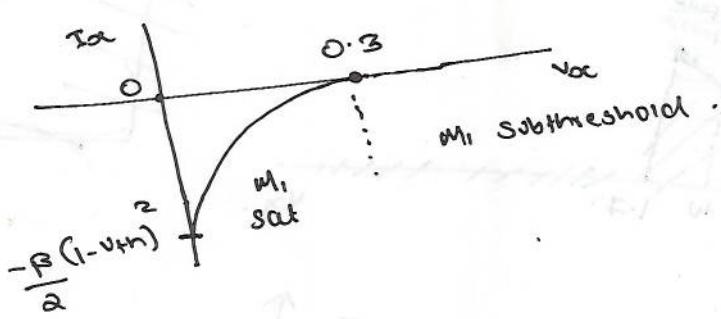


Device starts in saturation at $V_D > V_{G-SAT}$

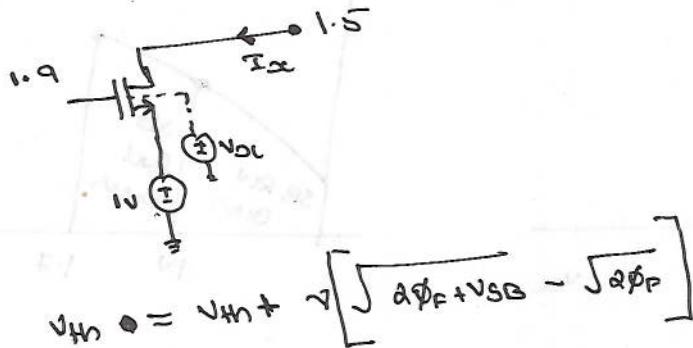
$$I_{DS} = -\frac{\beta}{2}(1 - V_{OC} - V_{TH})^2 = -I_D; g_m = \frac{2I_D}{2VGS} = \beta(1 - V_{OC} - V_{TH})$$

Device goes off at $V_{OC} = 1 - V_{TH} \approx 0.3V$ at which

point $I_{DS} = 0$



Q.5e



$$V_{SB} = (1 - V_{OC})$$

$$V_{TH} = V_{TH0} + \sqrt{\sqrt{\alpha\phi_F + V_{SB}} - \sqrt{2\phi_F}} \quad \text{--- (1)}$$

i) $0 \leq V_{OC} \leq 1 \Rightarrow V_{TH} > V_{TH0} \Rightarrow$ device M1 in saturation

\Rightarrow BB O100n is reverse biased

$$\Rightarrow V_{TH_{max}} \text{ for } V_{OC} = 0, \gamma = 0.45, 2\phi_F = 0.9 = 0.893V$$

$$I_{DQ} = \frac{\beta}{2} (0.9 - V_{TH})^2$$

$$g_m = \beta (0.9 - V_{TH})$$

ii) $1 \leq v_x \leq 1.7 \Rightarrow v_{SB}$ in negative & $v_{TH} < v_{TH0}$

\Rightarrow SB Diode in forward branch.

$$\Rightarrow v_{TH_{min}} \quad \left\{ \begin{array}{l} v_{SB} = -0.7, \beta = 0.45, 2\beta F = 0.9 \\ = 0.47V \end{array} \right.$$

$\Rightarrow M_1$ is in saturation.

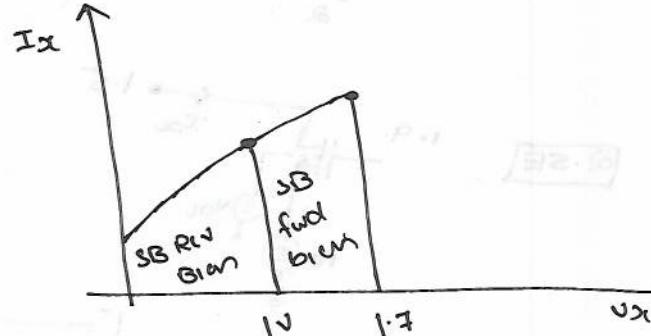
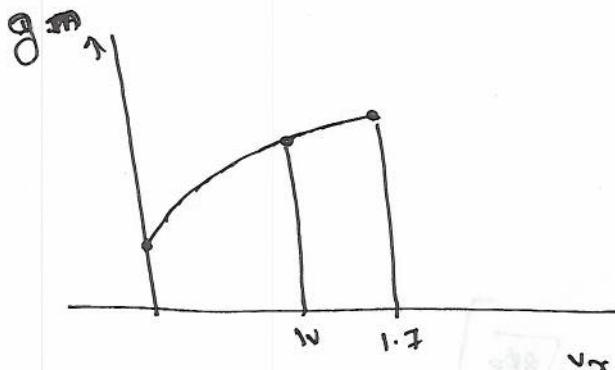
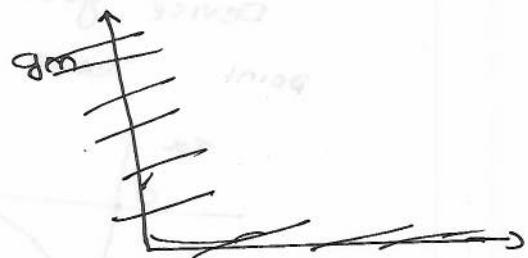
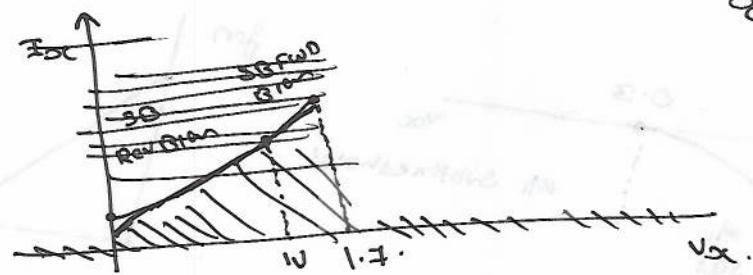
$$I_{DQ} = \frac{\beta}{2} (0.9 - V_{TH})^2 \quad g_m = \beta (0.9 - V_{TH})$$

forward branch

iii) $v_x \geq 1.7 \Rightarrow$ SB diode heavily forward biased

\Rightarrow Eqn ① no longer valid

\Rightarrow device will latch up



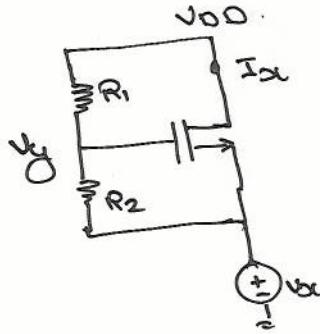
$$① \rightarrow I_{DQ} = \frac{\beta}{2} (0.9 - V_{TH})^2$$

Forward current is zero & $v_{TH} < v_{TH0}$ so $I_{DQ} = 0$

Forward current is zero & $v_{TH} < v_{TH0}$ so $I_{DQ} = 0$

$$I_{DQ} = (\beta = 0.45, V_{TH} = 0.47, V_{DD} = 0.9)$$

Q. 6b



$$V_y = \frac{V_{DD}R_2 + V_x R_1}{R_1 + R_2}$$

$$V_{GS} = \frac{[V_{DD} - V_x] R_2}{R_1 + R_2}$$

where M_1 is off.

i) starting condition $V_x = V_{DD}$ for convenience

$$\text{or } V_y = V_{DD} \quad V_{GS} = 0$$

when V_{GS}

$$\cancel{= V_{TH}}$$

Device will turn on

$$\Rightarrow \frac{(V_{DD} - V_x) R_2}{R_1 + R_2} = V_{TH}$$

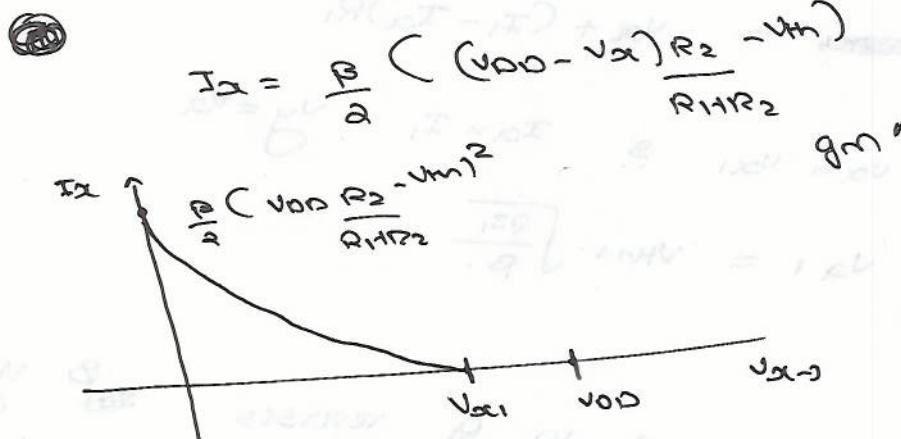
$$V_x = V_{DD} - \frac{V_{TH}(R_1 + R_2)}{R_2} = V_{DD} \text{ till which } I_{Dx} = 0$$

Exceeds to saturation.

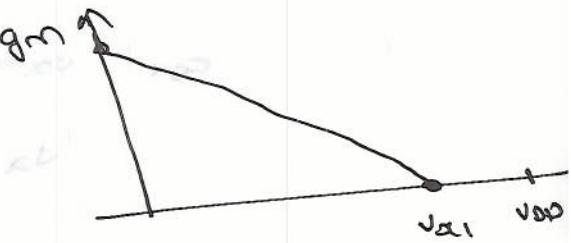
ii) since $V_g < V_d$

driven always to saturation.

$$\text{so for } V_x < V_{DD} - \frac{V_{TH}(R_1 + R_2)}{R_2}$$



$$g_m = \frac{\beta}{2} \frac{(V_{DD} - V_x) R_2}{R_1 + R_2} \quad - V_{TH}$$

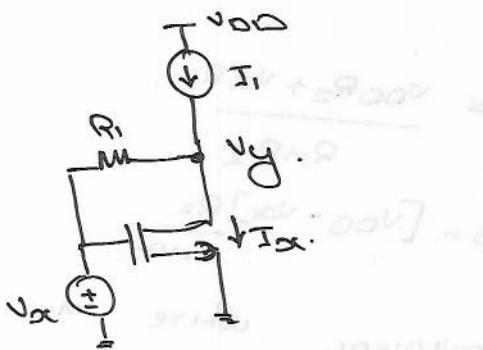


$$dV_{DS} = \alpha (R_2 - \frac{V_x}{R_1}) \text{ due to } Q_2$$

$$dI = \alpha (I - \frac{V_x}{R_1})$$

$$dV = \alpha (I^2 - \frac{V_x^2}{R_1^2} - \frac{2IV_x}{R_1})$$

2.6 e



- i) Starting at $V_x = 0$ for convenience.
 Device M_1 is off at this point $\Rightarrow V_y = 0 + I_1 R_1$
 As V_x increases V_y also increases on $V_x + I_1 R_1$.
 $I_{1x} = 0 \quad \& \quad V_y = V_x + I_1 R_1$ until $V_x = V_{th}$.

- ii) $V_x > V_{th}$ \Rightarrow Device turns on in saturation on $V_y > V_g - V_{th}$
 $\{V_y > V_x \text{ at } V_x = V_{th}\}$

$$I_{1x} = \frac{\beta}{2} (V_x - V_{th})^2 ; \quad g_m = \beta (V_x - V_{th})$$

hence $V_y = V_x + (I_1 - I_{1x}) R_1$

For $V_x = V_{x1} \quad \therefore \quad I_{1x} = I_1 \quad V_y = V_x$

$$V_{x1} = V_{th} + \sqrt{\frac{2I_1}{\beta}}$$

- iii) $V_x > V_{x1}$ current in R_1 reverses
 Device still in saturation until

$V_y < V_{d1}$
 $V_{d1} = V_{d12} \quad \& \quad V_d = V_g - V_{th}$

$$\Rightarrow V_{x2} + (I_1 - I_{1x}) R_1 = V_{d2} - V_{th}$$

$$\Rightarrow (I_{1x} - I_1) R_1 = V_{th}$$

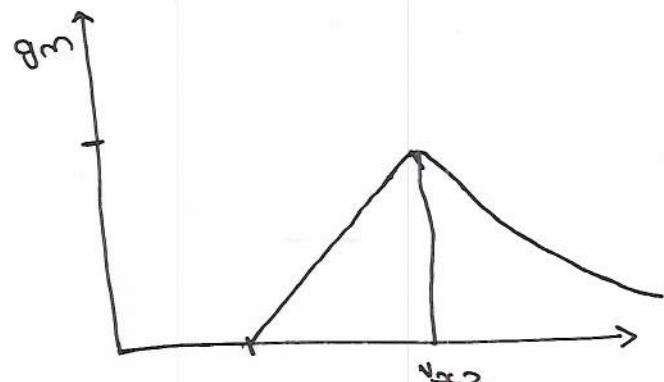
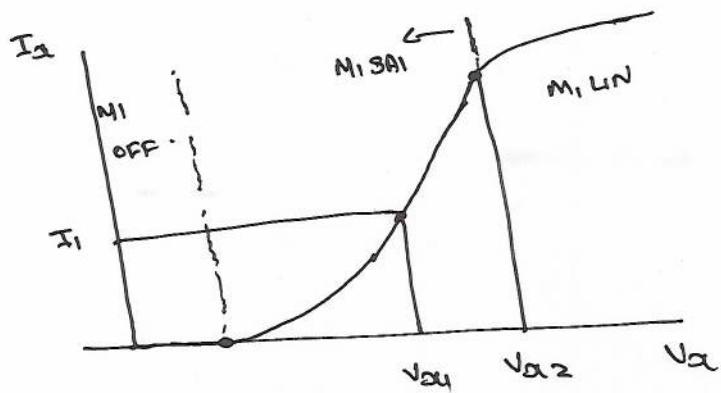
$$\Rightarrow \left(\frac{\beta}{2} (V_{d2} - V_{th})^2 - I_1 \right) R_1 = V_{th}$$

$$I_{\alpha} = \frac{\beta(v_{\alpha} - v_{th})^2}{2} \text{ in this region ; } g_m = \beta(v_{\alpha} - v_{th}).$$

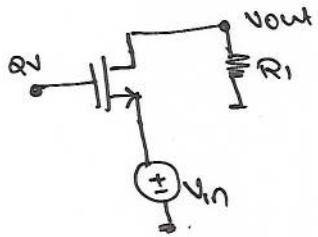
(N) $v_{\alpha} > v_{\alpha 2}$ device is in linear region.

$$I_{\alpha} = \beta \left[(v_{\alpha} - v_{th})(v_y) - \frac{v_y^2}{2} \right] \text{ where } v_y = v_{\alpha} + (I_1 - I_2) R_1$$

$$g_m = \cancel{\text{Beta}} \beta v_y.$$

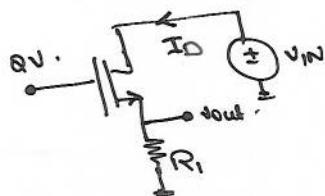


(2.7b) On V_{out} as function of $V_{in} \in [0, \infty)$



$$V_{in} > V_{out}$$

Reducing



- i) starting from $V_{in} = 0 \Rightarrow I_d = 0 \Rightarrow V_{out} = 0$.
Device starts in linear region

$$I_d = \beta \left(\frac{(2 - V_{out})(V_{in} - V_{out}) - (V_{in} - V_{out})^2}{V_{th}} \right).$$

$$V_{out} = I_d R_1$$

~~when~~ this is valid until $V_{in} = V_{in1} = 2 - V_{th}$

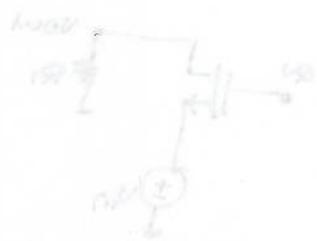
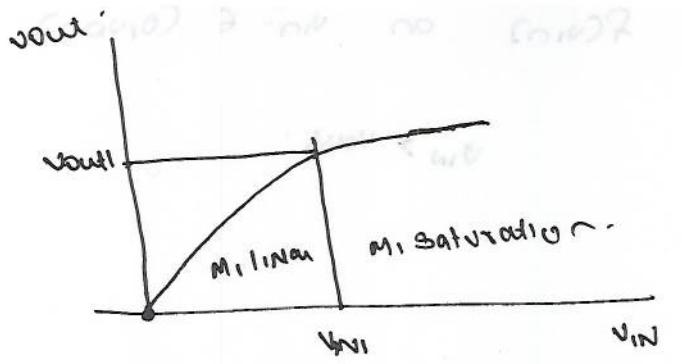
$$V_{in1} = V_{th} + \sqrt{2\phi_F + V_{out} - \sqrt{2\phi_F}}$$

- ii) when $V_{in} > V_{in1}$ device is in saturation.

$$I_d = \frac{\beta}{2} (2 - V_{out} - V_{th})^2 \quad \& \quad V_{out} = I_d R_1 = V_{out1}$$

$$= \frac{\beta}{2} (2 - \cancel{I_d R_1} - V_{th})^2$$

For high V_{in} ~~as~~ V_{out} increases slightly due to channel modulation but otherwise stays constant.



Q. 7 c) Same as 2.7 b with $V_{bg} = 0V$.



$\rightarrow O = \text{diss} \ L = \text{diss} \ G \rightarrow O = \text{int} \ \text{noisy} \ \text{product} \quad \text{D}$
 $\text{source} \ \text{noise} \ \text{in} \ \text{drain} \ \text{noise}$

$$\frac{O}{G} = \frac{(A_{v_{out}} - A_{v_{out}-\text{noise}})(A_{v_{out}-\text{noise}})}{1 + A_{v_{out}}^2} \quad \text{if} = \text{diss}$$

$$\text{ratio} \quad \frac{O}{G} = \frac{\text{int} \ \text{noise}}{\text{drain} \ \text{noise}} \quad \text{if} = \text{diss}$$

$$\left[\frac{O}{G} = \frac{A_{v_{out}}^2}{1 + A_{v_{out}}^2} \right] \quad \text{if} = \text{diss}$$

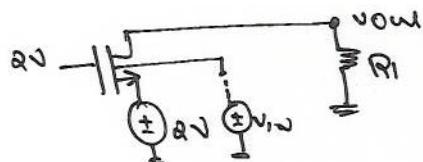
$$\text{Vibration} \ \text{noise} \ \text{in} \ \text{drain} \ \text{noise} \ \text{ratio} \quad \frac{O}{G} = \frac{A_{v_{out}}^2}{1 + A_{v_{out}}^2} \quad \text{if} = \text{diss}$$

$$(A_{v_{out}} = A_{v_{out}} - \text{noise}) \quad \frac{O}{G} = \text{diss}$$

$$(A_{v_{out}} = A_{v_{out}} - \text{noise}) \quad \frac{O}{G} = \text{diss}$$

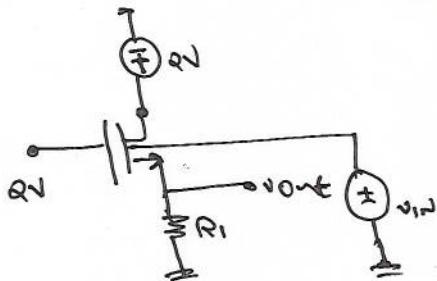
get optimum operating point \Rightarrow no noise if
 $V_{bg} = V_{th}$ \Rightarrow minimum noise at
 $A_{v_{out}}$ \Rightarrow minimum noise at V_{out}

(Q.8c)



$$V_{out} < 2V.$$

@ Redrawing



① Starting with $V_{IN}=0$
Device is in saturation

$$V_{TH} = V_{TH0} + \sqrt{2\phi_F + (V_{out} - V_{IN})} - \sqrt{2\phi_F}$$

⇒ ②

$$V_{out} = I_D R_1$$

$$I_D = \frac{\beta}{2} (2 - V_{out} - V_{TH})^2$$

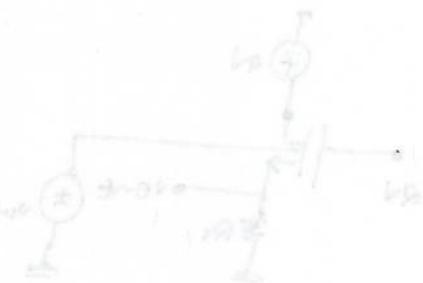
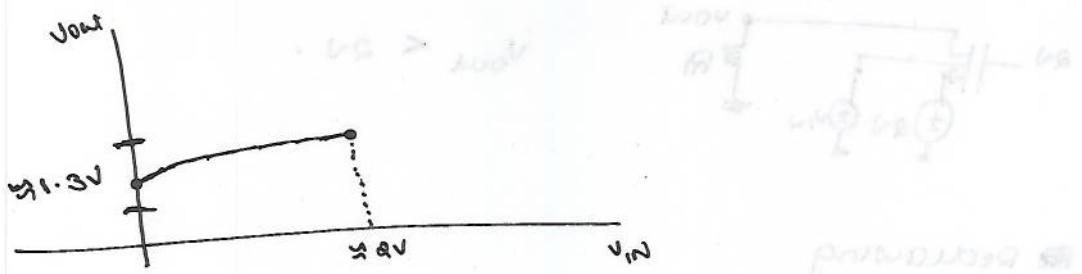
(ignoring channel width modulation)

As V_{IN} increases ~~V_{TH}~~

As V_{IN} increases V_{TH} reduces increasing I_D
 & hence V_{out} slowly increases due to negative feedback via V_{TH} & R_1
 increase in much slow compared to V_{IN} .
 due to negative feedback it will catch up with V_{out}

ii) As V_{IN} increases at some point (V_{out} stays in the vicinity of $2 - 0.7 = 1.3V$) and beyond which it will forward bias taking V_{TH} below V_{TH0} .

iii) For large values of V_{IN} forward biased \neq equation of V_{out} be heavily be valid



~~Output voltage~~ $V_{out} = \frac{1}{2} \cdot V_{AC}$

$$\text{Total current} = \text{Load current} + \text{Leakage current}$$

$$I_{load} = I_{load}$$

$$(I_{load} - I_{leakage}) \frac{R}{L} = \frac{dI}{dt}$$

between ωt and $\omega t + \Delta\omega$

Parasitic current $I_{leakage}$

at ωt parasitic current $I_{leakage}$ is zero

and $\omega t + \Delta\omega$ parasitic current $I_{leakage}$ is ∞

at ωt the current I_{load} is I_{load} and $\frac{dI}{dt} = 0$

at $\omega t + \Delta\omega$ the current I_{load} is I_{load} and $\frac{dI}{dt} = 0$

Problem off or open circuit $\frac{dI}{dt} \neq 0$ (i)

work off $\frac{dI}{dt} \neq 0$ \rightarrow $I_{load} \neq 0$

and ωt work off $\frac{dI}{dt} \neq 0$ (ii) \rightarrow $I_{load} \neq 0$

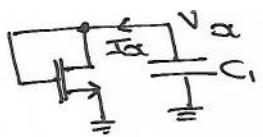
power $P = I_{load}^2 \cdot R_L$

max power $P = I_{load}^2 \cdot R_L$ \rightarrow $I_{load} = \sqrt{\frac{P}{R_L}}$

total $\frac{dI}{dt}$ constant \rightarrow max current I_{load} \rightarrow (iii)

leakage $I_{leakage}$

Q.9b



$t = 0 \quad V_{dA} = 3V \quad$ Device is in saturation

$$t > 0 \quad I_d = \frac{\beta(V_{dA} - V_{th})^2}{2} = -\frac{C_1 dV_{dA}}{dt}$$

$$\frac{dV_{dA}}{(V_{dA} - V_{th})^2} = -\frac{\beta dt}{2C_1}$$

Integration \Rightarrow

$$\frac{(V_{dA} - V_{th})^{-1}}{-1} = -\frac{\beta t}{2C_1} + K.$$

$$\frac{1}{V_{dA} - V_{th}} = \frac{\beta t - K}{2C_1}$$

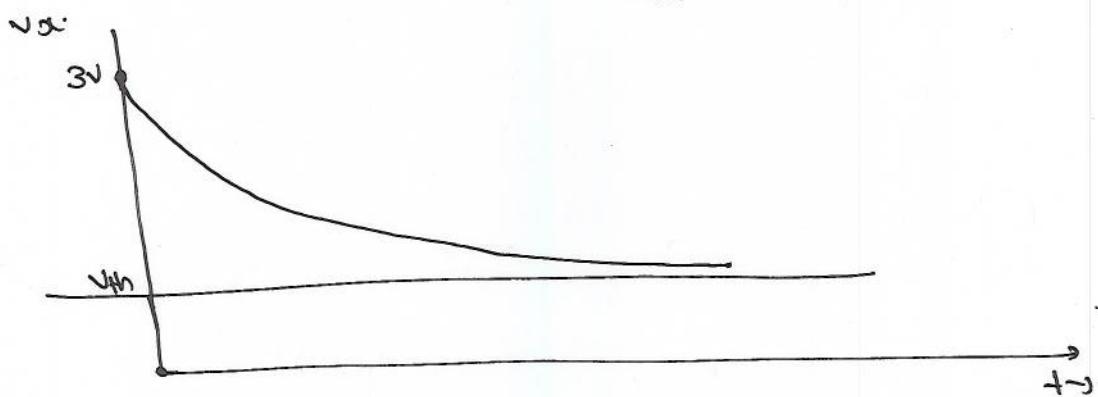
$$t = 0 \quad V_{dA} = 3 \quad \Rightarrow$$

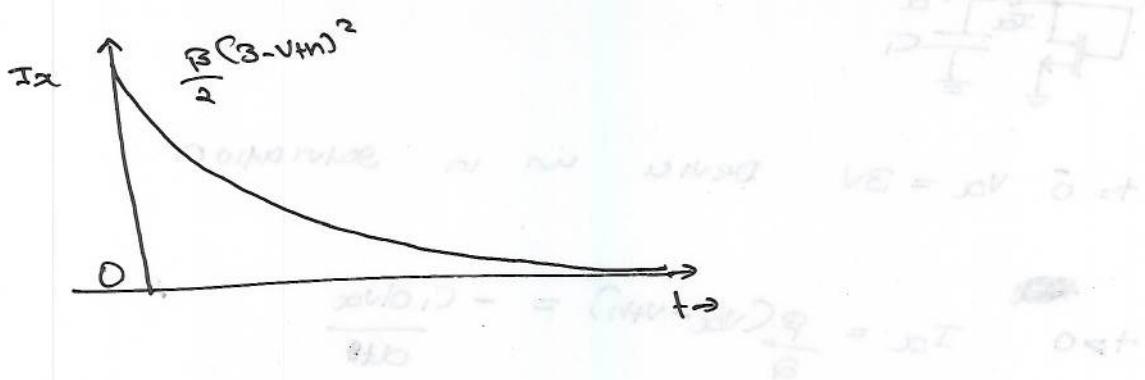
$$\frac{1}{3 - V_{th}} = -K.$$

$$\Rightarrow \frac{1}{V_{dA} - V_{th}} = \frac{\beta t}{2C_1} + \frac{1}{3 - V_{th}}$$

$$V_{dA} = V_{th} + \frac{1}{\frac{\beta t}{2C_1} + \frac{1}{3 - V_{th}}}.$$

$$I_d = \frac{1}{2} \left(\frac{1}{\frac{\beta t}{2C_1} + \frac{1}{3 - V_{th}}} \right)^2$$





$$\frac{20V - \eta}{12V} = \frac{20V - 29}{12V} = \frac{1}{2}$$

$$\frac{20V - \eta}{12V} = \frac{20V}{6(V_E - \eta)}$$

$$20 - \frac{\eta}{12} = \frac{20V}{6(V_E - \eta)}$$

$$\eta = \frac{10}{12} = \frac{1}{\frac{12V}{6(V_E - \eta)}}$$

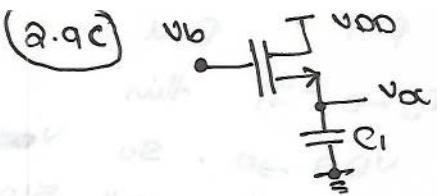
$$\text{For } E = 30V \quad D = +$$

$$21 = \frac{1}{\frac{12V}{6(V_E - \eta)}}$$

$$\frac{1}{12V - \eta} + \frac{10}{12V} = \frac{1}{12V - 30V}$$

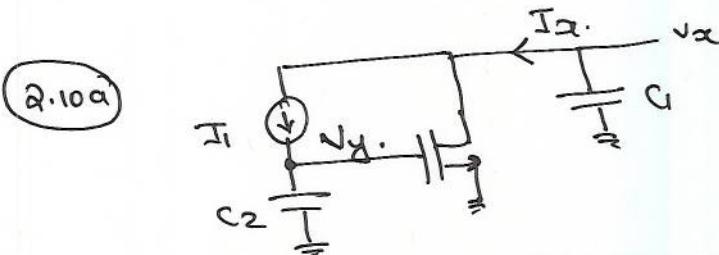
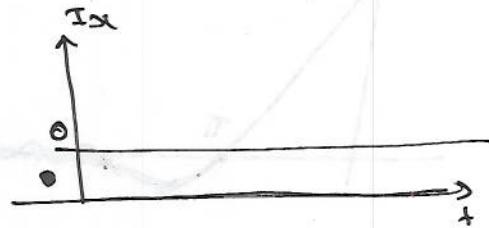
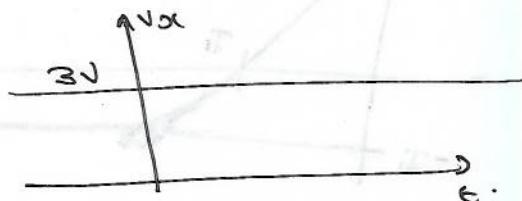
$$\frac{1}{12V - \eta} + \frac{10}{12V} = \frac{1}{-12V - 30V}$$





$$+ = 0 \quad V_x = 3V \quad \Rightarrow \text{device } M_1 \text{ is off} \quad \text{so } V_x \text{ stays}$$

at 3V. & $I_{Dx} = 0$



$$+ = 0 \quad V_y = 3 \quad V_x = 1$$

$+ > 0$

Device is in linear region

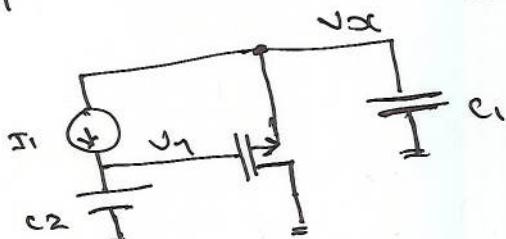
$$V_y = 3 + \frac{I_1 t}{C_1}$$

$$I_1 + \beta \left[(V_y - V_x - V_{th}) - \frac{V_x^2}{2} \right] + C_1 \frac{dV_x}{dt} = 0$$

$\Rightarrow V_x$ discharge toward zero.

$+ > T_1$ $V_x < 0 \Rightarrow$ source drain snap. Device still in linear region

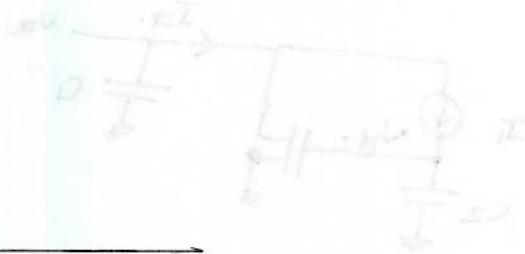
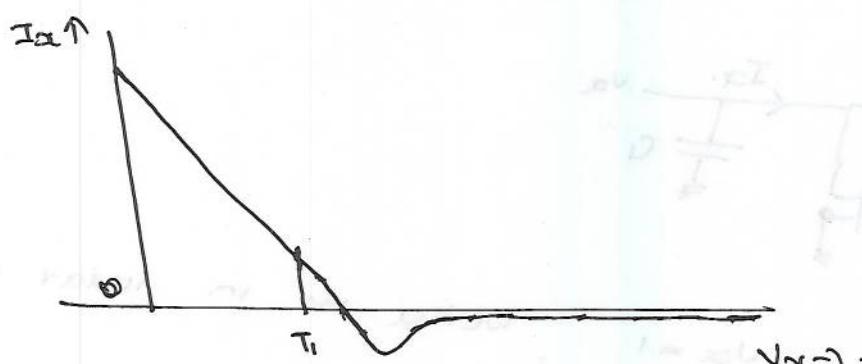
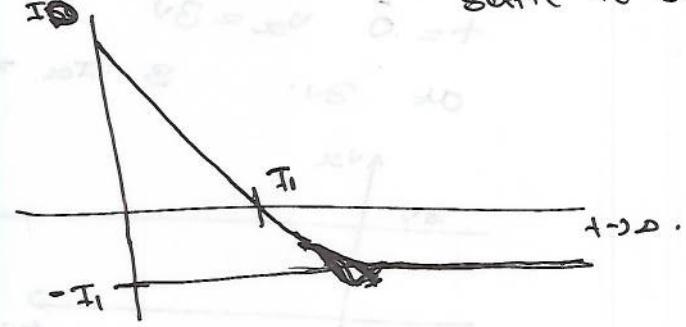
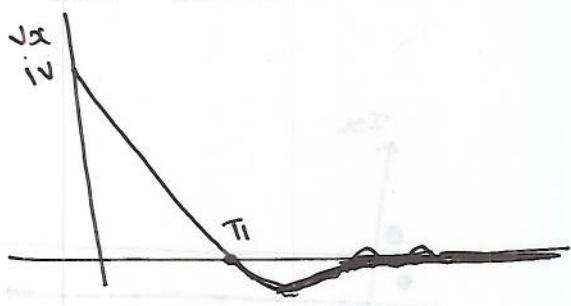
$$V_y = 3 + I_1 t$$



$$I_1 = \beta \left[(V_y - V_x - V_{th}) (-V_x) - \frac{V_x^2}{2} \right] + C_1 \frac{dV_x}{dt} = 0$$

clearly V_x will reduce ~~to~~ & I_D will tend to I_1 at steady state.

Solving this we can get a plot. But intuitively at $t = \infty$ $V_{DS} = \infty$ $I_D \rightarrow -I_1$, this will be achieved by a small $V_{DS} \rightarrow 0$. So V_{DS} will overshoot to a negative value & then will slowly settle to zero.



$$V_{DS} = \frac{I_D}{R_D}$$

$$\frac{dI_D}{dt} + \frac{I_D}{T_1} = B$$

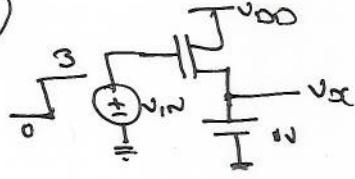
$$I_D = \frac{B_0}{R_D} e^{-\frac{t}{T_1}} + \left[\frac{B_0}{R_D} - \frac{B_0}{R_D} e^{-\frac{t}{T_1}} \right] e^{\frac{t}{T_1}}$$



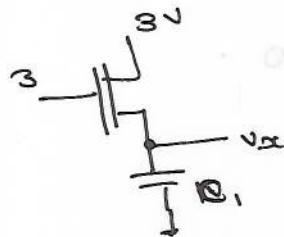
$$I_D = \frac{B_0}{R_D} e^{-\frac{t}{T_1}} + \left[\frac{B_0}{R_D} - \frac{B_0}{R_D} e^{-\frac{t}{T_1}} \right] e^{\frac{t}{T_1}}$$

It's best way of a better understanding. More about this

(2.11a)



$t < 0$ M_1 is off as $v_{DS} < 0$

 $t > 0$ 

ignoring body effect \Rightarrow channel length modulation.

$$\frac{\beta}{2} (3 - v_{TH} - v_D)^2 = \frac{C_1 dv_D}{dt}.$$

$$\frac{dv_D}{(3 - v_{TH} - v_D)^2} = + \frac{\beta dt}{2C_1}.$$

$$\frac{1}{+ (3 - v_{TH} - v_D)} = + \frac{\beta t}{2C_1} + K.$$

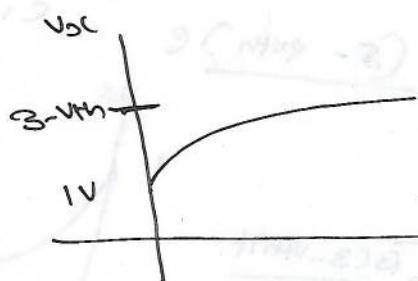
$$t=0 \quad v_D = 1$$

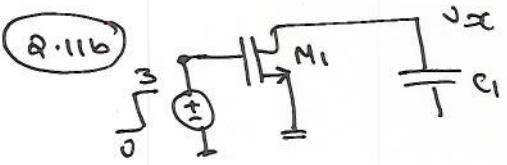
$$K = \frac{1}{2 - v_{TH}}$$

$$3 - v_{TH} - v_D = \frac{1}{\frac{\beta t}{2C_1} + K}.$$

$$v_D = 3 - v_{TH} - \frac{1}{\frac{\beta t}{2C_1} + K}$$

$$= 3 - v_{TH} - \frac{1}{\frac{\beta t}{2C_1} + \frac{1}{2 - v_{TH}}}$$





$t=0$ $V_{QS} = 0$ M_1 is off.

$t>0$ $V_{QS} = 3V$ M_1 is on in linear region

$$\beta \left[(3 - V_{TH}) V_x - \frac{V_x^2}{2} \right] + C_1 \frac{dV_x}{dt} = 0$$

$$\frac{dV_x}{\frac{\beta(3-V_{TH})V_x - V_x^2}{2C_1}} = -\frac{\beta dt}{2C_1}$$

$$\frac{dV_x}{\frac{\beta(3-V_{TH})}{2C_1} \left[\frac{1}{V_x} + \frac{1}{\beta(3-V_{TH}) - V_x} \right]} = -\frac{\beta dt}{2C_1}$$

integrating ... unk

$$\int \frac{V_x}{\frac{\beta(3-V_{TH})}{2C_1} \left[\frac{1}{V_x} + \frac{1}{\beta(3-V_{TH}) - V_x} \right]} = -\frac{\beta(3-V_{TH})t}{C_1} - \frac{\beta(3-V_{TH})t}{C_1}$$

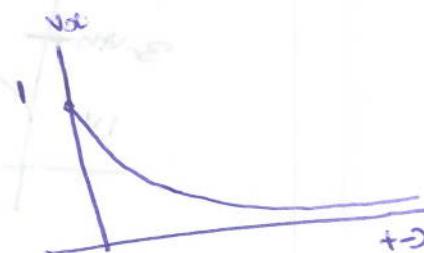
$$\frac{V_x}{\frac{\beta(3-V_{TH})}{2C_1} \left[\frac{1}{V_x} + \frac{1}{\beta(3-V_{TH}) - V_x} \right]} = K e^{-\frac{\beta(3-V_{TH})t}{C_1}}$$

$$t=0 \quad V_x = 1 \Rightarrow K = \frac{1}{S - \sqrt{V_{TH}}}$$

$$\frac{V_x}{\frac{\beta(3-V_{TH})}{2C_1} \left[\frac{1}{V_x} + \frac{1}{\beta(3-V_{TH}) - V_x} \right]} = \frac{1}{S - \sqrt{V_{TH}}} e^{-\frac{\beta(3-V_{TH})t}{C_1}}$$

$$\frac{\beta(3-V_{TH})}{2C_1} - 1 = \frac{(S - \sqrt{V_{TH}}) e^{-\frac{\beta(3-V_{TH})t}{C_1}}}{(S - \sqrt{V_{TH}})^2}$$

$$V_x = \frac{\beta(3-V_{TH})}{1 + (S - \sqrt{V_{TH}})^2} e^{-\frac{\beta(3-V_{TH})t}{C_1}}$$



(Q.26) $t=0^-$ $v_x = v_{DD}$, $I_D = I_1$, $v_y = v_{yo} = \frac{v_{DD} - v_{th}}{\beta} = \frac{v_{DD} - v_{th} - \sqrt{2I_1}}{\beta}$.

$t=0^+$ voltage across capacitors can't change instantly so the step out v_{in} propagates to $v_x \approx v_y$

$$v_y(0^+) = v_{yo} + v_0 \quad \& \quad v_x(0^+) = v_{yo} + v_0$$

$t > 0$ since $I_{C2} = 0$ $v_x - v_y = \text{constant} = v_{DD} - v_{yo}$.

This means v_{gs} or drain current is constant and same as $t=0^-$. However since $v_0 > v_{th}$ device enters linear region & $I_D < I_1$ given by:

$$I_D = \beta \left[(v_{DD} - v_{yo} - v_{th})(v_{DD} - v_y) - \frac{(v_{DD} - v_y)^2}{2} \right].$$

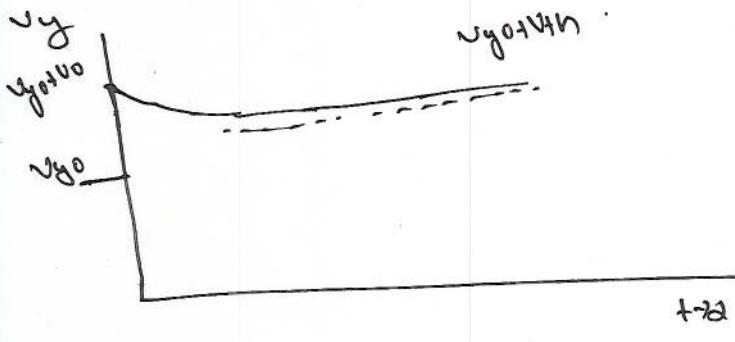
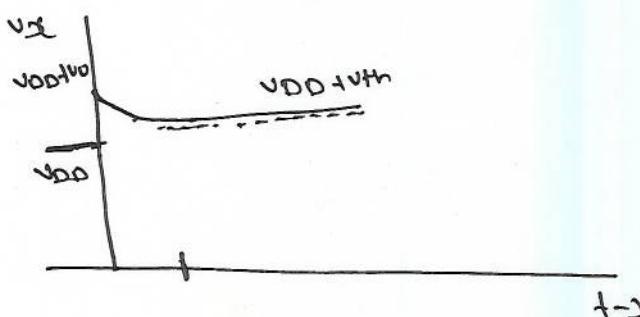
Thus will lead to discharge of node v_y .

as $C_2 \frac{d(v_0 - v_y)}{dt} = I_1 - I_D$.

As v_y falls this will continue at which point

$$I_D = I_1. \text{ The drain current hence known}$$

v_x tracks the change until v_x reaches saturation voltage at v_y is $v_{yo} + v_{th}$ to $v_{yo} + v_{th}$. in steady state.



Q.26b

$$t=0^+ \quad v_x = v_{DD} \quad v_y = v_{DD} = v_{DD} - v_{Th} = \frac{2v_D}{\beta} + v_{Th}$$



At $t=0^+$ $v_x = v_{DD}$ and $v_y = v_{DD} = v_{DD} - v_{Th}$

$$t=0^+ \quad v_x = v_{DD} - v_0$$

$$v_y = v_{DD} - v_0.$$

$$t>0 \quad \text{since } I_{C1} = 0$$

$$v_x - v_y = v_{DD} - v_{DD} = 0.$$

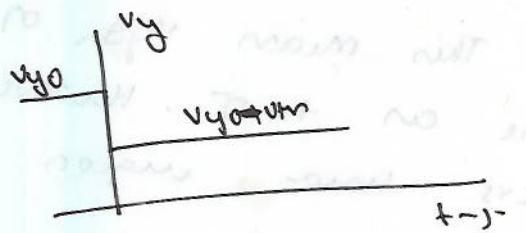
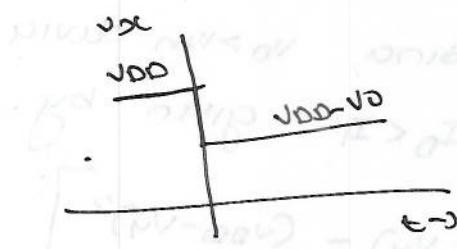
Since drain

in saturation.

so no change in

$$I_D = I_1 \Rightarrow I_{C2} = 0$$

voltages after initial step



After some time, the voltage v_x reaches a steady state value.

At $t=0^+$, $v_x = v_{DD}$ and $v_y = v_{DD} - v_{Th}$.

After some time, v_x reaches a steady state value.

At $t=0^+$, $v_x = v_{DD}$ and $v_y = v_{DD} - v_{Th}$.

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After some time, v_x reaches a steady state value.