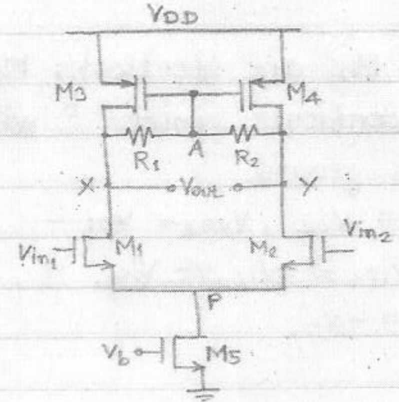
Total 28.

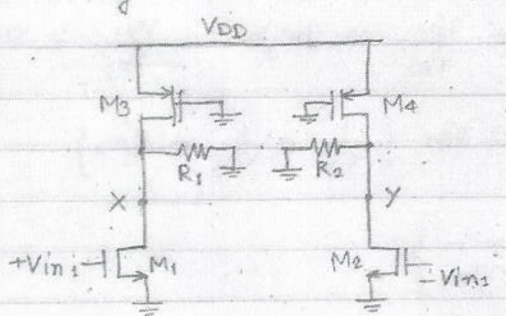
Problem 4.18

Fig. 4.38(d)



If M_1 & M_2 are identical and so are M_3 & M_4 and if $R_1 = R_2$ the circuit is symmetric and half-circuit concept can be used. Since circuit is symmetric, point A will be at AC ground. P is also at AC ground.

Using the half-circuit concept,



$$\frac{V_x}{V_{in_1}} = -g_{m_1}(r_{o_3} \,||\, r_{o_1} \,||\, R_1)$$

$$\frac{V_y}{-V_{in_1}} = -g_{m_2}(r_{o_4} \,||\, r_{o_2} \,||\, R_2)$$

$$\therefore\ V_x - V_y = -g_{m_1}(r_{o_3} \,||\, r_{o_1} \,||\, R_1)V_{in_1} - g_{m_2}(r_{o_4} \,||\, r_{o_2} \,||\, R_2)V_{in_1}$$

$$\therefore\ \frac{V_x - V_y}{2V_{in_1}} = -g_m(r_{o_3} \,||\, r_{o_1} \,||\, R_1)$$

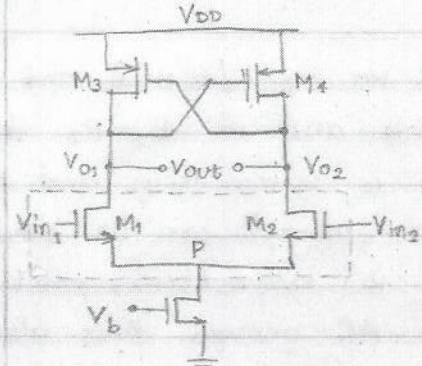
$$\therefore\ \frac{V_{out}}{V_{in}} = A_v = -g_m(r_{o_3} \,||\, r_{o_1} \,||\, R_1)$$ ... small signal differential voltage gain

1 mark for half ckt

2 mark for solution ⇒ give one mark for correct approach.

total = 3.

Fig. 4.39(c)



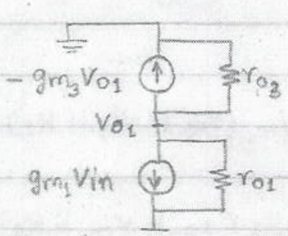
If M_1, M_2 are identical; M_3, M_4 are identical, point P will be an ac ground.

$V_{GS_3} = V_{o2}$, $V_{GS_4} = V_{o1}$

If $V_{in_1} = -V_{in_2} = V_{in_1}$,

$V_{o1} = -V_{o2}$

The small signal equivalent circuit can be drawn as follows:



$$g_m V_{in} + \frac{V_{o1}}{r_{o1}} - g_{m_3} V_{o1} + \frac{V_{o1}}{r_{o3}} = 0$$

$$-g_{m_1} V_{in} = V_{o1} \left\{ \frac{1}{r_{o1}} + \frac{1}{r_{o3}} - g_{m_3} \right\}$$

$$\therefore \frac{V_{o1}}{V_{in1}} = -g_{m_1} \left\{ \frac{1}{\frac{1}{r_{o1}} + \frac{1}{r_{o3}} - g_{m3}} \right\}$$

Also, similarly,

$$\frac{V_{o2}}{-V_{in_1}} = -g_{m_2} \left\{ \frac{1}{\frac{1}{r_{o2}} + \frac{1}{r_{o4}} - g_{m4}} \right\}$$

$$\Rightarrow \frac{V_{o1} - V_{o2}}{2 V_{in_1}} = -g_{m_1} \left\{ \frac{1}{r_{o1}^{-1} + r_{o3}^{-1} - g_{m_3}} \right\}$$

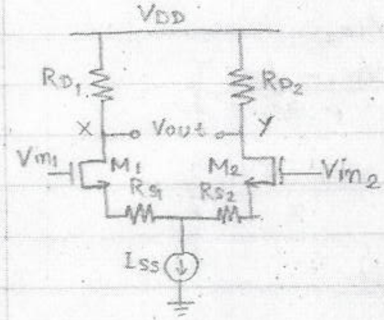
$$\Rightarrow \frac{V_o}{V_{in}} = A_v = -\frac{g_{m1}}{\frac{1}{r_{o1}} + \frac{1}{r_{o3}} - g_{m3}}$$ ... small signal differential gain

1 mark half ck

2 mark for solution

total 3

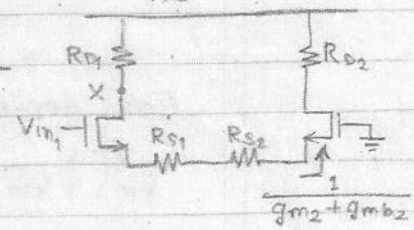
Problem 4.21



$\lambda = 0$

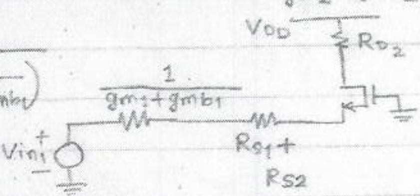
We use superposition theorem

From eq^n (3.55) if $r_o = \infty$,



$$\left.\frac{V_x}{V_{in_1}}\right|_{V_{in_2}=0} = \frac{-g_{m1} R_{D1}}{1+(g_{m1}+g_{mb1})\left(R_{S1}+R_{S2}+\frac{1}{g_{m2}+g_{mb2}}\right)}$$

From eq^n if $r_o = \infty$,



$$\left.\frac{V_y}{V_{in_1}}\right|_{V_{in_2}=0} = \frac{(g_{m2}+g_{mb2})\cdot R_{D2}}{1+(g_{m2}+g_{mb2})\left(R_{S1}+R_{S2}+\frac{1}{g_{m1}+g_{mb1}}\right)}$$

total 4 marks
→ give partial credits & marks
→ if approach is correct.

Similarly,

$$\left.\frac{V_y}{V_{in_2}}\right|_{V_{in_1}=0} = \frac{-g_{m2} R_{D2}}{1+(g_{m2}+g_{mb2})\left(R_{S1}+R_{S2}+\frac{1}{g_{m1}+g_{mb1}}\right)}$$

$$\& \left.\frac{V_x}{V_{in_2}}\right|_{V_{in_1}=0} = \frac{(g_{m1}+g_{mb1})\cdot R_{D1}}{1+(g_{m1}+g_{mb1})\left(R_{S1}+R_{S2}+\frac{1}{g_{m2}+g_{mb2}}\right)}$$

$$V_x = \underbrace{\frac{-g_{m1} R_{D1}}{1+(g_{m1}+g_{mb1})\left(R_{S1}+R_{S2}+\frac{1}{g_{m2}+g_{mb2}}\right)}}_{A} V_{in_1} + \underbrace{\frac{(g_{m2}+g_{mb2})\cdot R_{D2}}{1+(g_{m2}+g_{mb2})\left(R_{S1}+R_{S2}+\frac{1}{g_{m1}+g_{mb1}}\right)}}_{B} \times V_{in_2}$$

$\therefore V_x = A V_{in_1} + B V_{in_2}$

$$V_y = \underbrace{\frac{-g_{m2} R_{D1}}{1+(g_{m2}+g_{mb2})\left(R_{S1}+R_{S2}+\frac{1}{g_{m1}+g_{mb1}}\right)}}_{D} V_{in_2} + \underbrace{\frac{(g_{m1}+g_{mb1})\cdot R_{D2}}{1+(g_{m1}+g_{mb1})\left(R_{S1}+R_{S2}+\frac{1}{g_{m2}+g_{mb2}}\right)}}_{C} V_{in_1}$$

$\therefore V_y = C V_{in_1} + D V_{in_2}$

$$\therefore\ V_x - V_y = (A - C)\, V_{in_1} + (B - D)\, V_{in_2}$$

$$\therefore\ V_x - V_y = (A - C)\left\{\frac{V_{in_1} + V_{in_2}}{2} + \frac{V_{in_1} - V_{in_2}}{2}\right\}$$

$$+ (B - D)\left\{\frac{V_{in_1} + V_{in_2}}{2} - \frac{V_{in_1} - V_{in_2}}{2}\right\}$$

$$\therefore\ V_x - V_y = \{A - C + B - D\}\,\frac{V_{in_1} + V_{in_2}}{2}$$

$$+ \{A - C - B + D\}\,\frac{V_{in_1} - V_{in_2}}{2}$$

$$\therefore\ V_{out} = A_{cm} V_{cm} + A_{dm} V_{dm}$$

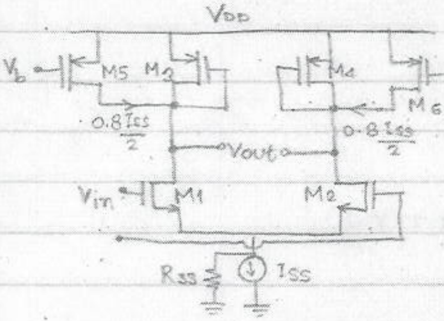
Considering only the differential mode gain,

$$\frac{V_{out}}{V_{in_1} - V_{in_2}} = (A - C - B + D)$$

$$= \frac{-(g_{m1})(R_{D1} + R_{D2}) - g_{mb2} R_{D2}}{1 + (g_{m1} + g_{mb1})\left(R_{S1} + R_{S2} + \frac{1}{g_{m2} + g_{mb2}}\right)}$$

$$+ \frac{-(g_{m2})(R_{D1} + R_{D2}) - g_{mb1} R_{D1}}{1 + (g_{m2} + g_{mb2})\left(R_{S1} + R_{S2} + \frac{1}{g_{m1} + g_{mb1}}\right)}$$

Problem 4.26



$$A_{V_{DM}} = -g_{m_1} R_D$$

$$A_{CM-DM} = \frac{-g_{m1}\,\Delta R_D}{(g_{m1}+g_{m2})R_{SS}+1} = \frac{-g_{m_1}\Delta R_D}{1+2g_{m_1}R_{SS}}$$

$$CMRR = \left|\frac{A_{DM}}{A_{CM-DM}}\right| = \left|\frac{1+2g_{m_1}R_{SS}}{\Delta R_D/R_D}\right| \quad \ldots (1)$$

$$R_D = r_{o5} \,\|\, r_{o3} \,\|\, \frac{1}{g_{m3}} \,\|\, r_{o1} \approx \frac{1}{g_{m3}}$$

$$\frac{\Delta R_D}{R_D} = \frac{1}{R_D}\cdot\frac{\partial R_D}{\partial g_{m3}}\Delta g_{m3} = \frac{1}{R_D}\cdot\frac{-1}{g_{m3}^2}\Delta g_{m3}$$

$$\Rightarrow \frac{\Delta R_D}{R_D} = -\frac{\Delta g_{m3}}{g_{m3}} \quad \ldots (ii)$$

$$g_{m3} = \sqrt{2\mu_p C_{ox}\left(\frac{W}{L}\right)_3 I_{D3}}$$

$$\therefore \frac{\partial g_{m3}}{\partial I_{D3}} = \sqrt{2\mu_p C_{ox}\left(\frac{W}{L}\right)_3}\,\frac{1}{2\sqrt{I_{D3}}}$$

$$\therefore \frac{\Delta g_{m3}}{g_{m3}} = \frac{\partial g_{m3}}{\partial I_{D3}}\frac{\Delta I_{D3}}{g_{m3}} = \Delta I_{D3}\cdot\sqrt{2\mu_p C_{ox}\left(\frac{W}{L}\right)_3}\cdot\frac{1}{2\sqrt{I_{D3}}}\cdot\frac{1}{\sqrt{2\mu_p C_{ox}\left(\frac{W}{L}\right)_3 I_{D3}}}$$

total 4 marks

⇒ 2 marks if approach is correct.

$$\Rightarrow \frac{\Delta g_{m3}}{g_{m3}} = \frac{\Delta I_{D3}}{2I_{D3}} \quad \ldots (iii)$$

$$I_{D5} + I_{D3} = \frac{I_{SS}}{2} \Rightarrow \Delta I_{D5} = -\Delta I_{D3}$$

$$\because I_{D5} = 0.8\,\frac{I_{SS}}{2} \Rightarrow I_{D5} = 4I_{D3}$$

$$\frac{\Delta I_{D5}}{I_{D5}} = -\frac{\Delta I_{D3}}{I_{D3}}\cdot\frac{I_{D3}}{I_{D5}} = -\frac{1}{4}\,\frac{\Delta I_{D3}}{I_{D3}} \quad \ldots (iv)$$

$$I_{D5} = \mu_p C_{ox}\frac{W}{2L}(V_b - V_{DD} - V_{TH})^2$$

$$\Delta I_{D5} = \frac{\partial I_{D5}}{\partial V_{TH}}\cdot\Delta V = -\frac{\mu_p C_{ox} W}{2L}\,2(V_b - V_{DD} - V_{TH})$$

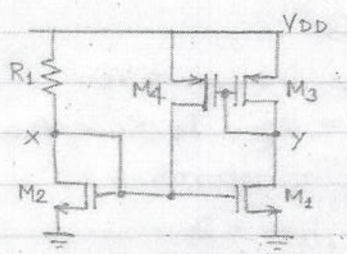
$$\therefore \frac{\Delta I_{D5}}{I_{D5}} = -\frac{2\Delta V}{V_b - V_{TH} - V_{DD}} \quad \ldots (v)$$

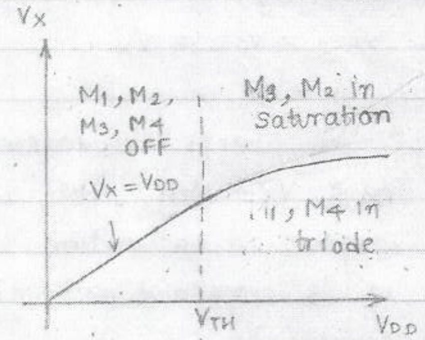
(i), (ii), (iii), (iv), (v) $\Rightarrow$

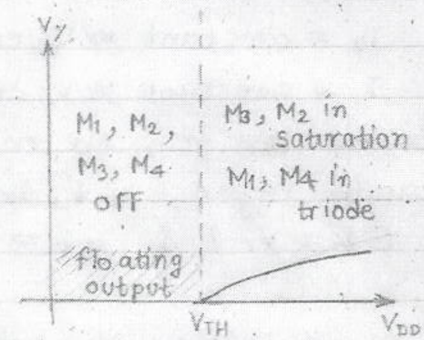
$$CMRR = \left| \frac{1 + 2 g_{m_1} R_{SS}}{\left(-\frac{1}{2}\right)(-4)(-2)\frac{\Delta V}{V_b - V_{DD} - V_{TH}}} \right|$$

$$\therefore \; CMRR = \frac{(1 + 2 g_{m_1} R_{SS}) \cdot (V_b - V_{DD} - V_{TH})}{4 \Delta V}$$

Problem 5.10(a)







1 mark for each plot.

2 mark for the explanation

⇒ if explanation is wrong dont give marks for plot.

total: 4.

Initially, when $V_{DD} = 0$ assume M_3 & M_4 are OFF.
$\therefore$ For $V_{DD} < V_{TH_{1,2}}$, $V_X = V_{DD}$ and M_1 & M_2 are OFF and Y is a floating output since it is isolated from V_{DD} and GND.

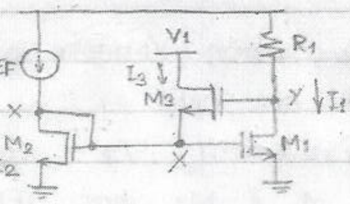
When $V_X \geq V_{TH1,2}$, M_2 & M_1 turn ON.
M_2 & M_3 are always in saturation.
At that moment, M_1 turns ON and pulls Y to GND, thus, turning ON M_3 & M_4.
As $V_{DD}\uparrow$, $V_X = V_{GS_2}$ & $V_Y = V_{DD} - V_{GS3}$
$\therefore V_X > V_Y$, M_1 & M_4 remain in triode always.

As $V_{DD}\uparrow$, $V_Y\uparrow$ but not as fast as V_{DD} because M_1 still tries to pull it to ground.
So, effectively $|V_Y - V_{DD}|\uparrow$ i.e. $|V_{GS_3}|\uparrow$, so, $I_{D3}\uparrow$
$\therefore$ M_1 is in triode $I_{D_1} = I_{D_3} \propto [2(V_{GS_1} - V_{TH})V_{DS_1} - V_{DS_1}^2]$ small. $\therefore I_{D_1} \propto [2(V_X - V_{TH})V_Y - V_Y^2]$. So, if $V_Y\uparrow$, $V_X\uparrow$ to increase I_{D1}.

When $V_x > V_{th12}$ its not possible for M_4 & M_1 to be in saturation. since in that case we would need

$I_3 = I_2 = I_1 = I_4 + I_{R1}$ to be equal to I_4. so one of M_1 or M_4 has to be in triode ⇒ $V_x > V_y + V_{th}$ ⇒ the other is in triode as well.

Problem 5.12(b)



Initially let $V_1 = V_{DD}$

$\therefore V_Y = V_{DD} - I_1 \cdot R_1 < V_{DD}$

$\therefore$ M_3 in saturation.

$I_2 = I_{ref} + I_3$

Assuming $\lambda = 0$,

$V_{GS_2} = V_{GS_1} = V_X$

$\therefore I_1 = I_2$

If V_1 starts decreasing

$V_1 \geq V_Y - V_{TH_3}$, M_3 will remain in saturation

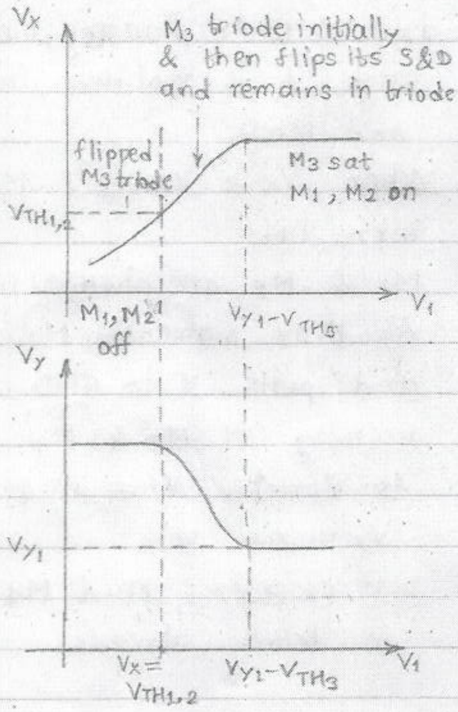
$\Rightarrow I_3$ constant $\Rightarrow I_1 = I_2$ constant.

I_2 = constant $\Rightarrow V_X$ const.

I_1 = constant $\Rightarrow V_Y$ const.

Beyond this pt., M_3 enters triode region. $I_3 \downarrow$, $I_2 \downarrow$, $I_1 \downarrow$

$I_1 \downarrow \Rightarrow V_Y \uparrow$ & $I_2 \downarrow \Rightarrow V_X \downarrow$



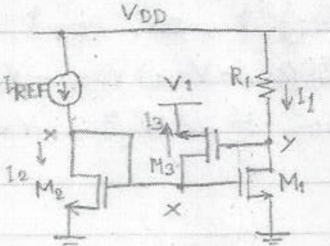
1 mark for each region

$\frac{1}{2}$ for plots

$\frac{1}{2}$ for explanation

total 3.

At a point where $V_1 < V_X$ & $V_Y - V_1 > V_{TH}$, M_3 turns ON in opposite direction. Now $I_3 \uparrow$, $I_2 \downarrow$, $I_1 \downarrow$. V_Y continues to rise & V_X continues to drop.



When $V_X < V_{TH_{1,2}}$, M_1 & M_2 are OFF.

$I_3 = I_{REF}$. $I_1 = I_2 = 0$. $V_Y = V_{DD}$.

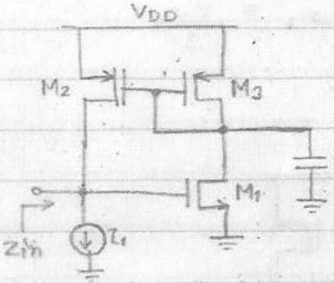
Now as $V_1 \downarrow$, $I_3 = I_{REF}$, $\because V_Y$ = const.

$V_{GS_3} \uparrow$. In triode region, if V_{DS} is small

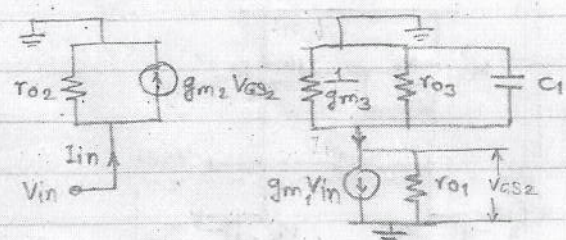
$I_3 \propto (V_{GS} - V_{TH}) V_{DS}$. So, if $V_{GS} \uparrow$, $V_{DS_3} \downarrow$ to maintain I_3 constant. $\because V_{DS} \downarrow$ i.e.

$(V_X - V_1) \downarrow \Rightarrow V_X \downarrow$

Problem 5.19



Small-signal equivalent model



1 mark for small signal

2 mark for solution ⇒ 1 if approach is correct.

total 3.

$$V_{GS2} = -g_{m1} V_{in} \left(\frac{1}{g_{m3}} \parallel r_{o3} \parallel r_{o1} \parallel \frac{1}{sC_1} \right)$$

If $\lambda = 0$,

$$V_{GS2} = \frac{-g_{m1} V_{in}}{g_{m3} + sC_1}$$

$$Z_{in} = \frac{V_{in}}{I_{in}} = \frac{V_{in}}{g_{m2} V_{GS2}} = \frac{V_{in}}{\frac{-g_{m1} g_{m2} V_{in}}{g_{m3} + sC_1}}$$

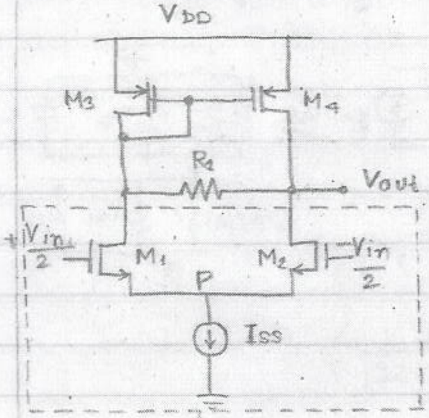
$$\therefore Z_{in} = -\left(\frac{g_{m3} + sC_1}{g_{m1} g_{m2}} \right)$$

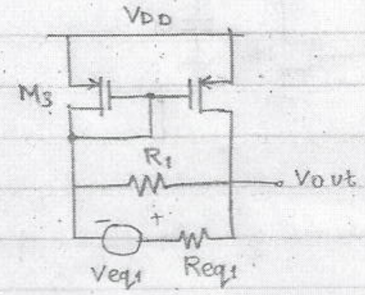
The negative input capacitive component is given by

$$\left(\frac{-sC_1}{g_{m1} g_{m2}} \right)$$

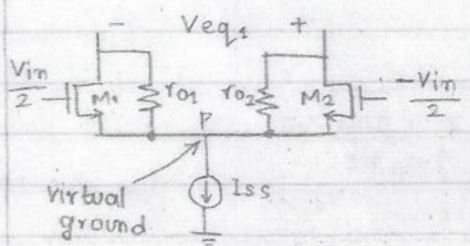
Problem 5.20(b)

Let $v_{in_1} = -v_{in_2} = \frac{v_{in}}{2}$





Substitution of input differential pair by a Thevenin equivalent



We substitute the input source and M_1, M_2 by a Thevenin equivalent. Because of symmetry, node P is a virtual ground, & a half-circuit equivalent yields

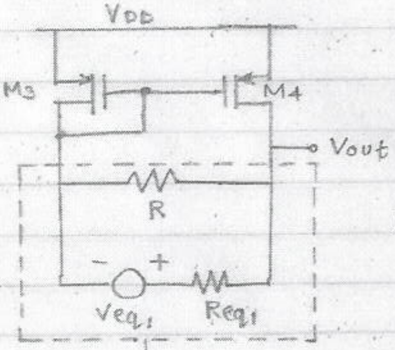
Total 4

2 marks if approach is correct.

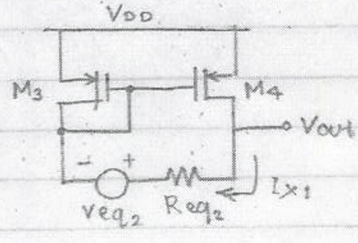
$V_{eq_1} = g_{m_{1,2}} r_{o_{1,2}} V_{in}$

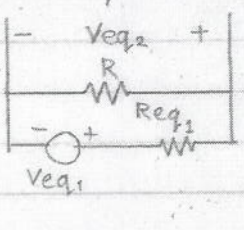
Moreover $R_{eq} = r_{o1} + r_{o2} = 2 r_{o_{1,2}}$

Simplified circuit:



Substituting a Thevenin equivalent for boxed circuit

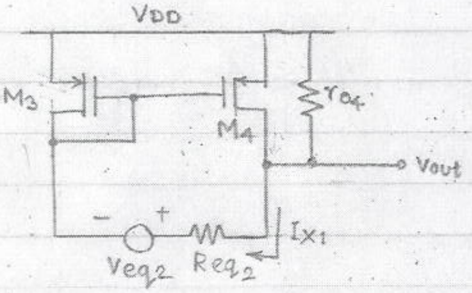




$$V_{eq_2} = \frac{V_{eq_1} R}{R + R_{eq_1}} = \frac{g_{m_{1,2}} r_{o_{1,2}} V_{in} \; R}{R + 2 r_{o_{1,2}}}$$

$$R_{eq_2} = 2 r_{o_{1,2}} \parallel R$$

Simplified circuit :



This circuit is similar to fig. 5.27(b) in the textbook.

Current through Req_2 is

$$I_{x_1} = \frac{V_{out} - \dfrac{g_{m1,2}\, r_{o1,2}\, V_{in} \cdot R}{R + 2 r_{o1,2}}}{2 r_{o1,2} \,||\, R + \dfrac{1}{g_{m3}} \,||\, r_{o3}}$$

The fraction of this current that flows through $1/g_{m3}$ is mirrored into M_4 with unity gain;

i.e.
$$2\left\{ \frac{V_{out} - \dfrac{g_{m1,2}\, r_{o1,2}\, V_{in} \cdot R}{R + 2 r_{o1,2}}}{2 r_{o1,2} \,||\, R + \dfrac{1}{g_{m3}} \,||\, r_{o3}} \cdot \frac{r_{o3}}{r_{o3} + 1/g_{m3}} \right\} = -\frac{V_{out}}{r_{o4}}$$

$$\therefore V_{out} \left\{ \frac{r_{o3}}{r_{o3} + 1/g_{m3}} \cdot \frac{1}{2 r_{o1,2} \,||\, R + \frac{1}{g_{m3}} \,||\, r_{o3}} + \frac{1}{r_{o4}} \right\}$$

$$= \frac{g_{m1,2}\, r_{o1,2}\, R}{R + 2 r_{o1,2}} \cdot \frac{r_{o3}}{r_{o3} + 1/g_{m3}} \cdot \frac{1}{2 r_{o1,2} \,||\, R + \frac{1}{g_{m3}} \,||\, r_{o3}} \; V_{in}$$

$$A_v = \frac{V_{out}}{V_{in}} = \frac{\dfrac{g_{m1,2}\, r_{o1,2}\, R\, r_{o3}}{(R + 2 r_{o1,2})(r_{o3} + \frac{1}{g_{m3}})} \cdot \dfrac{1}{2 r_{o1,2} \,||\, R + \frac{1}{g_{m3}} \,||\, r_{o3}}}{\dfrac{r_{o3}}{r_{o3} + \frac{1}{g_{m3}}} \cdot \dfrac{1}{2 r_{o1,2} \,||\, R + \frac{1}{g_{m3}} \,||\, r_{o3}} + \dfrac{1}{r_{o4}}}$$

$$\therefore A_v = \frac{g_{m1,2}\, r_{o1,2}\, R\, r_{o3} r_{o4}}{R + 2 r_{o1,2}} \left\{ \frac{1}{r_{o3} r_{o4} + \left(r_{o3} + \frac{1}{g_{m3}}\right)\left(2 r_{o1,2} \,||\, R + \frac{1}{g_{m3}} \,||\, r_{o3}\right)} \right\}$$