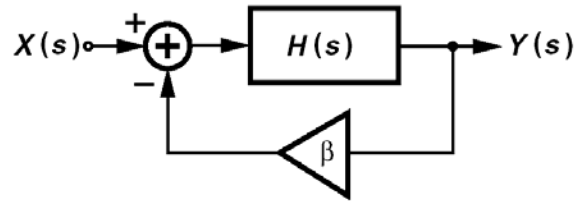


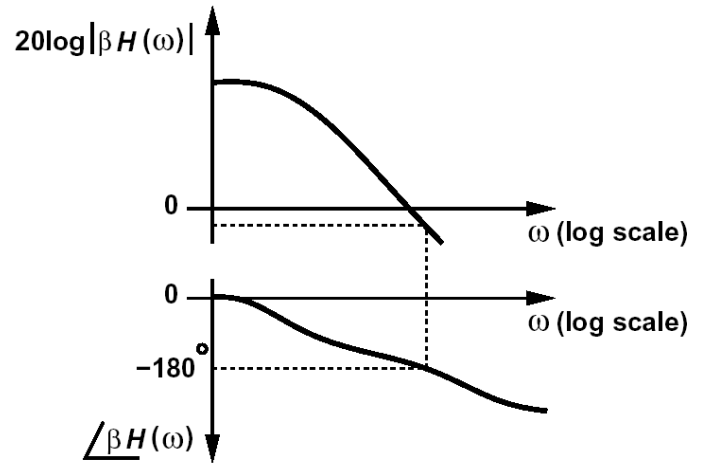
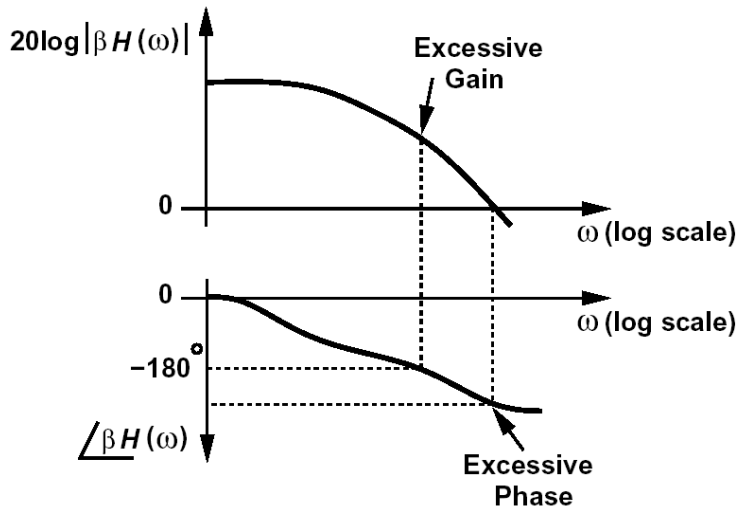
# Stability and Frequency Compensation

## When amplifiers go bad ...

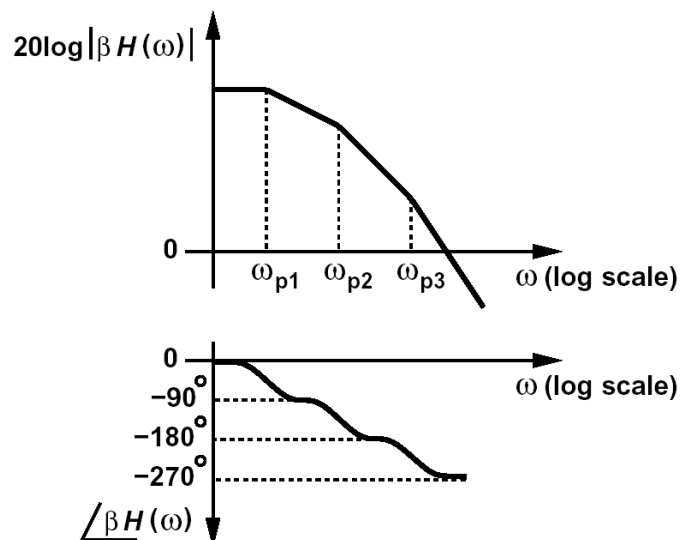
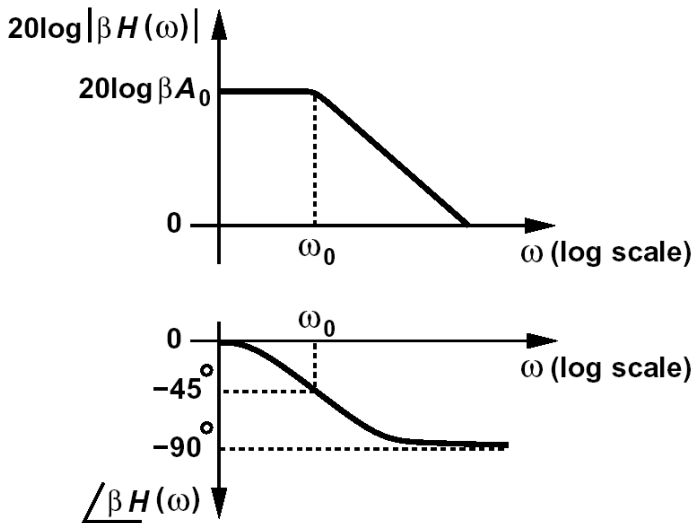
$$\frac{Y}{X}(s) = \frac{H(s)}{1 + \beta H(s)}$$



What happens if  $\beta H$  becomes equal to -1?

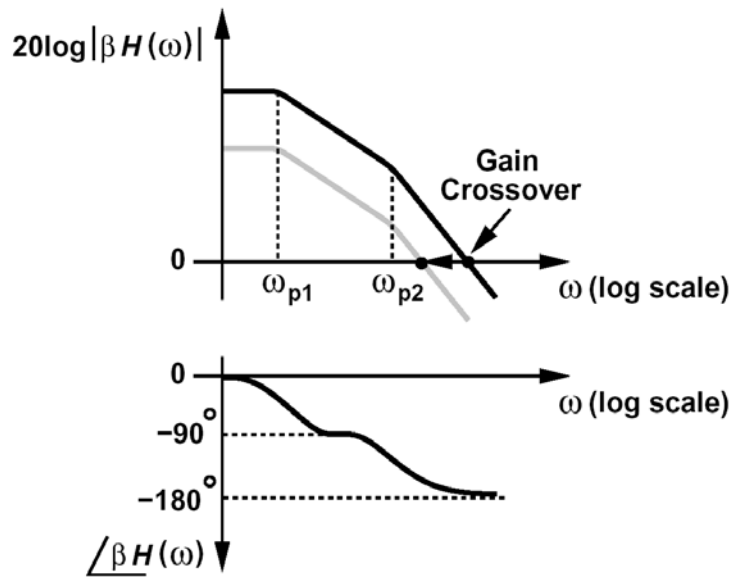


## One-Pole and Multiple-Pole Systems



**- Effect of Feedback Factor**

**We must consider the worst case:  $\beta = 1$ .**

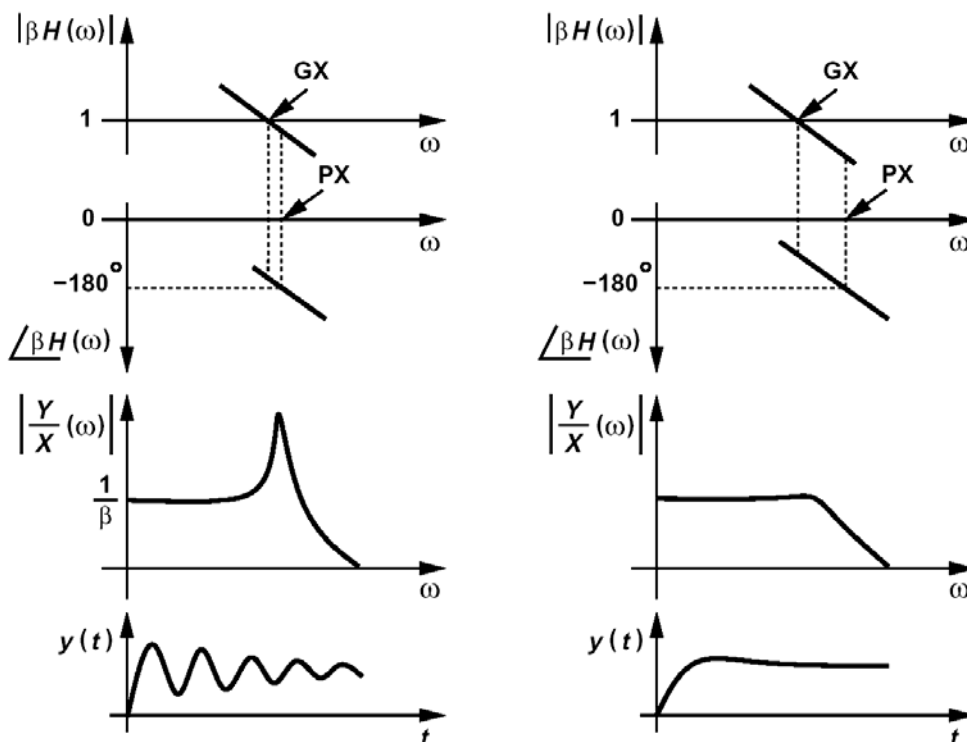


**Review of Bode Approximations**

- The slope of the magnitude changes by +20dB/dec at every zero frequency and by -20 dB/dec at every pole frequency.
- The phase begins to change at one-tenth of the pole (zero) frequency, changes by -45 degrees (+45 degrees) at the pole (zero), and approaches a -90-degree (+90-degree) change at 10 times the pole (zero) frequency.

**Phase Margin**

How far do we want to be from the “phase cross-over” point?



**Example**

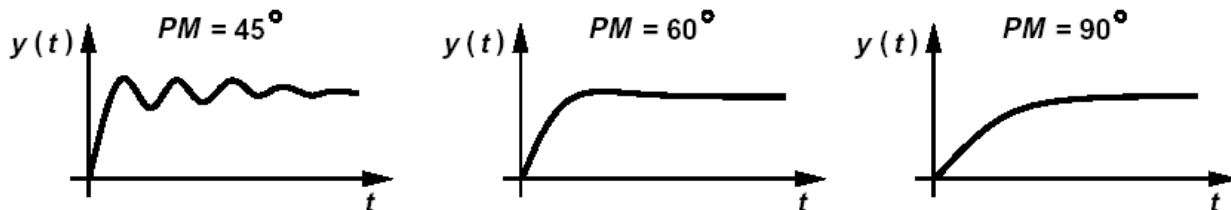
If the phase is equal to -175 degrees at GX, then  $\beta H(j\omega) = 1 \times \exp(-j175^\circ)$

$$\begin{aligned} \frac{Y}{X}(j\omega_1) &= \frac{H(j\omega_1)}{1 + \beta H(j\omega_1)} \\ &= \frac{1}{\beta} \frac{\exp(-j175^\circ)}{1 + \exp(-j175^\circ)} \\ &= \frac{1}{\beta} \cdot \frac{-0.9962 - j0.0872}{0.0038 - j0.0872} \end{aligned}$$

We define “phase margin” as 180 degrees +  $\angle \beta H(\omega = \omega_1)$

How much phase margin is adequate? If PM = 45 degrees, then

$$\begin{aligned} \frac{Y}{X} &= \frac{H(j\omega_1)}{1 + 1 \times \exp(-j135^\circ)} \\ &= \frac{H(j\omega_1)}{0.29 - 0.71j} \end{aligned}$$



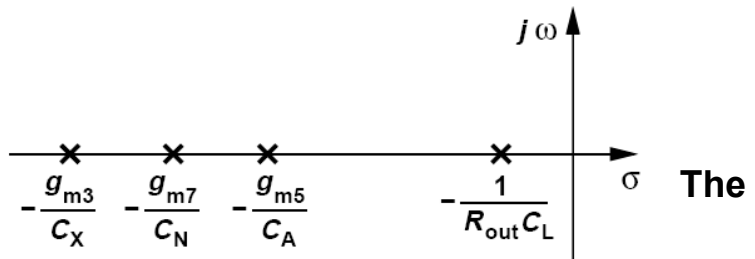
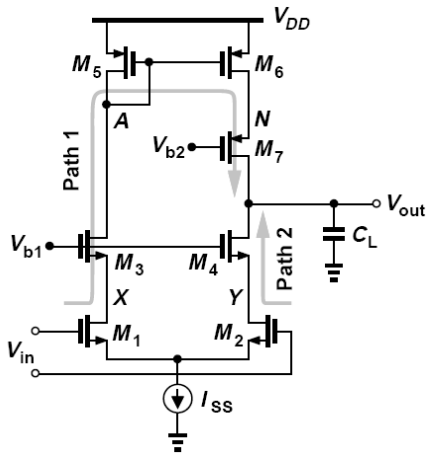
In practice, the phase margin does not reveal large-signal settling effects. So for most time-domain circuits, we use the phase margin only as a rough guideline.

**Frequency Compensation**

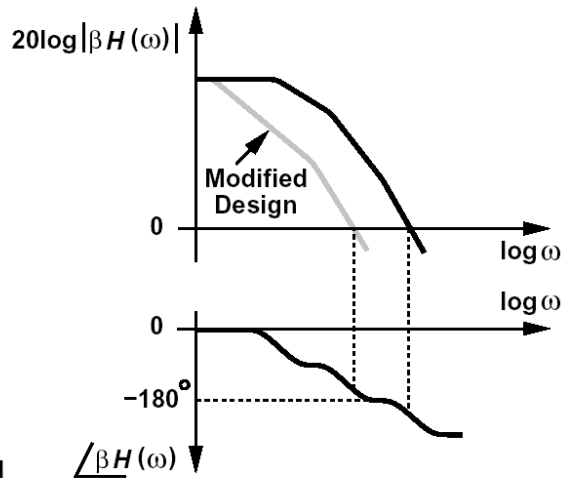
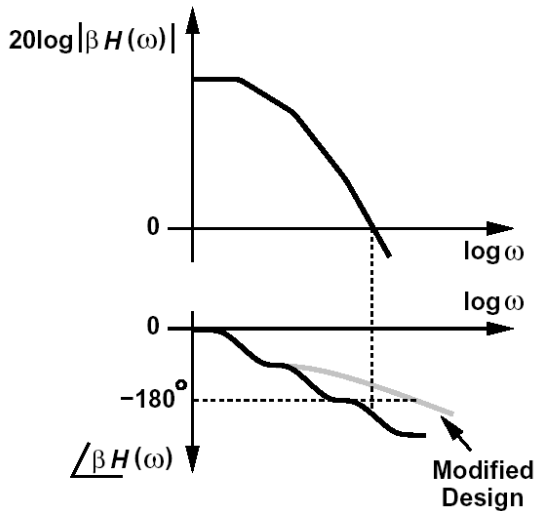
In order to achieve stability with acceptable frequency and time response, we must:

Minimize the number of poles while maintaining one dominant pole so that the phase shift near gain cross-over frequency is small.

**Example**

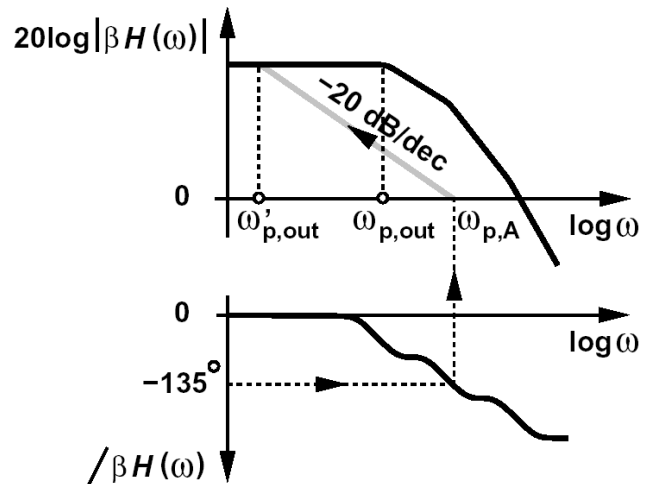


Frequency compensation means bring the dominant pole closer to the origin so that the gain drops sufficiently.

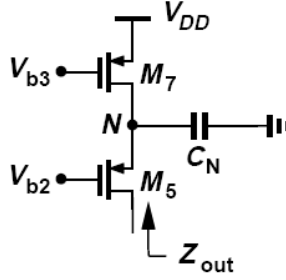
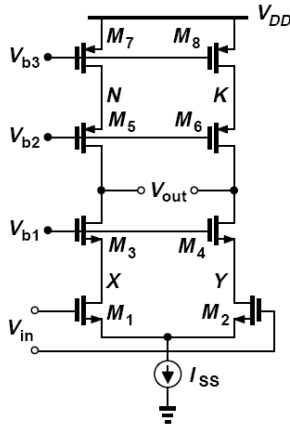


**Example**

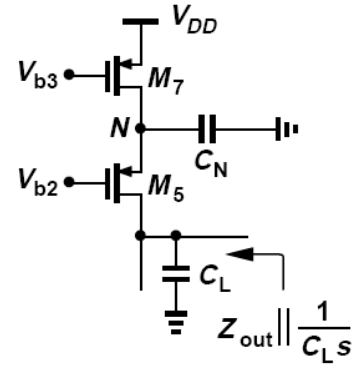
Compensate an op amp for a phase margin of 45 degrees.



- Does a telescopic op amp need compensation?



(a)

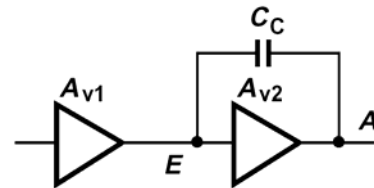
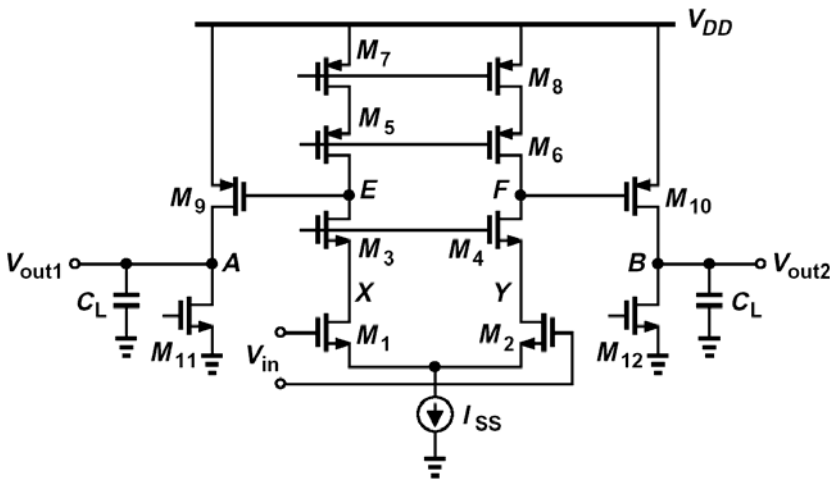


(b)

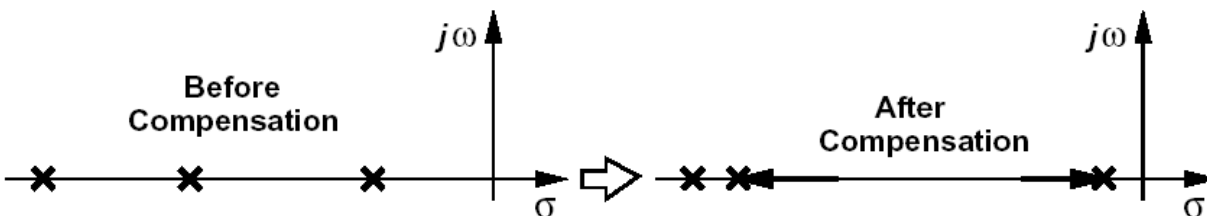
$$Z_{out} \parallel \frac{1}{C_L s} = \frac{(1 + g_{m5} r_{O5}) \frac{r_{O7}}{r_{O7} C_N s + 1} \cdot \frac{1}{C_L s}}{(1 + g_{m5} r_{O5}) \frac{r_{O7}}{r_{O7} C_N s + 1} + \frac{1}{C_L s}}$$

$$= \frac{(1 + g_{m5} r_{O5}) r_{O7}}{[(1 + g_{m5} r_{O5}) r_{O7} C_L + r_{O7} C_N] s + 1}$$

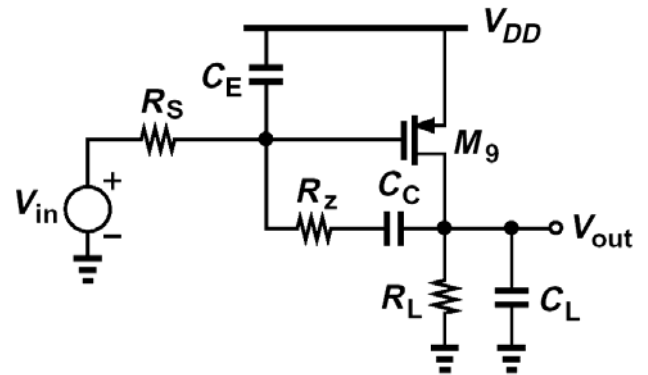
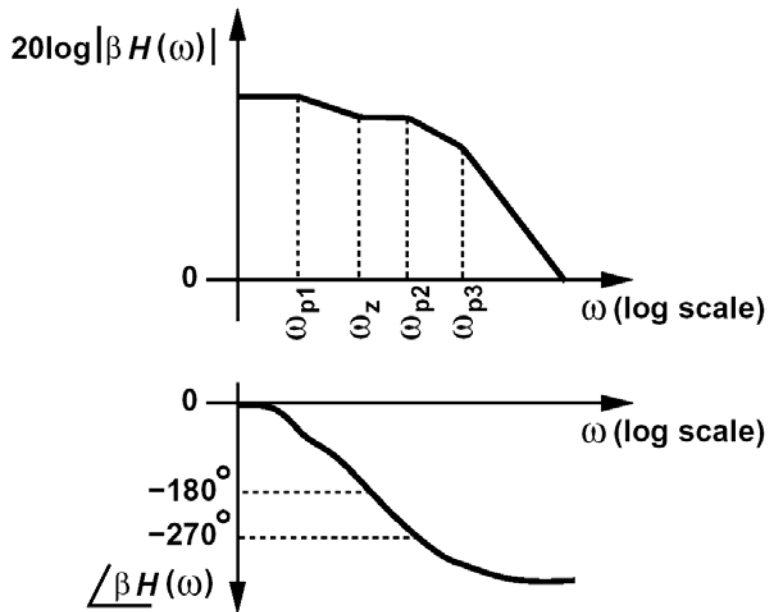
**Compensation of Two-Stage Op Amps**



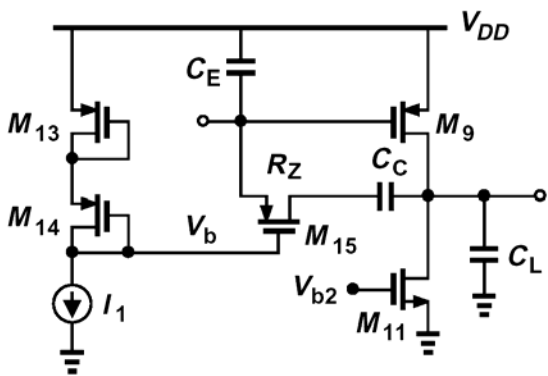
**Pole Splitting:**



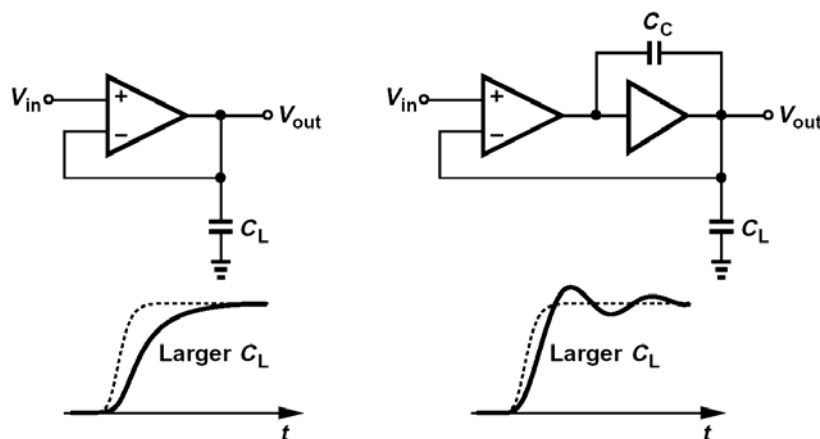
**Problem of Right-Half-Plane Zero**



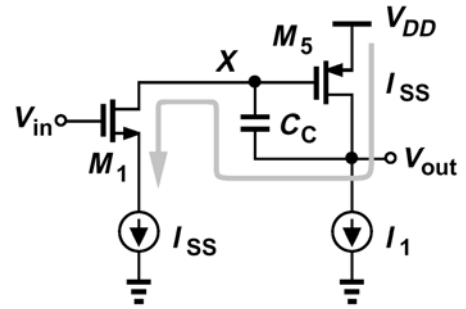
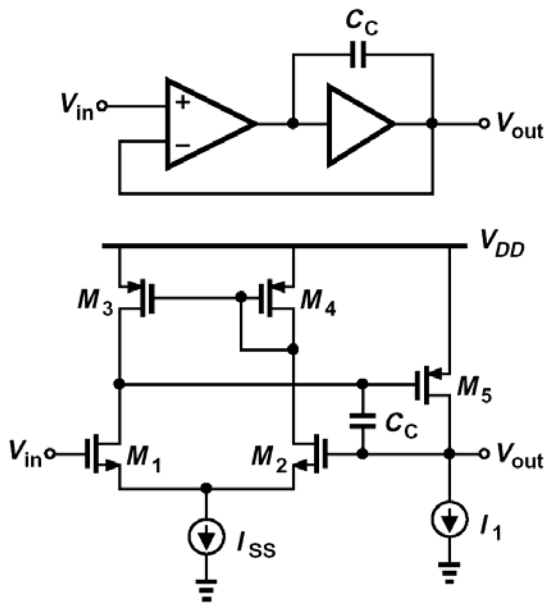
**How do we make  $R_z$  track transistors?**



**Effect of Load Capacitance**



### Slewing in Two-Stage Op Amps



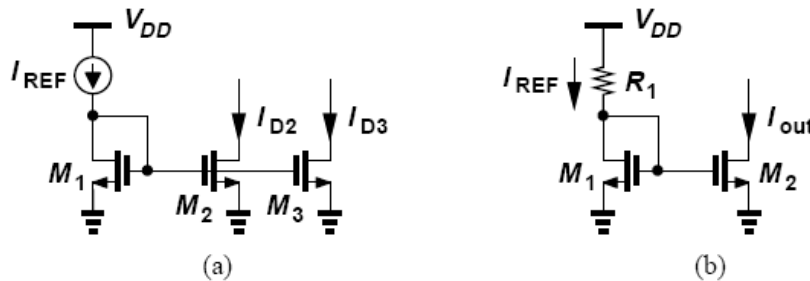
# Bandgap References

## Applications

- Bias currents and voltages in analog circuits: mirrors, diff pairs, common-mode levels, etc.
- Reference voltage or current in A/D and D/A converters.

## Reference Generation

The goal is to establish an ac voltage or current that is independent of supply voltage and process parameters and has a well-defined dependence on temperature.



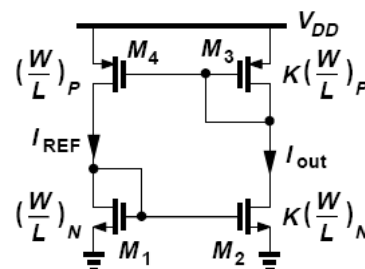
The dependence on temperature assumes one of two forms:

- proportioned to absolute temperature (PTAT)
- temperature independent (TI)

Once we generate a PTAT or TI voltage, we can use this configuration to produce corresponding current:

Unfortunately, on-chip resistors experience 15-20% variation.

## Supply-Independent Biasing

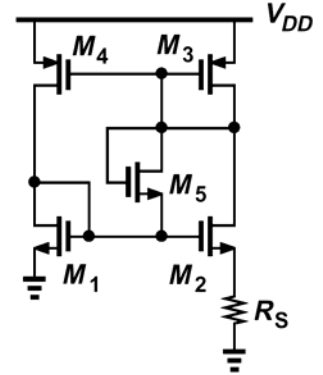


$$\sqrt{\frac{2I_{out}}{\mu_n C_{ox} (W/L)_N}} + V_{TH1} = \sqrt{\frac{2I_{out}}{\mu_n C_{ox} K (W/L)_N}} + V_{TH2} + I_{out} R_S.$$



Channel-length modulation results in a finite supply rejection → use long devices.

- Problem of Startup



We will see other supply-independent schemes later.

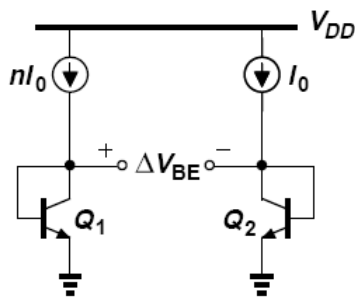
Temperature-Independent Reference

Basic Idea: Combine two phenomena that vary with T in opposite directions, e.g., add two voltages with positive and negative TCs, such that the sum has zero TC:

$$V_{REF} = \alpha_1 V_1 + \alpha_2 V_2 \quad \alpha_1 \partial V_1 / \partial T + \alpha_2 \partial V_2 / \partial T = 0$$

- Positive TC Voltage

$\Delta V_{BE}$  of two bipolar transistors operating at different current densities:



$$\begin{aligned} \Delta V_{BE} &= V_{BE1} - V_{BE2} \\ &= V_T \ln \frac{nI_0}{I_{S1}} - V_T \ln \frac{I_0}{I_{S2}} \\ &= V_T \ln n. \end{aligned}$$

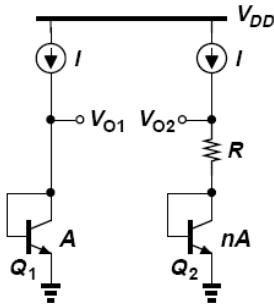
What happens if emitter area of Q<sub>2</sub> is m times that of Q<sub>1</sub>?

- Negative TC Voltage

$$\begin{aligned} \frac{\partial V_{BE}}{\partial T} &= \frac{V_T}{T} \ln \frac{I_C}{I_S} - (4 + m) \frac{V_T}{T} - \frac{E_g}{kT^2} V_T \\ &= \frac{V_{BE} - (4 + m)V_T - E_g/q}{T}. \end{aligned}$$

Since  $V_T$  has a TC = 0.087 mV/°K, we need a total of  $\sim 17.2V_T$  to add to  $V_{BE}$  so that the net TC  $\rightarrow 0 \rightarrow V_{REF} \approx V_{BE} + 17.2V_T \approx 1.25 \text{ V}$ .

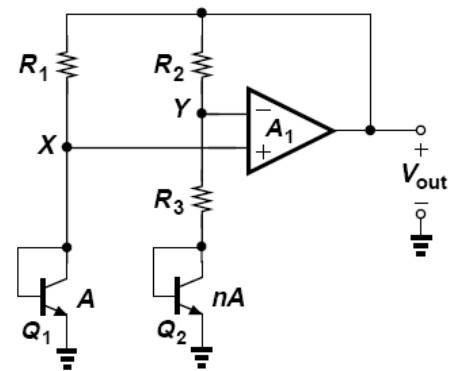
**Conceptual Scheme: Suppose  $V_{O1}$  and  $V_{O2}$  are somehow made equal:**



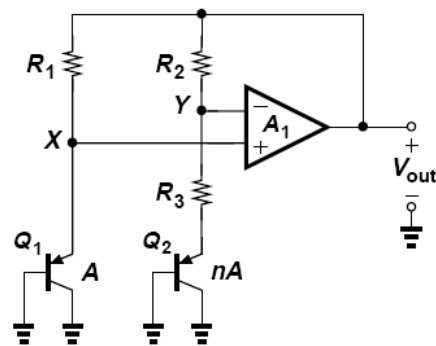
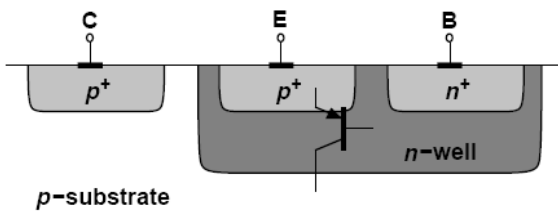
**How do we make the  $V_{O1}$  and  $V_{O2}$  equal?  
How do we avoid  $\ln n = 17.2$ ?**

$$V_{out} = V_{BE2} + \frac{V_T \ln n}{R_3} (R_3 + R_2)$$

$$= V_{BE2} + (V_T \ln n) \left(1 + \frac{R_2}{R_3}\right)$$

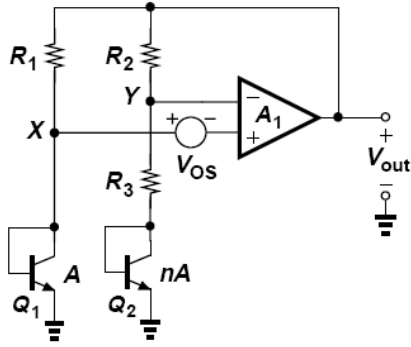


**Bipolar characteristics are essential here. Attempts at building purely CMOS precision references have not been very successful.  
How do we build this in CMOS?**



### Effect of Nonidealities

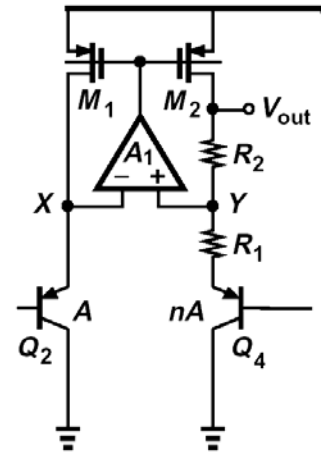
**The op amp nonidealities introduce various errors here. Let's consider the effect of offset.**



How do we alleviate this issue?

Is the feedback positive or negative?!

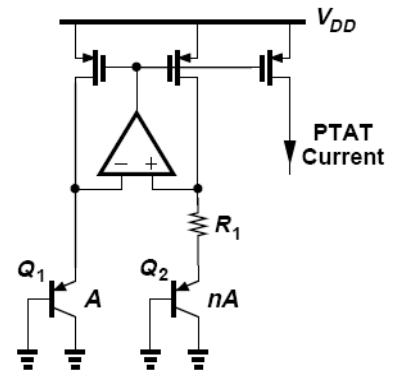
How about the output impedance of the op amp?



How about the supply rejection of the op amp itself?

Why is it called “bandgap?”  $V_{REF} = \frac{E_g}{q} + (4 + m)V_T$

PTAT Current Generation



## Low-Voltage Bandgap

How do generate a bandgap voltage less than 1.25 V?  
[Banba, JSSC, May 99] Assume  $R_3 = R_2$ .

