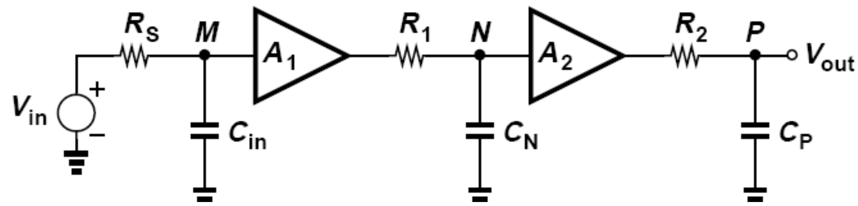
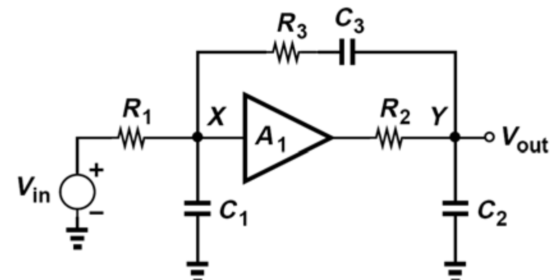
B. Razavi

Frequency Response of Amplifiers

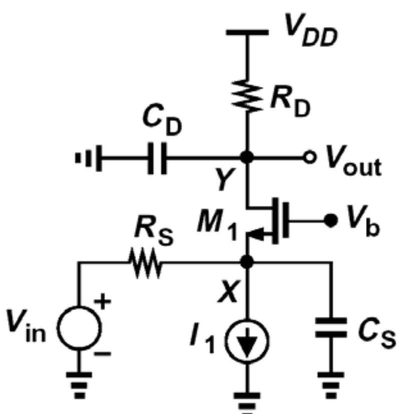
Association of Poles with Nodes



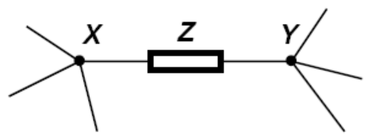
$$\frac{V_{out}}{V_{in}}(s) = \frac{A_1}{1+R_S C_{in} s}\cdot\frac{A_2}{1+R_1 C_N s}\cdot\frac{1}{1+R_2 C_P s}$$



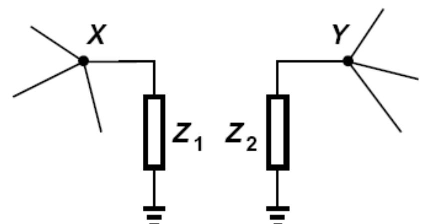
Example



Miller Effect



(a)

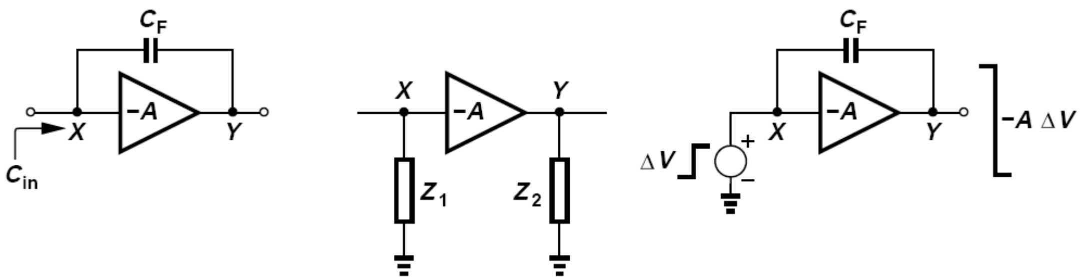


(b)

$$Z_1 = \frac{Z}{1 - \frac{V_Y}{V_X}} \quad Z_2 = \frac{Z}{1 - \frac{V_X}{V_Y}}$$

- **Strictly speaking, the gain must be calculated at the frequency of interest.**
- **Not every circuit lends itself to Miller decomposition.**

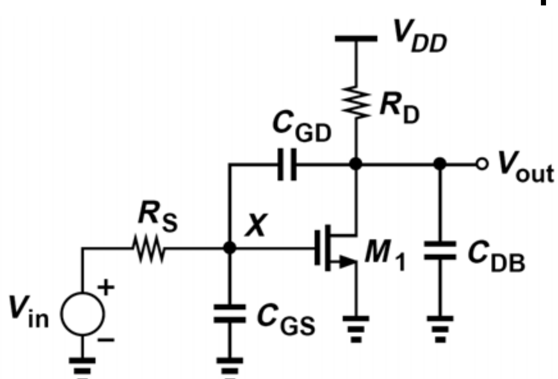
- **Example**



(a) (b) (c)

Does the circuit have two poles?!

<u>Common- Source Stage</u>
(Also half circuit of a differential pair) Use of Miller's Theorem:

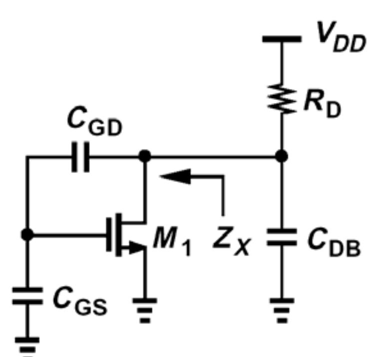


$$\omega_{in} = \frac{1}{R_S[C_{GS} + (1 + g_m R_D)C_{GD}]}.$$

$$\omega_{out} = \frac{1}{R_D(C_{DB} + C_{GD})}$$

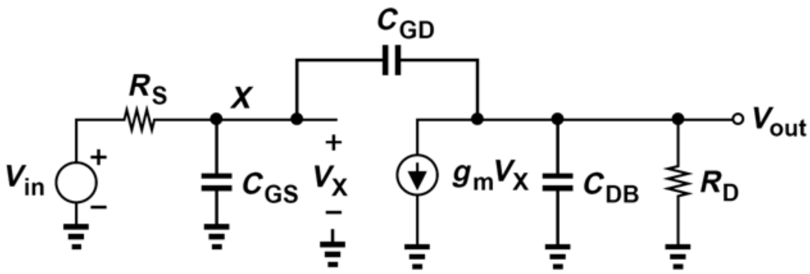
If R_s is relatively large,

$$Z_X = \frac{1}{C_{eq}s} || \left(\frac{C_{GD}}{C_{GD} + C_{GS}} \cdot \frac{1}{g_{m1}} \right)$$



$$\omega_{out} = \frac{1}{[R_D || \left(\frac{C_{GD}}{C_{GD}+C_{GS}} \cdot \frac{1}{g_{m1}}\right)](C_{eq}+C_{DB})}$$

Exact Analysis:



$$\frac{V_{out}}{V_{in}(s)} = \frac{(C_{GD}s - g_m)R_D}{R_S R_D \xi s^2 + [R_S(1+g_m R_D)C_{GD} + R_S C_{GS} + R_D(C_{GD}+C_{DB})]s + 1}$$

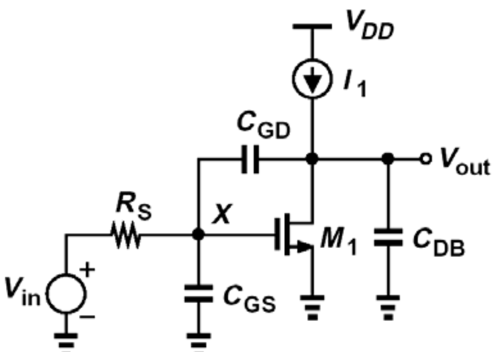
$$\xi = C_{GS}C_{GD} + C_{GS}C_{DB} + C_{GD}C_{DB}$$

The circuit has one zero and two poles even though there are three caps.

Dominant Pole Approximation:

$$\omega_{p1} = \frac{1}{R_S(1+g_m R_D)C_{GD} + R_S C_{GS} + R_D(C_{GD}+C_{DB})}$$

<u>Example</u>



$$\omega_2 \approx \frac{(1+g_m R_S)C_{GD} + C_{DB}}{R_S(C_{GD}C_{GS} + C_{GS}C_{DB} + C_{GD}C_{DB})}$$

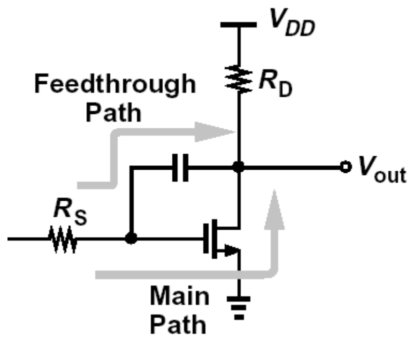
A few notes on CS response:

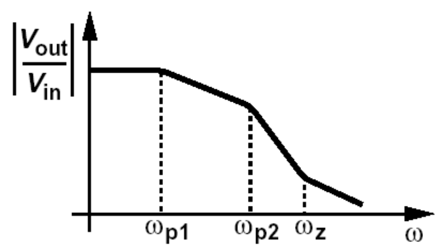
1. Second pole can also be found using the dominant pole appr.:

$$\begin{aligned}\omega_{p2} &= \frac{1}{\omega_{p1}} \cdot \frac{1}{R_S R_D (C_{GS}C_{GD} + C_{GS}C_{DB} + C_{GD}C_{DB})} \\ &= \frac{R_S(1+g_m R_D)C_{GD} + R_S C_{GS} + R_D(C_{GD}+C_{DB})}{R_S R_D (C_{GS}C_{GD} + C_{GS}C_{DB} + C_{GD}C_{DB})}\end{aligned}$$

If C_{GS} is large enough,

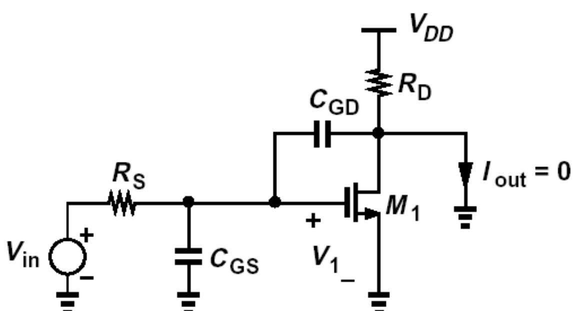
2. The zero arises from the direct coupling through C_{GD}:





This right-half plane zero causes trouble in two-stage op amp design.

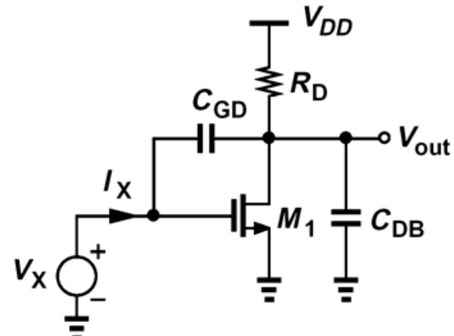
The zero can also be calculated by noting that



3. The input impedance can be estimated using Miller's Theorem:

$$Z_{in} = \frac{1}{[C_{GS} + (1+g_m R_D)C_{GD}]s}.$$

Or more precisely:



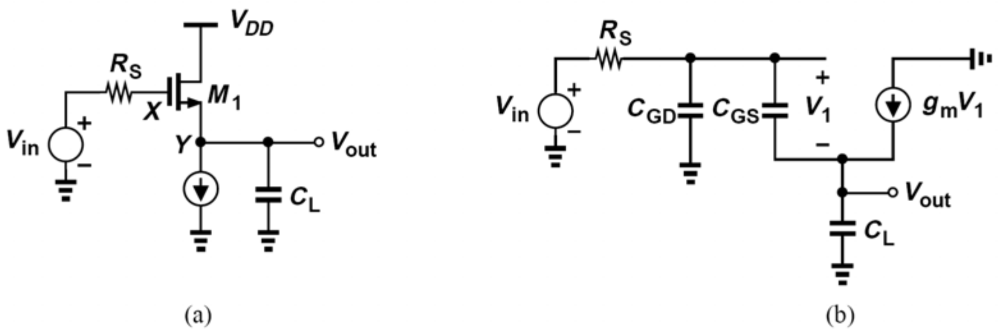
$$\frac{V_X}{I_X} = \frac{1 + R_D(C_{GD}+C_{DB})s}{C_{GD}s(1 + g_m R_D + R_D C_{DB} s)}$$

If the frequency is low enough such that:

$$|R_D(C_{GD}+C_{DB})s| \ll 1 \text{ and } |R_D C_{DB} s| \ll 1 + g_m R_D$$

then Miller multiplication works. What happens if C_{GD} is very large?

Source Follower



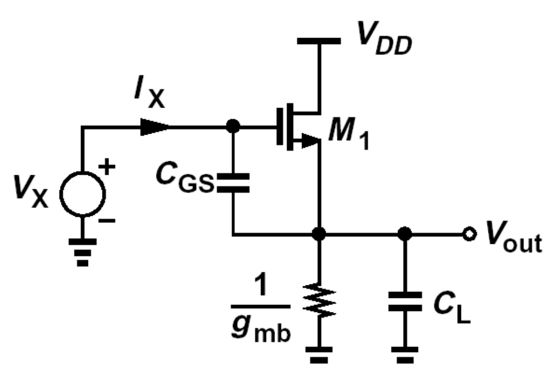
$$\frac{V_{out}}{V_{in}}(s) = \frac{g_m + C_{GS}s}{R_S(C_{GS}C_L + C_{GS}C_{GD} + C_{GD}C_L)s^2 + (g_m R_S C_{GD} + C_L + C_{GS})s + g_m}$$

The zero is in the left-half plane (why?). Can use dominant pole appr. to estimate the poles (but in practice does not happen often):

$$\omega_{p1} \approx \frac{g_m}{g_m R_S C_{GD} + C_L + C_{GS}} = \frac{1}{R_S C_{GD} + \frac{C_L + C_{GS}}{g_m}}.$$

With Rs = 0,

- Input Impedance



$$V_X = \frac{I_X}{C_{GS}s} + \left(\frac{I_X}{C_{GS}s} + \frac{g_m I_X}{C_{GS}s}\right)\left(\frac{1}{g_{mb}} \| \frac{1}{C_L s}\right)$$

$$Z_{in} = \frac{1}{C_{GS}s} + \left(1 + \frac{g_m}{C_{GS}s}\right)\frac{1}{g_{mb} + C_L s}$$

Special cases:

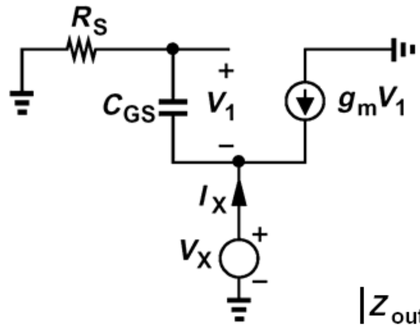
1. At low freqs:

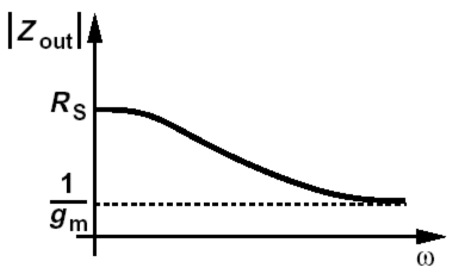
$$Z_{in} \approx \frac{1}{C_{GS}s}(1+\frac{g_m}{g_{mb}})+\frac{1}{g_{mb}}.$$

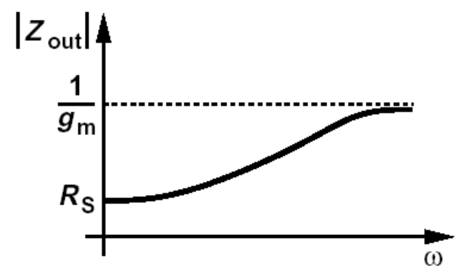
2. At high freqs:

$$Z_{in} \approx \frac{1}{C_{GS}s}+\frac{1}{C_Ls}+\frac{g_m}{C_{GS}C_Ls^2}$$

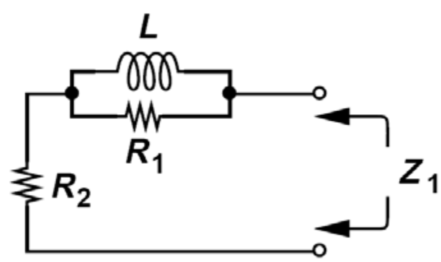
- Output Impedance







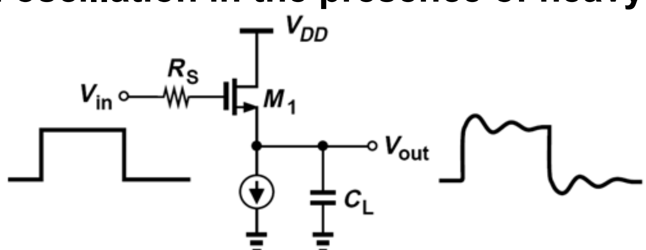
Can we represent Z_{out} with a passive RL network? Since Z_{out} is of first order, it can have only one inductor and no other storage element. Considering the cases at f = o and f = ∞, we arrive at this equivalent circuit:



$$Z_{out}-\frac{1}{g_m}=\frac{C_{GS}s(R_S-\frac{1}{g_m})}{g_m+C_{GS}s}$$

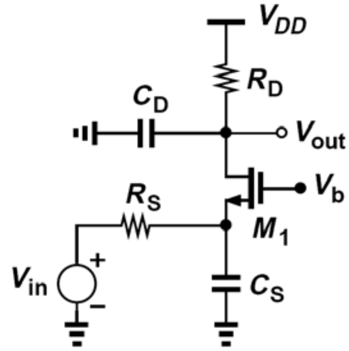
$$L=\frac{C_{GS}}{g_m}(R_S-\frac{1}{g_m})$$

The inductive behavior resulting from finite source impedance may cause significant ringing or even oscillation in the presence of heavy load capacitance.



Common-Gate Stage

(No Miller effect)

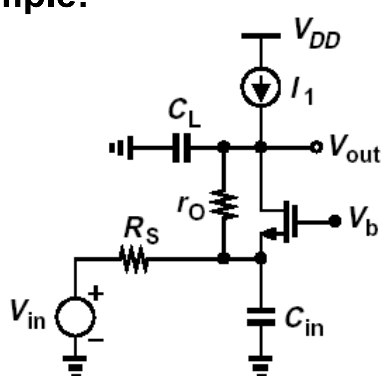


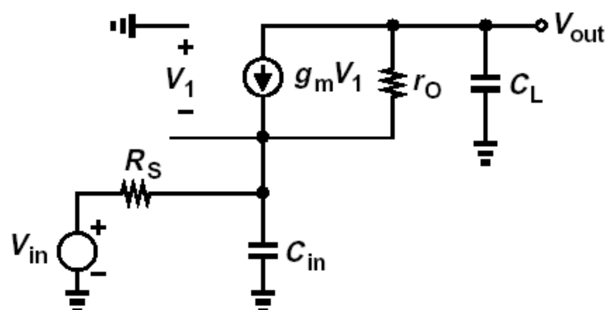
$$\frac{V_{out}}{V_{in}}(s) = \frac{(g_m + g_{mb})R_D}{1 + (g_m + g_{mb})R_S}\frac{1}{(1 + \frac{C_S}{g_m + g_{mb} + R_S^{-1}}s)(1 + R_D C_D s)}$$

Input impedance in the presence of channel-length modulation:

$$Z_{in} \approx \frac{Z_L}{(g_m + g_{mb})r_O} + \frac{1}{g_m + g_{mb}}$$

Example:



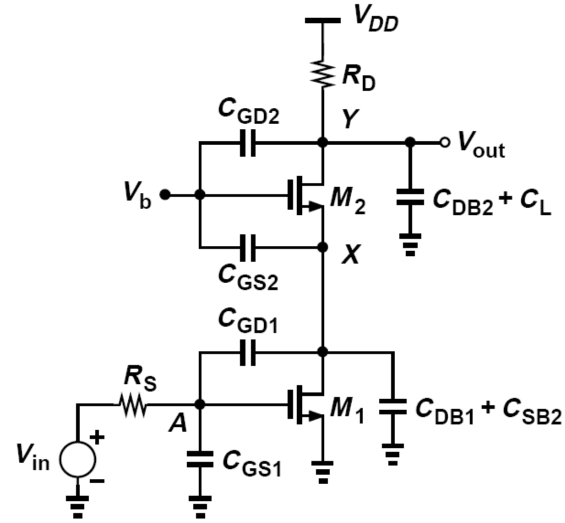


$$(-V_{out}C_L s + V_1 C_{in} s)R_S + V_{in} = -V_1 \qquad V_1 = -\frac{-V_{out}C_L s R_S + V_{in}}{1 + C_{in}R_S s}$$

$$\frac{V_{out}}{V_{in}}(s) = \frac{1 + g_m r_O}{r_O C_L C_{in} R_S s^2 + [r_O C_L + C_{in} R_S + (1 + g_m r_O)C_L R_S]s + 1}$$

$$Z_{in} = \frac{1}{g_m + g_{mb}} + \frac{1}{C_L s}\cdot\frac{1}{(g_m + g_{mb})r_O}.$$

Why does Zin become independent of C_L at high frequencies?



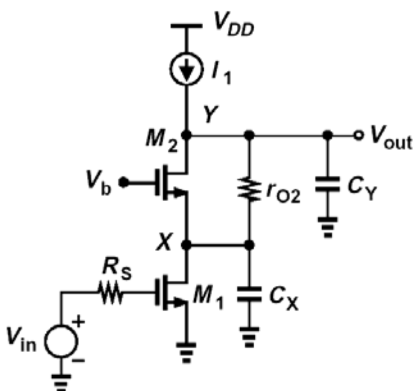
Cascode Stage

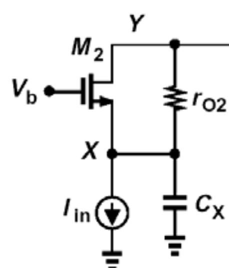
Miller effect much less significant here. Why?

$$\omega_{p,A} = \frac{1}{R_S[C_{GS1} + (1 + \frac{g_{m1}}{g_{m2} + g_{mb2}})C_{GD1}]}$$

Poles at X and Y:

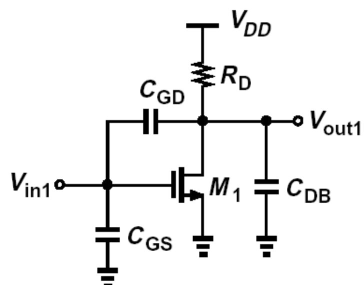
Example:

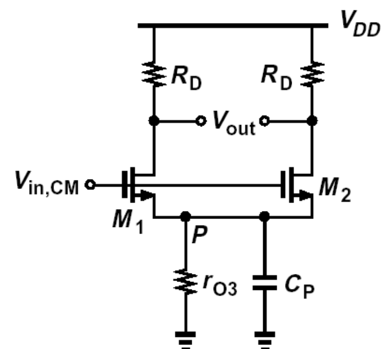


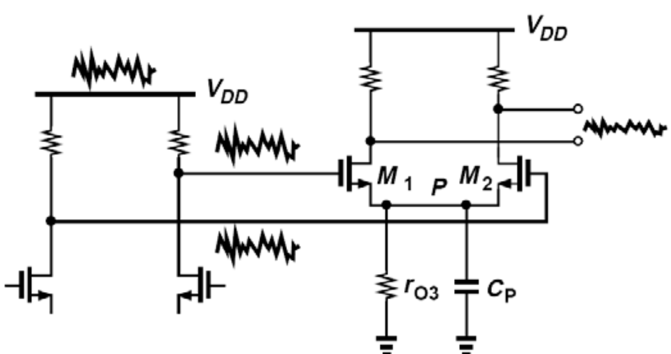


$$\frac{V_{out}}{I_{in}} = -\frac{g_{m2}r_{O2} + 1}{C_X s} \cdot \frac{1}{\frac{C_Y}{C_S} g_{m2} r_{O2} + C_Y r_{O2} s}$$

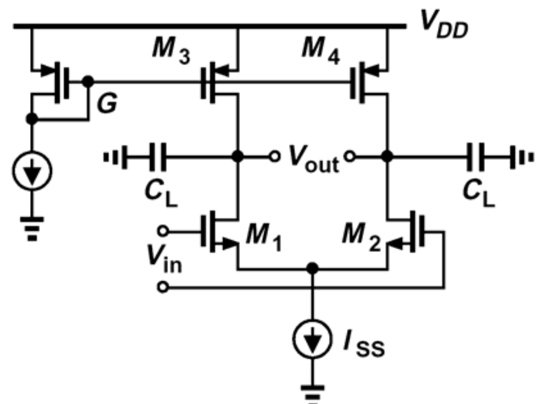
Differential Pair



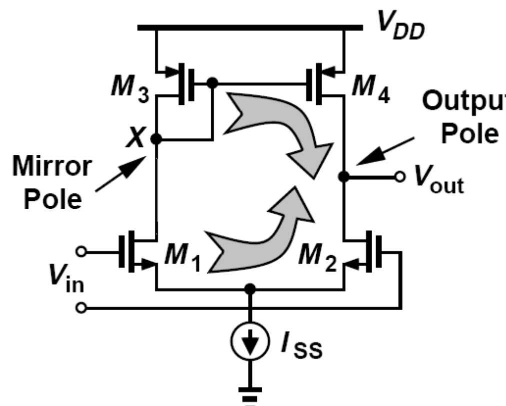




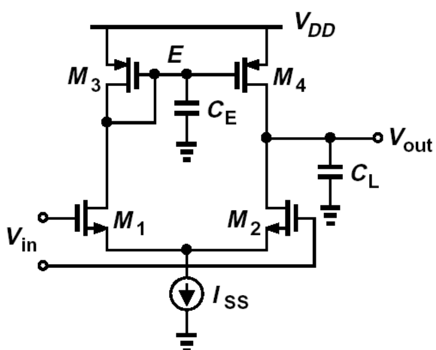
Diff pair with current-source loads:

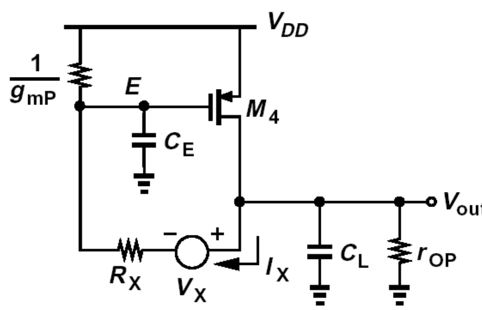


<u>Diff Pair with Active Load</u>



Mirror pole is not very far, degrading phase margin of op amps.





Comparison of fully-differential pair and circuit with active load:

Two poles:	Three poles:
- Miller effect at input	- Miller effect at V_{in2}
- Miller effect at input	- Load cap at V_{out}
- Load cap at output	- Time constant at X