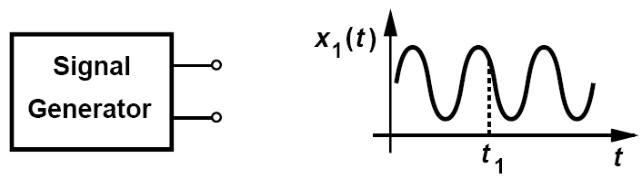
Noise

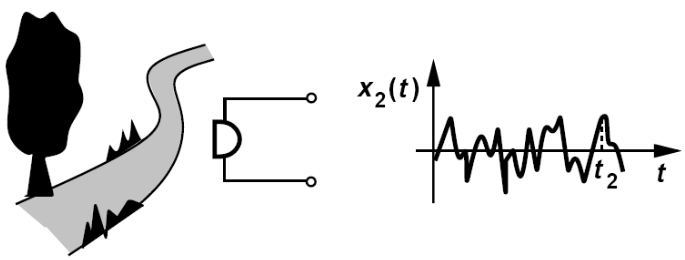
What is Noise?

Noise is a random (more accurately a stochastic) process. We consider a phenomenon random because we do not know everything about it, or simply because we do not need to know everything about it.

Since the instantaneous noise amplitude is not known, we resort to "statistical" models, i.e., some properties that can be predicted.



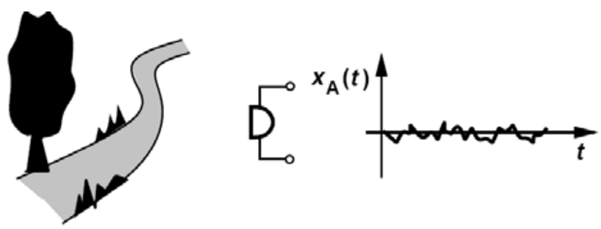
(a)



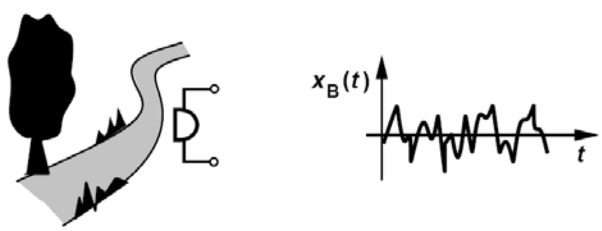
(b)

Average Power

Larger fluctuations mean that the noise is "stronger."

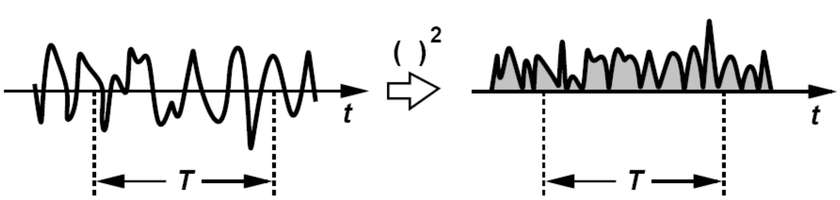


(a)



(b)

$$P_{av} = \lim_{T\to\infty} \frac{1}{T} \int_{-T/2}^{+T/2} \frac{x^2(t)}{R_L} dt,$$



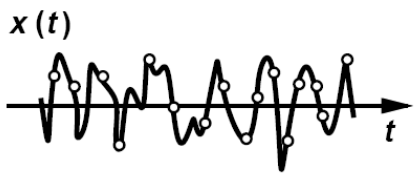
Normalized average power:

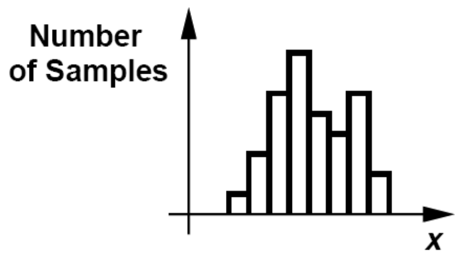
$$P_{av} = \lim_{T\to\infty} \frac{1}{T} \int_{-T/2}^{+T/2} x^2(t) dt,$$

Statistical Characterization

1. Time Domain

By sampling the time-domain waveform for a long time, we can construct a "probability density function" (PDF). The PDF in essence indicates "how often" the amplitude is between certain limits.

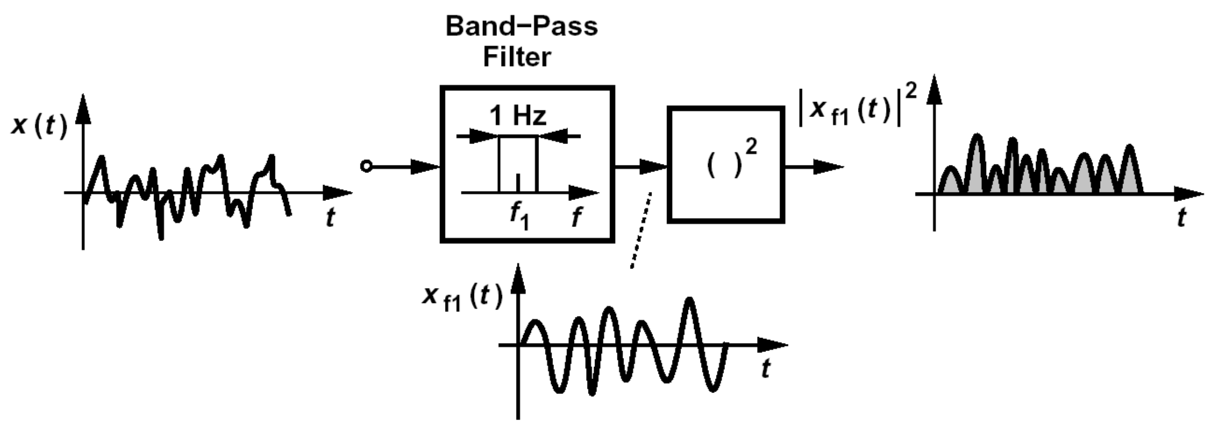




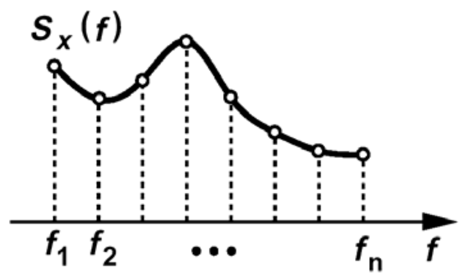
For example, a Gaussian distribution is defined by a mean and a standard deviation. We say the noise amplitude rarely exceeds 4σ.

2. Frequency Domain

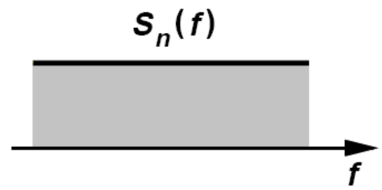
For random signals, the concept of Fourier transform cannot be directly applied. But we still know that men carry less high-frequency components in their voice than women do. The
We define the "power spectral density" (PSD) (also called the "spectrum") as:



The PSD thus indicates how much power the signal carries in a small bandwidth around each frequency.



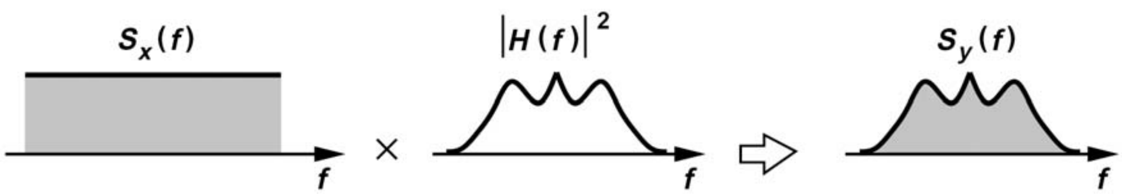
Example: Thermal Noise Voltage of a Resistor



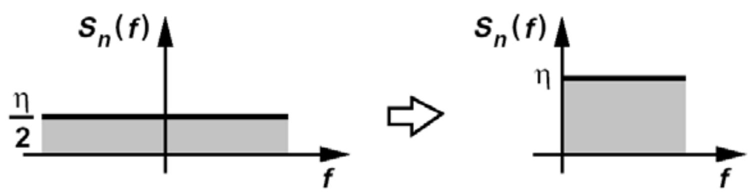
A flat spectrum is called "white."

- Is the total noise power infinite?
- What is the total noise power in 1 Hz?
- What is the unit of S(f)?

Important Theorem

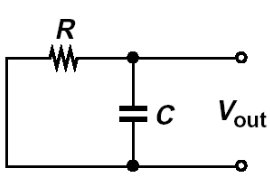


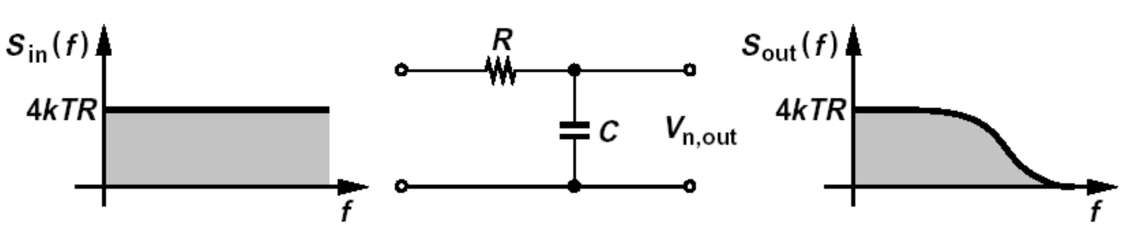
For mathematical convenience, we may "fold" the spectrum as shown here:



Example

Calculate the total rms noise at the output of this circuit.





Note:

- **The PDF and PSD generally bear no relationship:**

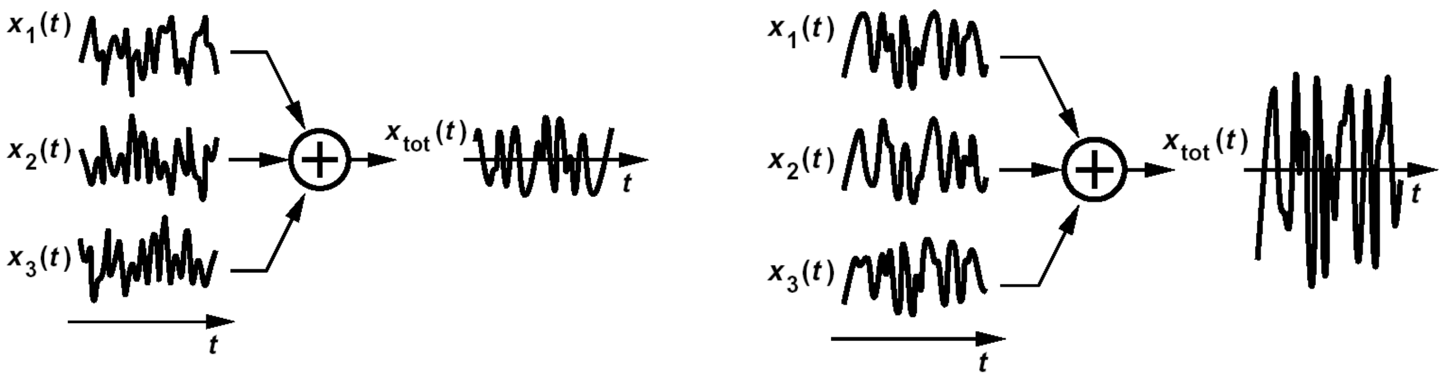
Thermal Noise: Gaussian, white
"Flicker" Noise: Gaussian, not white

Correlated and Uncorrelated Sources

Can we use superposition for noise components?

$$P_{av} = \lim_{T\to\infty} \frac{1}{T} \int_{-T/2}^{+T/2} [x_1(t) + x_2(t)]^2 dt$$

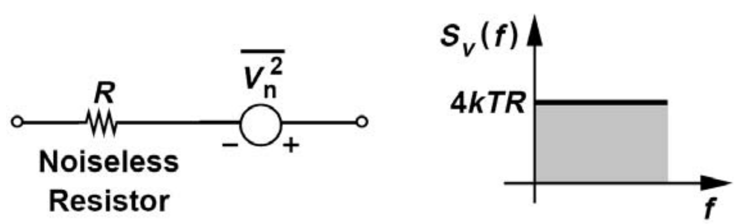
We occasionally encounter correlated sources:



Types of Noise

1. Thermal Noise

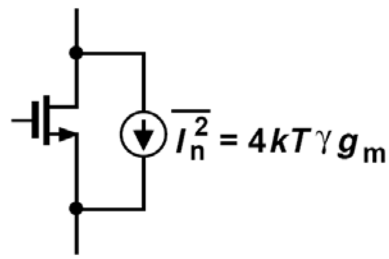
Random movement of charge carriers in a resistor causes fluctuations in the current. The PDF is Gaussian because there are so many carriers. The PSD is given by:



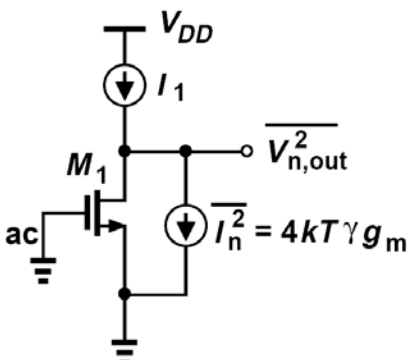
Note that the polarity of the voltage source is arbitrary.

- **Example: A 50-Ω resistor at room temperature exhibits an RMS noise voltage of .**
  If this resistor is used in a system with 1-MHz bandwidth, then it contributes a total rms voltage of .

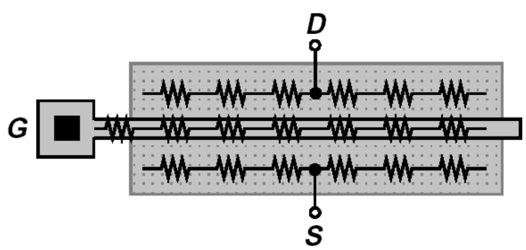
The ohmic resistances in transistors contribute thermal noise:

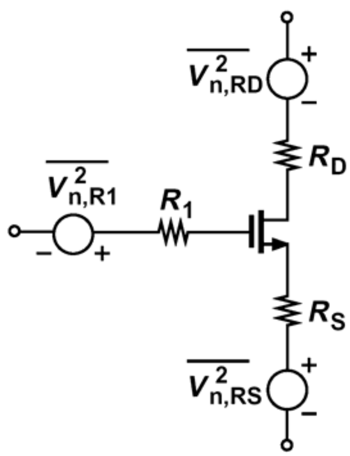


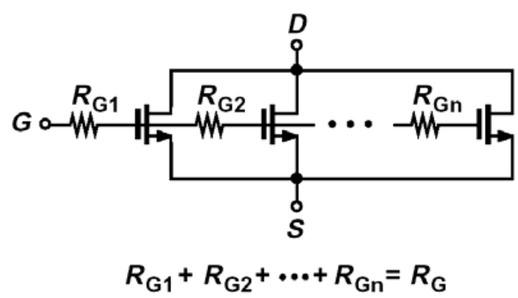
Example:



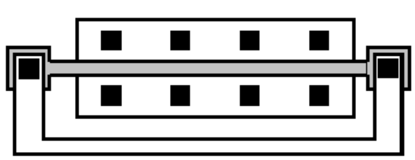
The ohmic sections also contribute thermal noise:

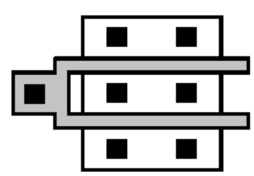






In a well-designed layout, only the channel thermal (and flicker) noise may be dominant:





2. Shot Noise

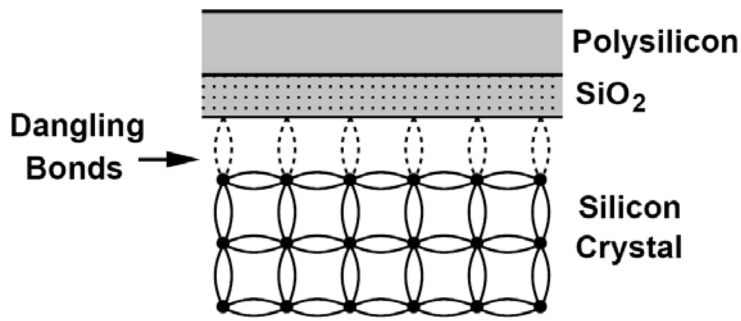
If carriers cross a potential barrier, then the overall current actually consists of a large number of random current pulses. . The random component of the current is called “shot noise” and given by:

Note that shot noise does not depend on the temperature.

Shot noise occurs in pn-junction diodes, bipolar transistors, and MOSFETs operating in subthreshold region.

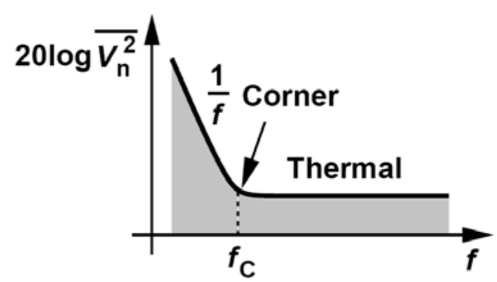
3. Flicker (1/f) Noise

In MOSFETs, the extra energy states at the interface between silicon and oxide trap and release carriers randomly and at different rates. The noise in the drain current is Gaussian, but its spectrum is given by:



Where k is a constant and its value heavily depends on how “clean” the process is. We often characterize the seriousness of 1/f noise by considering the 1/f “corner” frequency.

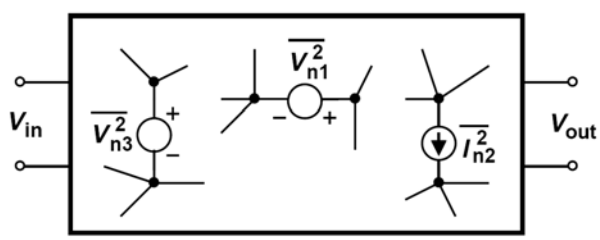
$$\overline{V_n^2} = \frac{K}{C_{ox}WL} \cdot \frac{1}{f}$$

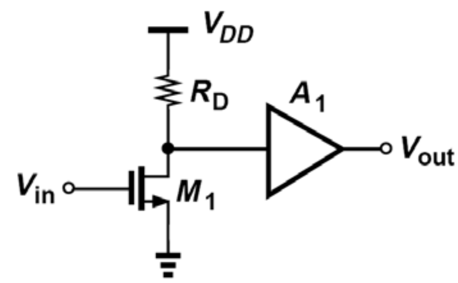


<u>Example</u>

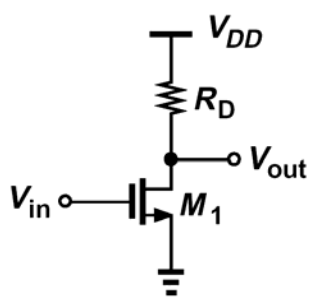
Calculate the total thermal and 1/f noise in the drain current of a MOSFET for a band from 1 kHz to 1 MHz.

Representation of Noise in Circuits



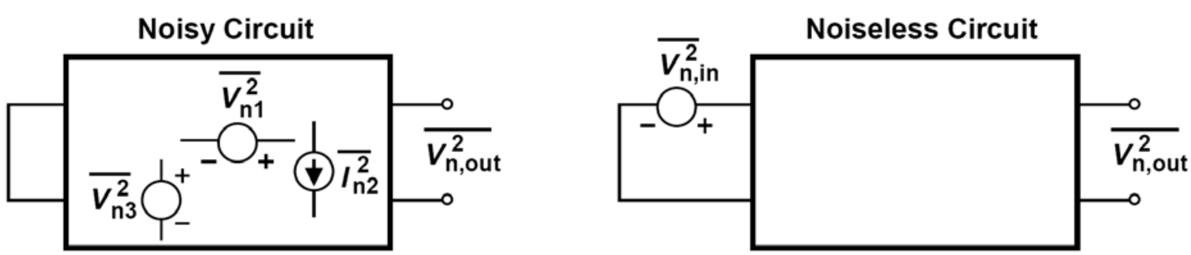


Example



Input-Referred Noise

Input-referred noise is the noise voltage or current that, when applied to the input of the noiseless circuit, generates the same output noise as the actual circuit does.

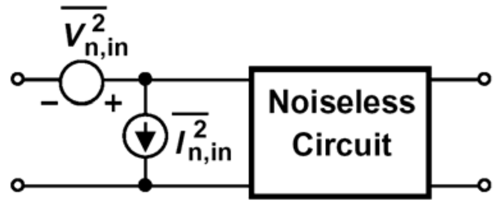


(a) (b)

Example

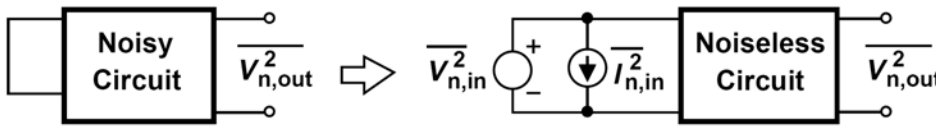
Find the input-referred noise voltage of the above circuit.

In general, we need both a voltage source and a current source at the input to model the circuit noise:

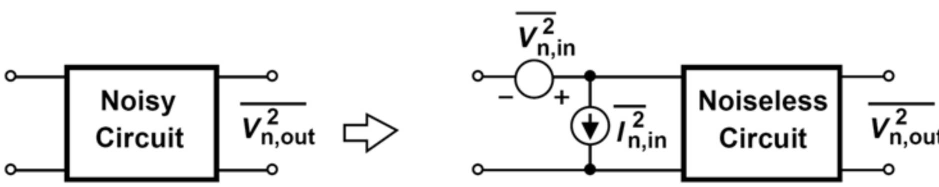


If the source impedance is high with respect to the input impedance of the circuit, then both must be considered.

- **How do we calculate the input-referred noise?**



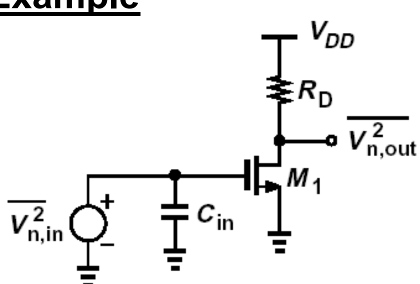
(a)



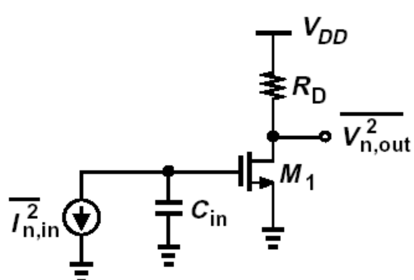
(b)

Important Note: These two components may be correlated in many cases.

<u>Example</u>



(a)



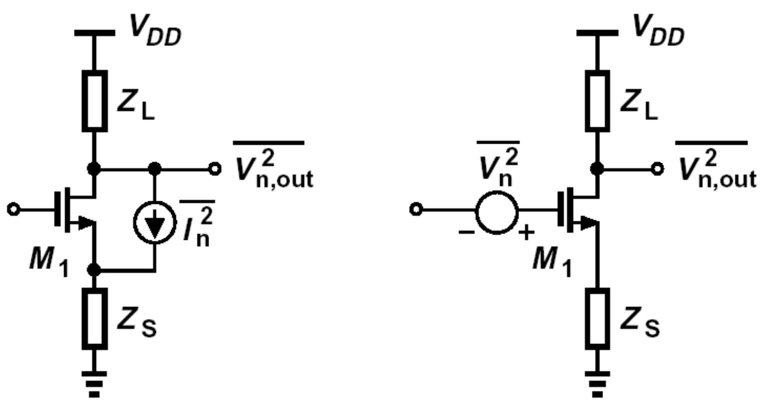
(b)

Note: Since the total noise depends on the bandwidth, we wish to minimize the bandwidth of each circuit → low-noise design becomes more difficult as the required speed goes up.

Noise in Single-Stage Amps

Useful Lemma

The following two circuits are equivalent if $\overline{V_n^2} = \overline{I_n^2}/g_m^2$



This lemma allows us to place the source of the noise at the gate, simplifying many calculations.

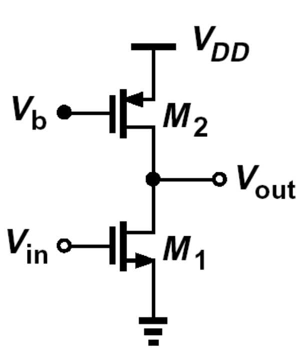
Common-Source Stage

As calculated before: $$\overline{V_{n,in}^2} = 4kT(\frac{2}{3g_m} + \frac{1}{g_m^2 R_D}) + \frac{K}{C_{ox}WL}\frac{1}{f}$$

Why does the noise decrease as R_D increases?

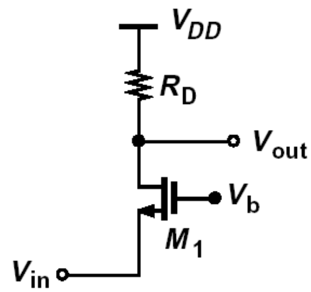
How to reduce the noise?

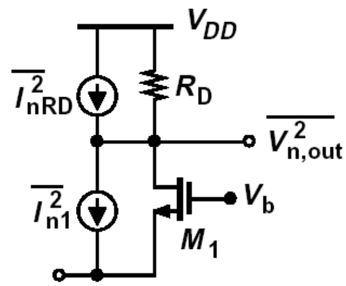
Example: Determine the input-referred thermal and 1/f noise.



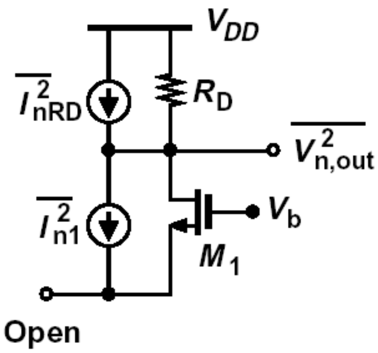
Common-Gate Stage

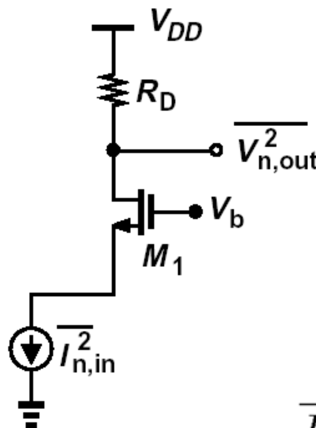
Need to find input-referred noise voltage and current.





$$\overline{V^2_{n,in}} = \frac{4kT(2g_m/3 + 1/R_D)}{(g_m + g_{mb})^2}$$

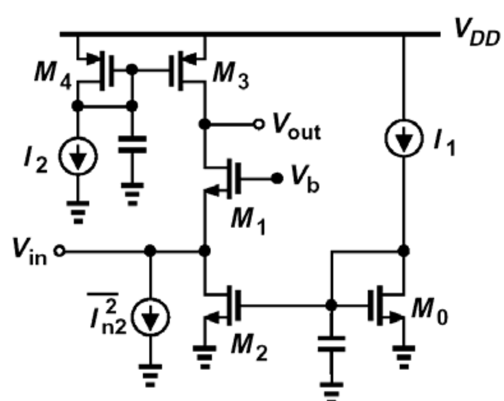




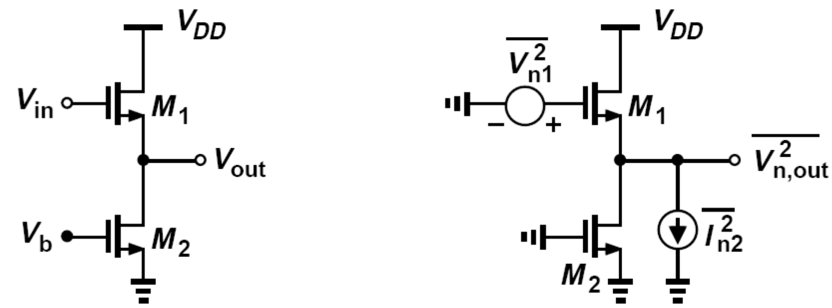
$$\overline{I^2_{n,in}} = \frac{4kT}{R_D}$$

The bias current source often contributes significant noise.

Example



Source Follower

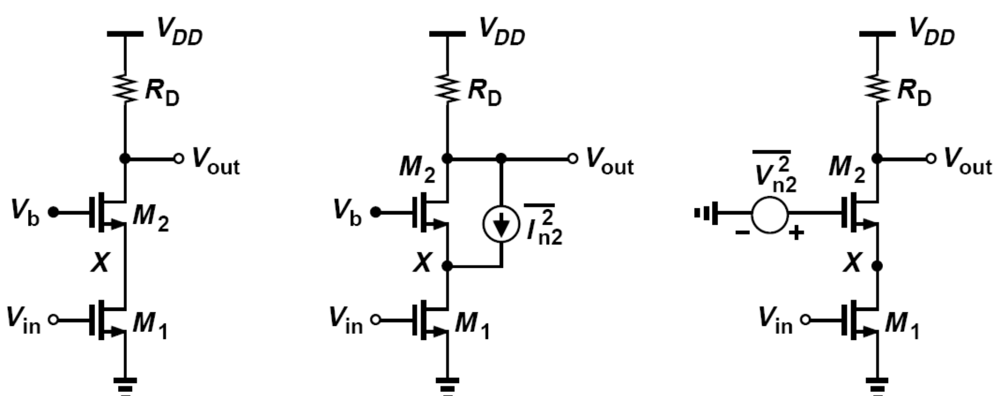


$$\overline{V_{n,out}^2}|_{M2} = \overline{I_{n2}^2}(\frac{1}{g_{m1}}||\frac{1}{g_{mb1}}||r_{O1}||r_{O2})^2$$

$$\begin{aligned}\overline{V_{n,in}^2} &= \overline{V_{n1}^2} + \frac{\overline{V_{n,out}^2}|_{M2}}{A_v^2} \\ &= 4kT\frac{2}{3}(\frac{1}{g_{m1}} + \frac{g_{m2}}{g_{m1}^2})\end{aligned}$$

Another reason not to use source followers.

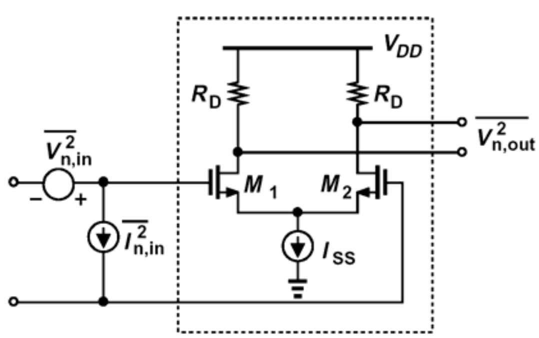
Cascode



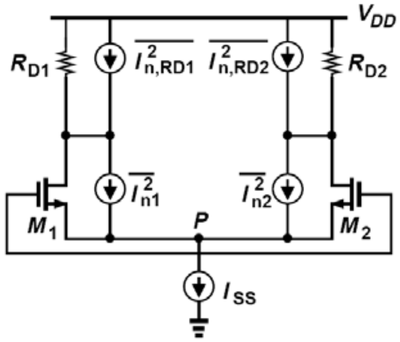
(a) (b) (c)

What if there is capacitance at node X?

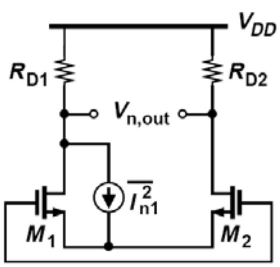
Differential Pair



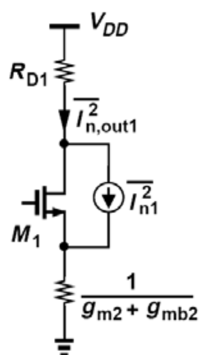
Since the four noise generators are uncorrelated, we can use superposition for the powers.



(a)



(b)



(c)

$$\overline{V_{n,in}^2} = 8kT(\frac{2}{3g_m} + \frac{1}{g_m^2 R_D})$$

Thus, the input-referred noise voltage of a diff pair is 40% larger than that of a common-source stage – probably the only disadvantage of differential operation.

- **Note that noise of I_{ss} appears as a common-mode disturbance.**

<u>Example</u>