EE215A

Final Exam

Fall 2010

Name:....Solutions....

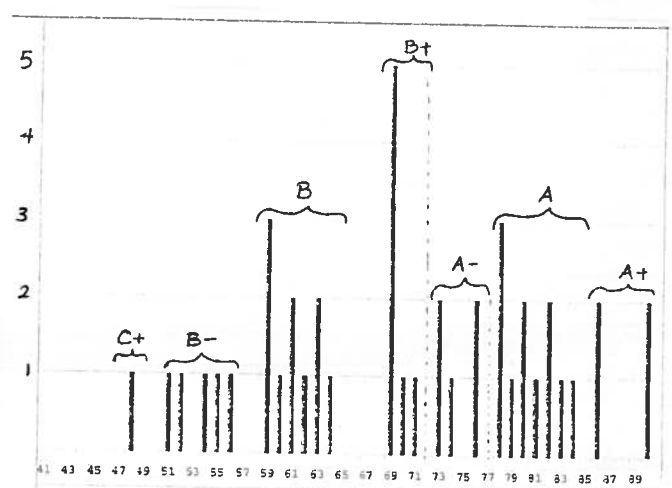
Time Limit: 3 Hours

Open Book, Open Notes

Each problem has 20 points.

All transistors are in saturation.

1. 20
2. 20
3. 20
4. 20
5. 20

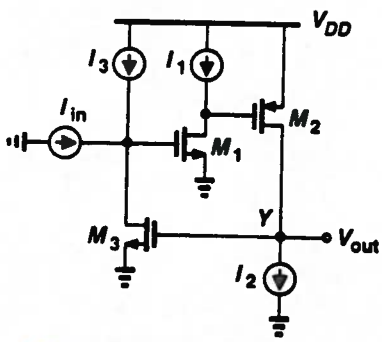


1. A transimpedance amplifier is shown below. Assume I_1, I_2, and I_3 are ideal. Also, $\lambda \neq 0$. Use feedback techniques in parts (a) and (b).

(a) Determine the open-loop transimpedance gain and input and output impedances.

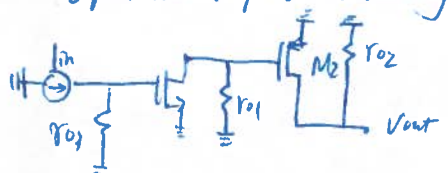
(b) Determine the closed-loop transimpedance gain and input and output impedances. Simplify the expressions for $\lambda \to 0$.

(c) Assuming $\lambda = 0$, compute the input-referred thermal noise current of the closed-loop circuit.



(a) V–C Feedback: use Y model: $Y_{21} = g_{m3} = \beta$. $Y_{22} = 1/r_{o3}$, $Y_{11} = 0$

Open the loop with loading: (small signal)



$\Rightarrow$

$$\boxed{\begin{aligned} R_{o,open} &= g_{m1} g_{m2} r_{o1} r_{o2} r_{o3} \\ \beta &= g_{m3} \\ R_{in,open} &= r_{o3} \\ R_{out,open} &= r_{o2} \end{aligned}}$$

(b) Close-loop: $R_{o,close} = \dfrac{R_{o,open}}{1 + R_{o,open}\, g_{m3}} = \boxed{\dfrac{g_{m1} g_{m2} r_{o1} r_{o2} r_{o3}}{(1 + g_{m1} g_{m2} g_{m3} r_{o1} r_{o2} r_{o3})}}$

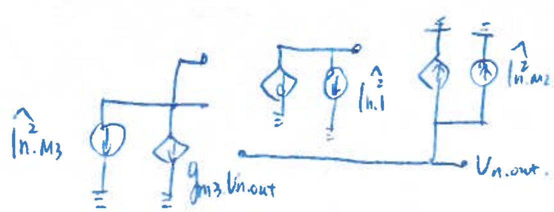
$R_{in,close} = \boxed{\dfrac{r_{o3}}{[1 + g_{m1} g_{m2} g_{m3} r_{o1} r_{o2} r_{o3}]}}$

$R_{out,close} = \boxed{\dfrac{r_{o2}}{[1 + g_{m1} g_{m2} g_{m3} r_{o1} r_{o2} r_{o3}]}}$

for $\lambda \to 0$,

$$\Rightarrow \boxed{\begin{aligned} R_{o,close} &= 1/g_{m3} \\ R_{in,close} &= 0 \\ R_{out,close} &= 0 \end{aligned}}$$

(c) $\lambda = 0$, $\Rightarrow$



$g_{m3} V_{n,out} = I_{n,M3}$

$\overline{V^2_{n,out}} = \overline{I^2_{n,M3}} / g_{m3}^2 = \overline{I^2_{n,in}} \underbrace{\left(\dfrac{1}{g_{m3}}\right)^2}_{R_{o,close},\ \lambda=0}$

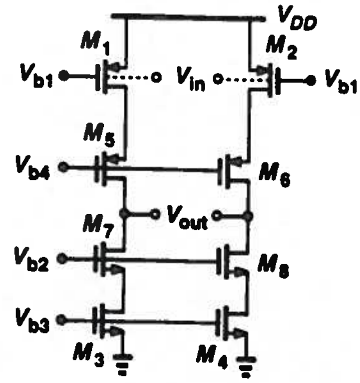
$\Rightarrow \boxed{\overline{I^2_{n,in}} = \overline{I^2_{n,M3}}}$

2. It is possible to use the bulk terminal of PMOS devices as an input. Consider the amplifier shown below. Assume $\lambda \neq 0$ for all devices, $\gamma \neq 0$ for M_1 and M_2, and $\gamma = 0$ for other transistors. Also, assume the input common-mode level is equal to V_{DD}.

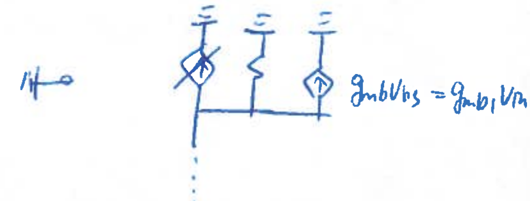
(a) Calculate the voltage gain.

(b) Calculate the input-referred thermal noise voltage of the amplifier. To avoid confusion, use Γ (rather than γ) for the thermal noise coefficient of MOSFETs, e.g., $\overline{I_n^2} = 4kT\Gamma g_m$.

(c) If the input common-mode level is raised to $V_{DD} + 0.5$ V, does the small-signal gain increase or decrease? Why?



(a) Fully symmetric $\Rightarrow$ use half circuit.



$G_m = g_{mb1}$

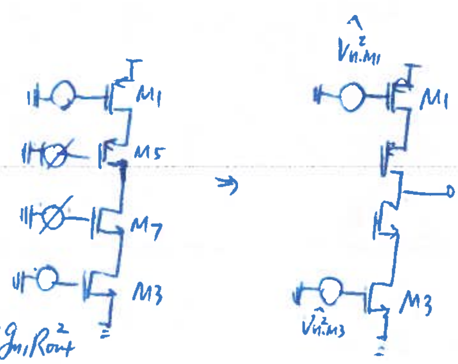
$R_{out} \approx (g_{m5}r_{o5}r_{o1}) \,//\, (g_{m7}r_{o7}r_{o3})$

$\boxed{|Gain| = |\, g_{mb1}\,(g_{m5}r_{o5}r_{o1} \,//\, g_{m7}r_{o7}r_{o3})\,|}$ #

(b) Calculate half circuit first:

the noise contribution of M_5 & M_7

are negligible $\Rightarrow$ calculate only M_1 & M_3:



for M_1: $\overline{V_{n,M1}^2} = 4kT\Gamma/g_{m1}$

Gain for $V_{n,M1}$ to output $\approx -g_{m1}R_{out}$ $\Big\}$ $\overline{V_{n,out,M1}^2} = 4kT\Gamma g_{m1}R_{out}^2$

for M_3: $\overline{V_{n,M3}^2} = 4kT\Gamma/g_{m3}$

Gain for $V_{n,M3}$ to output $\approx -g_{m3}R_{out}$ $\Big\}$ $\overline{V_{n,out,M3}^2} = 4kT\Gamma g_{m3}R_{out}^2$

$\therefore$ $\boxed{\overline{V_{n,in,total}^2} = 2\cdot 4\cdot kT\cdot \Gamma\cdot (g_{m1}+g_{m3})/g_{mb1}^2}$ #

(c) $g_{mb} = \mu_n C_{ox}(\frac{W}{L})(|V_{GS}|-|V_{TH}|)(\frac{\gamma}{2})\Big/\sqrt{|2\phi_F+V_{SB}|}$

$|V_{TH}|\uparrow$. $|2\phi_F+V_{SB}|\uparrow$. $g_{mb1}\downarrow$ $\Rightarrow$ $\boxed{|Gain| \text{ decrease.}}$ #

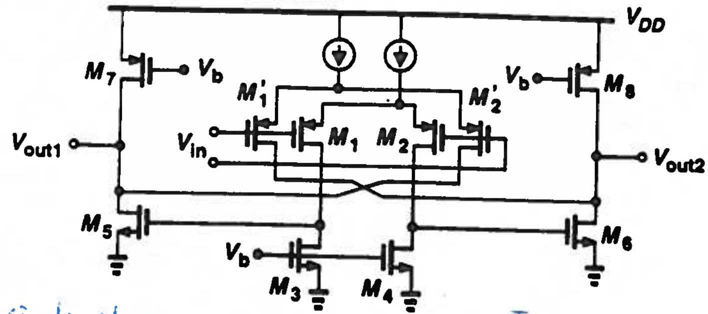
(actually $R_{out} = g_{m5}r_{o5}r_{o1} \,//\, g_{m7}r_{o7}r_{o3}$, changes slightly. $r_{o1} = 1\Big/\frac{1}{2}\mu_n C_{ox}\frac{W}{L}_1(|V_{GS}|-|V_{TH}|)^2\lambda$ increase and all the other terms remain the same, R_{out} will increase only slightly)

3. The op amp shown below employs a fast path in parallel with a slow path. Assume $\lambda \neq 0$.

(a) Identify the fast and slow paths and determine the number of poles in each path.

(b) Calculate the total small-signal differential gain of the op amp.

(c) Which transistors typically limit the output voltage swing?

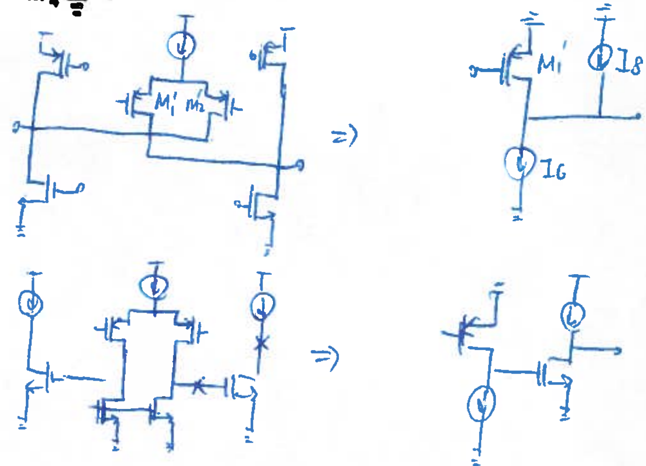


(a) fast path: $M_1' M_2'$, single stage

$\Rightarrow \boxed{\text{single pole}}$

slow path: $M_1 M_2$, two stages

$\Rightarrow \boxed{\text{two poles.}}$



(b)

Using superposition:

$V_{o1} = -V_{in}\, g_{m1}' (r_{o7} \parallel r_{o5} \parallel r_{o1}')$

$V_{o2} = -V_{in}\, g_{m1} (r_{o1} \parallel r_{o3})\, g_{m5} (r_{o7} \parallel r_{o5} \parallel r_{o1}')$

$V_o = V_{o1} + V_{o2} \Rightarrow \boxed{\text{Gain} = -\left[g_{m1}' + g_{m1} (r_{o1} \parallel r_{o3})\, g_{m5} \right] (r_{o1}' \parallel r_{o5} \parallel r_{o7})}$

(c)

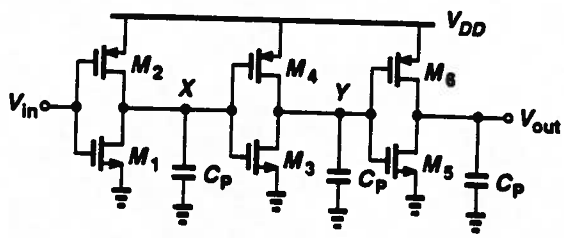
M_1' & M_2'

4. Consider the three-stage amplifier shown below. Assume all stages are idetical and neglect all other capacitances. Also, $\lambda \neq 0$.

(a) Using Bode approximations, plot the magnitude and phase of the circuit's transfer function and identify important points. Explain why the amplifier is not stable if used in a unity-gain feedback loop.

(b) Compensate the amplifier (for unity-gain feedback) by adding a large capacitor, C_C, from node Y to ground. Determine the required value of C_C for a phase margin of 45°. You may assume $C_C \gg C_P$. Estimate the -3-dB bandwidth and unity-gain bandwidth of the amplifier after compensation.

(c) Compensate the amplifier (for unity-gain feedback) by adding a large capacitor, C_C, from node Y to node X. Determine the required value of C_C for a phase margin of 45°. You may assume $C_C \gg C_P$. Estimate the -3-dB bandwidth and unity-gain bandwidth of the amplifier after compensation.

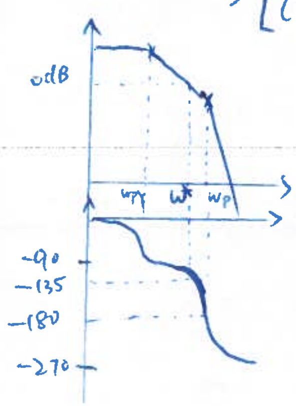


(a) $\omega_p = 1/C_p(r_{on}//r_{op})$

$A = A_0^3/(1+s/\omega_p)^3$, $\beta = 1$, where $|A_0| = |(g_{mn}+g_{mp})\cdot(r_{on}//r_{op})|$

Triple pole $\Rightarrow$ each stage has an identical pole. Suppose $A_0 \approx 10$, the unity gain BW will be one decade away from this pole, and the unity gain phase will be far below $-180°$ $\Longrightarrow$ Unstable!

(b) $A = \dfrac{A_0^3}{\left[(1+s/\omega_p)^2(1+s/\omega_{pY})\right]}$, where $\omega_p = 1/C_p(r_{on}//r_{op})$, $\omega_{pY} \approx 1/C_c(r_{on}//r_{op})$



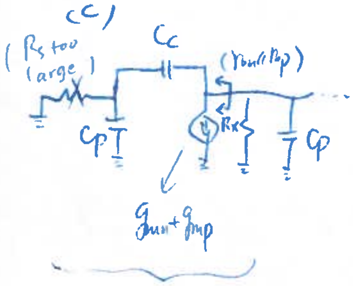
$|A_0| = |(g_{mn}+g_{mp})(r_{on}//r_{op})|$

assume ω^* the unity gain BW

$$\Rightarrow \begin{cases} -2\tan^{-1}(\omega^*/\omega_p) = -45° \\ \dfrac{\omega^*}{\omega_p} = 0.414 \end{cases} \Rightarrow \begin{cases} \dfrac{A_0^3}{\left(1+\frac{\omega^*}{\omega_p}\right)^2\sqrt{1+\left(\frac{\omega^*}{\omega_{pY}}\right)^2}} = 1 \\ \boxed{C_c = \dfrac{C_p}{0.414}\sqrt{\dfrac{A_0^6}{1.367}-1}} \end{cases}$$

$$\boxed{-3\,dB\,BW = 1/C_c(r_{on}//r_{op}),\quad \text{Unity gain BW} = 0.414/C_p(r_{on}//r_{op})}$$

(c)



Small sig M3 & M4

$A_0 = (g_{mn}+g_{mp})(r_{on}//r_{op})$

$R_X = \dfrac{C_c+C_p}{C_c(g_{mn}+g_{mp})}$

$\therefore \omega_{pX} = 1/C_c(1+A_0)(r_{on}//r_{op})$

$\omega_{pY} = \dfrac{1}{\left(\frac{C_c+C_p}{C_c(g_{mn}+g_{mp})}//r_{on}//r_{op}\right)\left(C_p+\frac{C_pC_c}{C_p+C_c}\right)}$

$\simeq (g_{mn}+g_{mp})/2C_p$

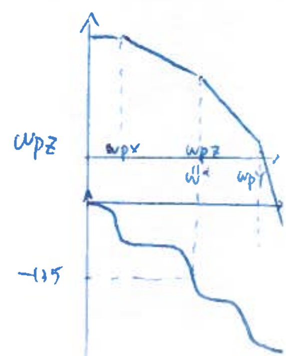
$\omega_{pZ} = 1/C_p(r_{on}//r_{op})$

pole splitting:

the unity gain freq $\omega^* = \omega_{pZ}$

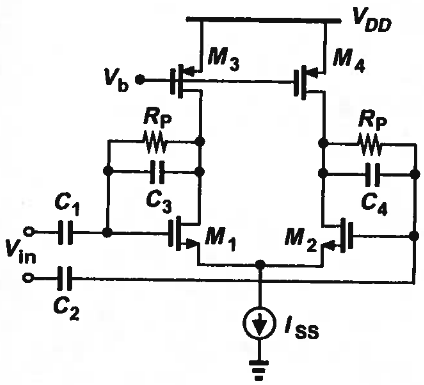
$H(s) = \dfrac{A_0^3}{\left(1+\frac{s}{\omega_{pX}}\right)\left(1+\frac{s}{\omega_{pY}}\right)\left(1+\frac{s}{\omega_{pZ}}\right)}$

$\dfrac{A_0^3}{\sqrt{2}\cdot\sqrt{1+\left(\frac{\omega_{pZ}}{\omega_{pX}}\right)^2}} = 1$



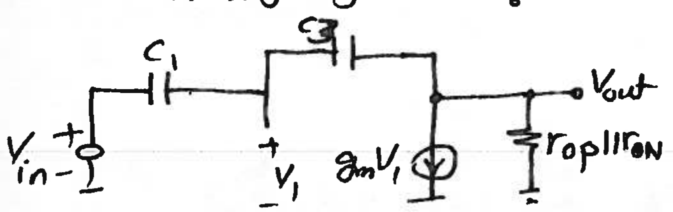
$$\boxed{C_c = \frac{C_p}{1+A_0}\sqrt{\frac{A_0^6}{2}-1},\quad -3\,dB\,BW = 1/[C_c(1+A_0)](r_{on}//r_{op}),\quad BW_u = 1/C_p(r_{on}//r_{op})}$$

5. Consider the amplifier depicted below, where the common-mode feedback is not shown. Assume $\lambda \neq, V_{DD} = 3$ V, and the output CM level is 1.5 V. Neglect the transistor capacitances and assume R_p is very large.
(a) What is the maximum allowable differential output voltage swing?
(b) Determine the gain error of the circuit.
(c) Determine the small-signal time constant of the amplifier.
(d) Explain why you would or would not choose this topology for your final project.



(a) Each output node can swing by $\pm V_{TH}$ around the CM level $\Rightarrow$ total diff. swing $= \pm 2V_{TH}$.

(b), (c) Using the following equivalent circuit, we obtain the transfer function:



$$\frac{V_{out}}{V_{in}} = \frac{C_1(C_3 s - g_m)(r_{op} \| r_{ON})}{(r_{op} \| r_{ON}) C_1 C_3 s + g_m (r_{op} \| r_{ON}) C_3 + C_1 + C_3}$$

$$\Rightarrow \tau = \frac{C_1 C_3 (r_{op} \| r_{ON})}{[g_m (r_{op} \| r_{ON}) + 1] C_3 + C_1}$$

At $s = 0$,
$$\frac{V_{out}}{V_{in}} = \frac{-g_m C_1 (r_{op} \| r_{ON})}{g_m (r_{op} \| r_{ON}) C_3 + C_1 + C_3} = -\frac{C_1}{C_3} \frac{1}{1 + \frac{C_1 + C_3}{g_m (r_{op} \| r_{ON}) C_3}}$$

$$\Rightarrow \text{Gain Error} = \frac{C_1 + C_3}{C_3} \frac{1}{g_m (r_{op} \| r_{ON})}$$

(d) Since the gain error is too large, this topology would not meet the project's specifications.

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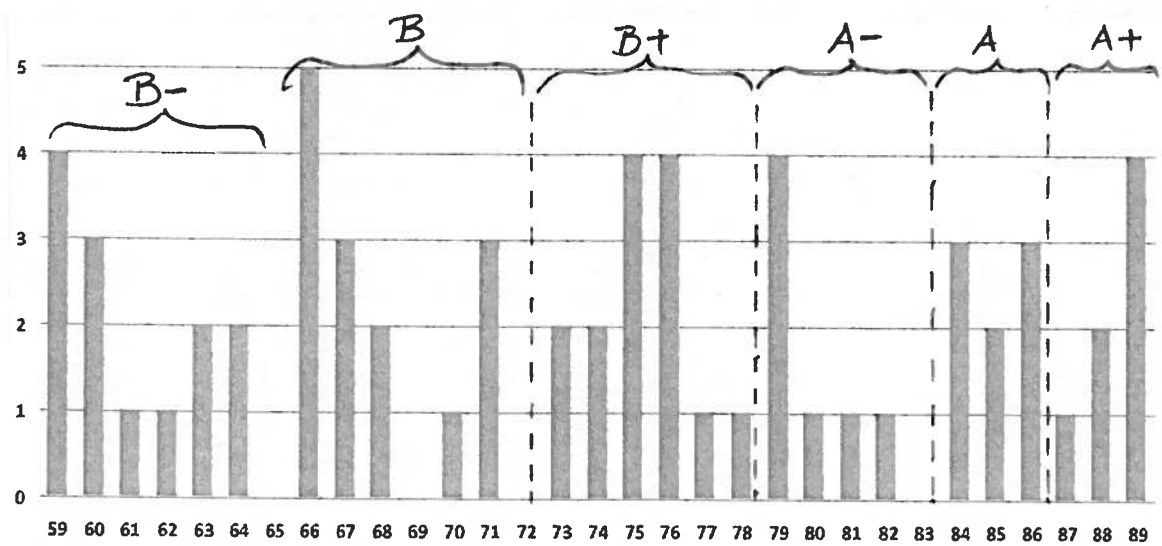
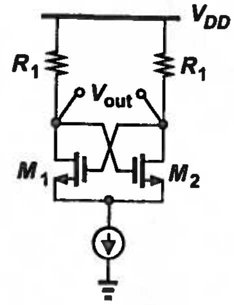
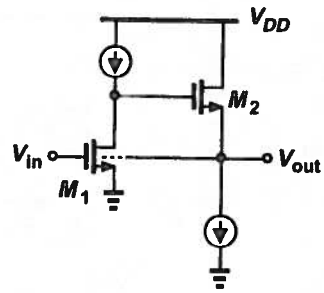


1. Determine the output thermal noise voltage in each circuit. Neglect device capacitances and channel-length modulation. In the circuit in part (I), M_1 and M_2 are identical and carry equal bias currents. In the circuit in part (II), the output node is tied to the body of M_1. Neglect the body effect of M_2 in this circuit. To avoid confusion between gammas, model the transistor noise as $4kT\Gamma g_m$.

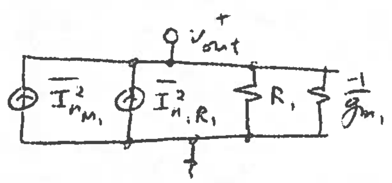


(I)



(II)

Since the circuit is symmetrical, use half circuit:



$$\overline{V^2_{n,out}} = 2 \times (\overline{I^2_{n,M_1}} + \overline{I^2_{n,R_1}})\, R^2_{out,half}$$

where:

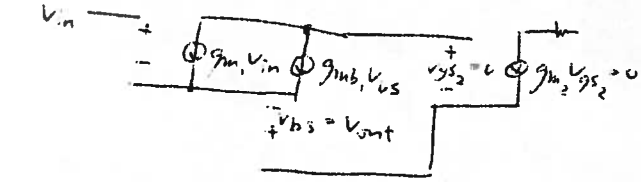
$$\overline{I^2_{n,M_1}} = 4kT\Gamma g_m$$

$$\overline{I^2_{n,R_1}} = 4kT / R_1$$

$$R^2_{out,half} = \left(R_1 \,\|\, \frac{-1}{g_{m_1}}\right)^2$$

Small signal analysis:

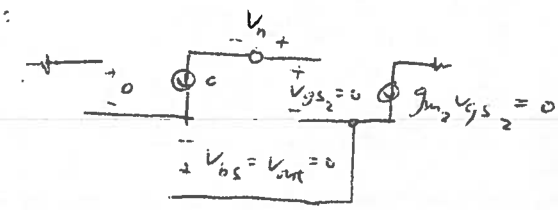
M_1:



$$g_{m_1} V_{in} = -g_{mb_1} V_{out}$$

$$V_{out} = \frac{g_{m_1}}{g_{mb_1}} V_{in}$$

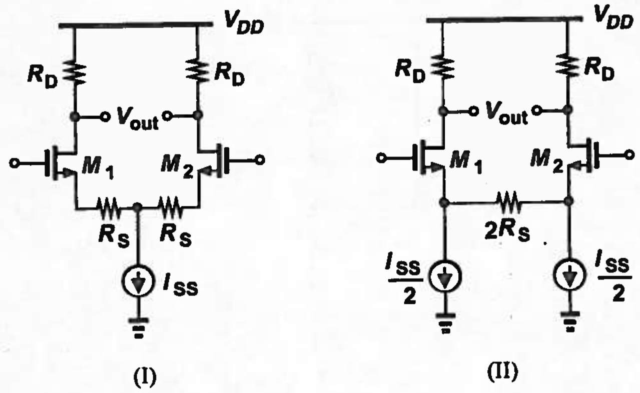
M_2:



noise of M_2 doesn't appear at the output (suppressed by the ∞ voltage gain from V_{out} to the drain voltage of M_1)

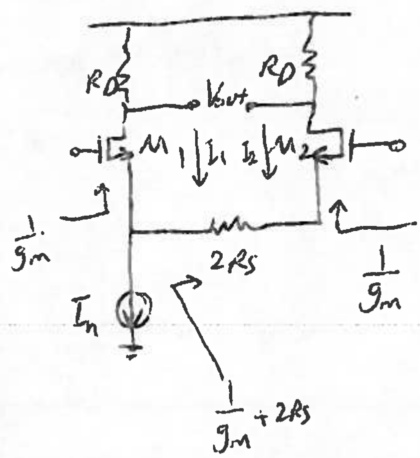
$$\overline{V^2_{n,out}} = \left(\frac{g_{m_1}}{g_{mb_1}}\right)^2 \frac{4kT\Gamma}{g_{m_1}}$$

2. A differential pair can be degenerated to achieve greater linearity. Consider the two topologies shown below.
(a) How much headroom does the tail current source lose as a result of degeneration in circuit (I)?
(b) If we denote the noise current of each tail current source in circuit (II) by I_n, determine the output noise voltage of this circuit only due to I_n.



a) $R_S \frac{I_{SS}}{2}$

b)



$$I_1 = I_n \times \frac{\frac{1}{g_m}+2R_S}{\frac{2}{g_m}+2R_S}$$

$$I_2 = I_n \times \frac{\frac{1}{g_m}}{\frac{2}{g_m}+2R_S}$$

$$V_{out} = (I_1 - I_2)R_D = I_n \frac{2R_S}{\frac{2}{g_m}+2R_S} R_D$$

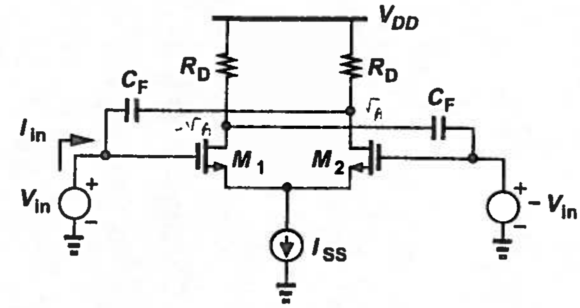
$$= I_n \frac{R_S R_D}{\frac{1}{g_m}+R_S}$$

Two uncorrelated Noise Sources : $\overline{V_{out,n}^2} = 2 \times \left(\frac{R_S R_D}{\frac{1}{g_m}+R_S}\right)^2 \overline{I_n^2}$

3. The differential pair shown below incorporates feedback capacitors to create a negative input capacitance. Neglect other capacitances and channel-length modulation.

(a) Determine the single-ended input impedance, defined as V_{in}/I_{in}. 10

(b) Express the result as an RC network. What happens as $C_F \to \infty$? Explain why this result is to be expected. 10



(a) KCL @ drain of M2: $\frac{V_A}{R_D} - g_m V_{in} + (V_A - V_{in}) C_F \cdot S = 0 \Rightarrow V_A = V_{in} \cdot \frac{g_m \cdot C_F}{C_F S + \frac{1}{R}}$

$I_{in} = (V_{in} - V_A) C_F \cdot S \Rightarrow Z_{in} = \frac{V_{in}}{I_{in}} = \frac{R_D}{1 - g_m R_D} + \frac{1}{C_F S (1 - g_m R_D)}$

$g_m R_D > 1 \Rightarrow$ negative cap

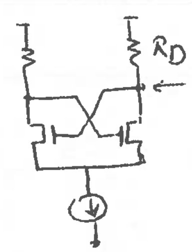
(b)

$C = C_F (1 - g_m R_D)$

$R = R_D \cdot \frac{1}{1 - g_m R_D}$

when $C_F \to \infty$, it's like a cross coupled pair

$C_F \to \infty \Rightarrow Z_{in} = \frac{R_D}{1 - g_m R_D}$



$Z = R_D \| -\frac{1}{g_m} = \frac{R_D}{1 - g_m R_D}$

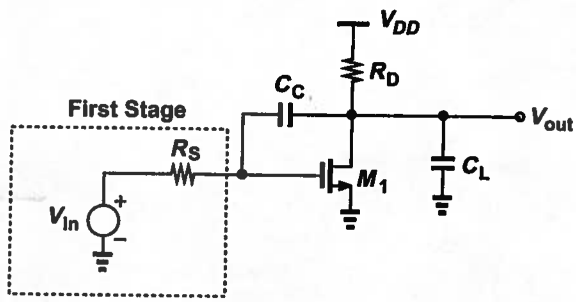
4. The circuit shown below is a simplified model of a two-stage op amp, where C_C is the compnesation capacitor. Neglect other capacitances and channel-length modulation.

(a) Assuming $g_m R_D \gg 1$, estimate ω_{p1}, ω_{p2}, and ω_z, where ω_z denotes the zero frequency.

(b) What condition is necessary for $|\omega_{p2}|$ to be equal to $|\omega_{z1}|$?

(c) Suppose $|\omega_{p2}| = |\omega_z|$. Plot the magnitude and phase of $\beta H(s)$ (with $\beta = 1$) as a function of frequency.

(d) Construct the magnitude and phase plots if C_C increases or decreases from the value assumed in part (c). What happens to the phase margin in each case?



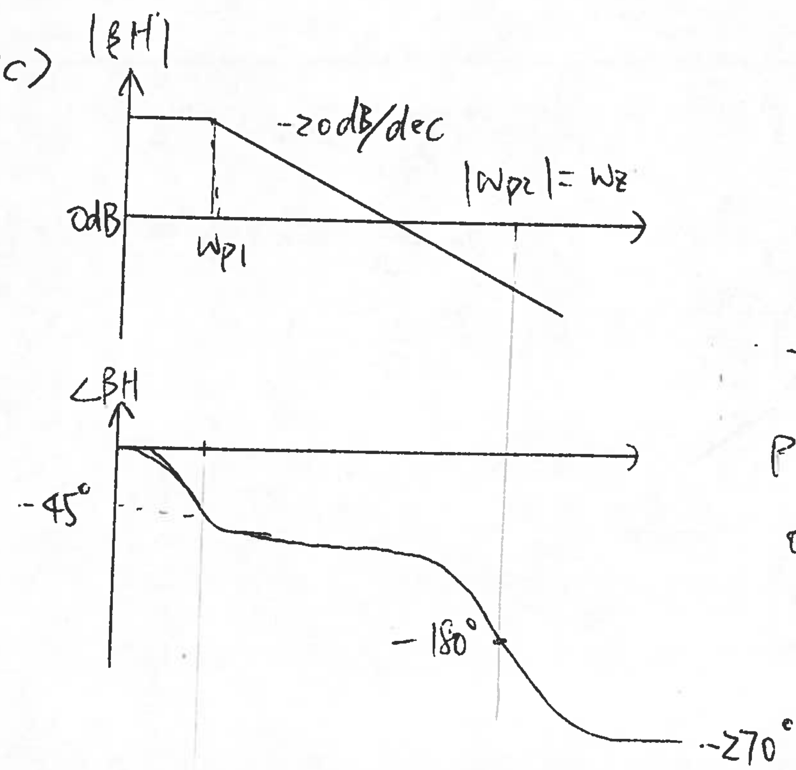
(a) $\omega_{p1} \approx -\dfrac{1}{R_S \cdot g_m R_D \cdot C_L}$

$\omega_{p2} \approx -\dfrac{g_m}{C_L}$

$\omega_z \approx \dfrac{g_m}{C_c}$

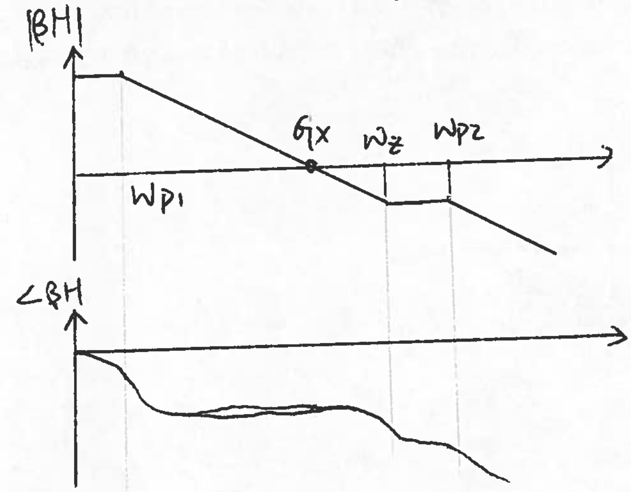
(b) $|\omega_{p2}| = |\omega_z| \Rightarrow C_c = C_L$

(c)



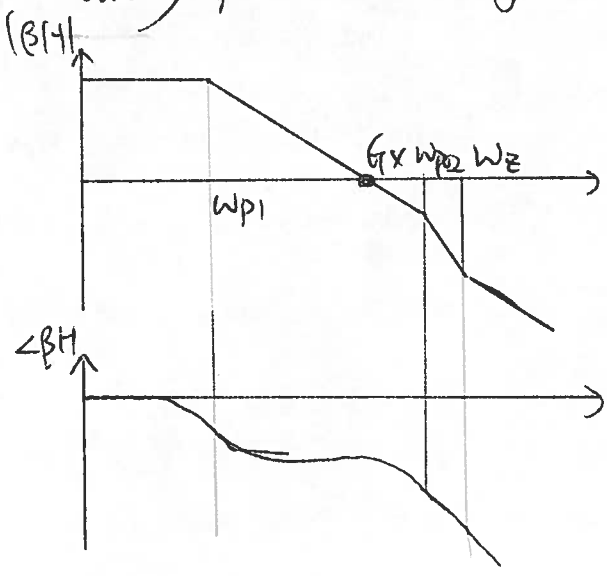
The 0dB point should be between ω_{p1} and ω_{p2}, so that we have a reasonable phase margin. (d) depends on this notion.

(d) As C_c increases, ω_{p1} and ω_z moves toward the origin while ω_{p2} stays.



The gain crossing point Gx decrease while $\tan^{-1}\frac{G_x}{\omega_z}$ remains the same. Furehermore, $\tan^{-1}\frac{G_x}{\omega_{p2}}$ decreases. Therefore the phase margin increases.

As C_c decreases, ω_{p1} and ω_z moves away from the origin while ω_{p2} stays.



1. $G_x \uparrow$
2. $\tan^{-1}\frac{G_x}{\omega_z}$ almost the same
3. $\tan^{-1}\frac{G_x}{\omega_{p2}}$ $\uparrow$

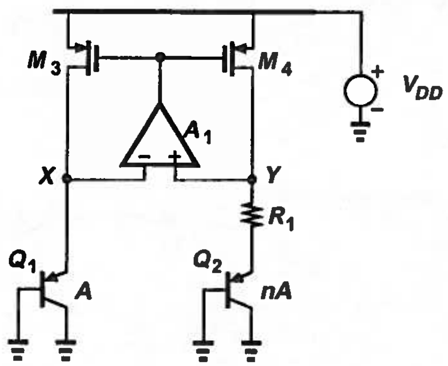
$\Rightarrow$ phase margin $\downarrow$

5. Consider the bandgap circuit shown below. We wish to study the noise in V_X and V_Y as a result of supply noise. Assume M_3 and M_4 are identical and model the bipolar devices by a resistance equal to $1/g_{mbi}$.

(a) If $A_1 = \infty$ and channel-length modulation is neglected, compute the small signal gain from V_{DD} to V_X and V_Y.

(b) If $A_1 = \infty$ and channel-length modulation is not neglected, compute the small signal gain from V_{DD} to V_X and V_Y.

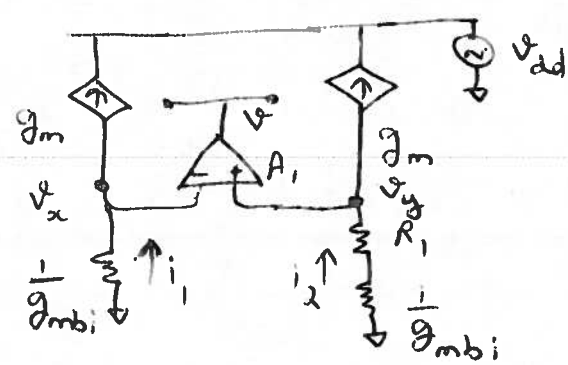
(c) If $A_1 < \infty$ and channel-length modulation is neglected, compute the small signal gain from V_{DD} to V_X and V_Y.



Points → 6 + 6 + 8

For each case, we study the effect of a small disturbance v_{dd} on V_{DD}.

a)



$A_1 = \infty$ no CLM

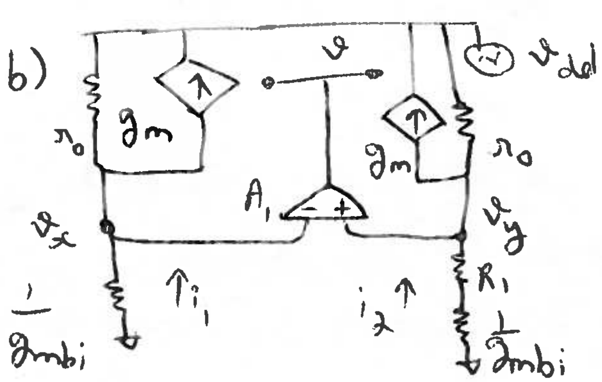
$i_1 = i_2 = g_m(v - v_{dd})$

But $v_x = v_y$ due to ∞ gain of op-amp.

$\Rightarrow i_1 = i_2 = 0 \quad (v = v_{dd})$

$\Rightarrow \boxed{v_x = v_y = 0}$

b)



$A_1 = \infty$ $\quad r_o \neq \infty$

Again $v_x = v_y$

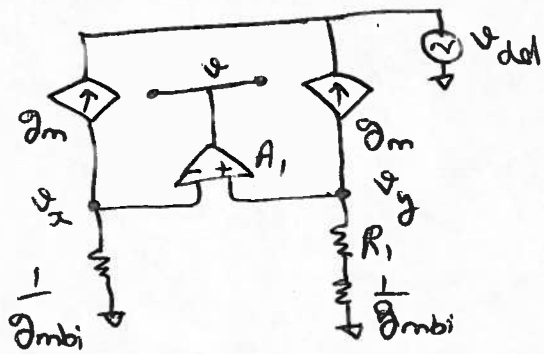
$\Rightarrow i_1/g_{mbi} = i_2\left(R_1 + \frac{1}{g_{mbi}}\right)$

$\Rightarrow \frac{g_m(v - v_{dd}) + \frac{(v_x - v_{dd})}{r_o}}{g_{mbi}} = \left[g_m(v - v_{dd}) + \frac{v_x - v_{dd}}{r_o}\right] \times \left[R_1 + \frac{1}{g_{mbi}}\right]$

P.T.O.

$$\Rightarrow \boxed{v_x = v_y = 0}$$

c)



$A_1 < \infty$ no CLM

In this case, v_x & v_y are not equal & v does not track v_{dd} exactly.

$$v_x = -g_m(v - v_{dd})\frac{1}{g_{mbi}}$$

$$v_y = -g_m(v - v_{dd})\left[R_1 + \frac{1}{g_{mbi}}\right]$$

$$v = A(v_y - v_x) = -g_m(v - v_{dd})R_1 A.$$

$$\Rightarrow v = v_{dd}\frac{g_m R_1 A}{g_m R_1 A + 1}.$$

$$\Rightarrow \boxed{v_x = v_{dd} \cdot \frac{g_m/g_{mbi}}{g_m R_1 A + 1} \qquad v_y = v_{dd} \cdot \frac{g_m\left[R_1 + \frac{1}{g_{mbi}}\right]}{g_m R_1 A + 1}}$$