EE215A

Midterm Exam
Fall 2011

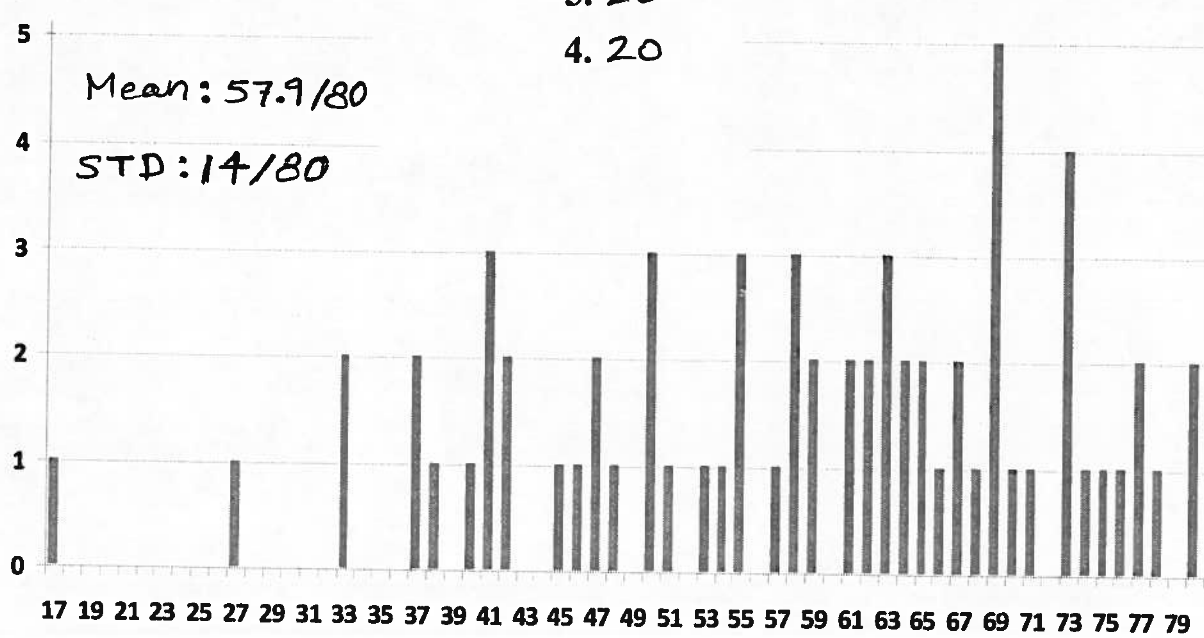
Time Limit: 2 hours
Open Book, Open Notes
Calculators are allowed.
Your Name:

Solutions

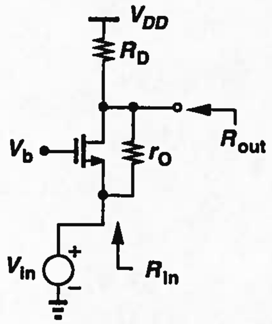
Name of Person to Your Left:

Name of Person to Your Right:

1. 20
2. 20
3. 20
4. 20



1. (a) Neglecting body effect, use Miller's theorem to determine R_{in} and R_{out} for the circuit shown below.
(b) Compare your results with our previous derivations and explain in detail.

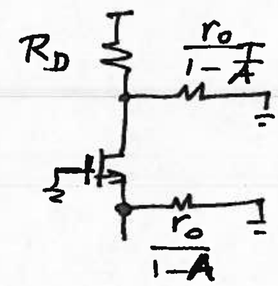


(a) With $\gamma = 0$, the voltage gain from the source to the drain is equal to

$$A = \frac{g_m r_o + 1}{r_o + R_D} R_D .$$ We divide r_o by $1 - A$ and place the result in parallel with $\frac{1}{g_m}$:

$$R_{in} = \frac{1}{g_m} \parallel \frac{r_o}{1-A}$$

$$= \frac{r_o + R_D}{1 + g_m r_o} .$$

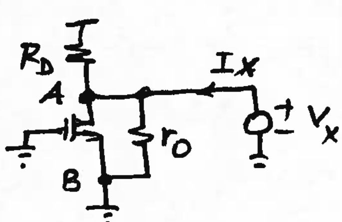


For R_{out}, if we place $\frac{r_o}{1-\frac{1}{A}}$ in parallel with R_D, we obtain

$$R_{out} = \frac{r_o}{1 - \frac{r_o + R_D}{(1+g_m r_o) R_D}} \parallel R_D$$

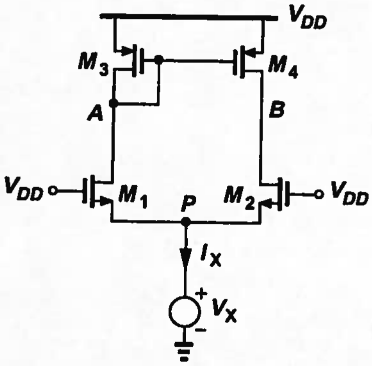
$$= \frac{(1 + g_m r_o) R_D}{g_m R_D - 1} \parallel R_D$$

(b) R_{in} is computed correctly. But, R_{out} is not. This is because for output resistance calculation, we apply the stimulus to the <u>output</u> node, altering the "gain" from the source to the drain:



We can still apply Miller's theorem here but with the "gain" from A to B (i.e., zero).

2. (a) Neglecting channel-length modulation and body effect, sketch I_X, V_A, and V_B as a function of V_X as V_X varies from zero to V_{DD}. Assume M_1 and M_2 are identical and so are M_3 and M_4.
(b) Write a quadratic equation that yields the value of V_B for $V_X = 0$. You need not solve the equation.



If M_1 and M_3 are in the saturation region,

$$\frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)_1 (V_{DD}-V_X-V_{TN})^2 = \frac{1}{2}\mu_p C_{ox}\left(\frac{W}{L}\right)_3 (V_{DD}-V_A-|V_{TP}|)^2$$

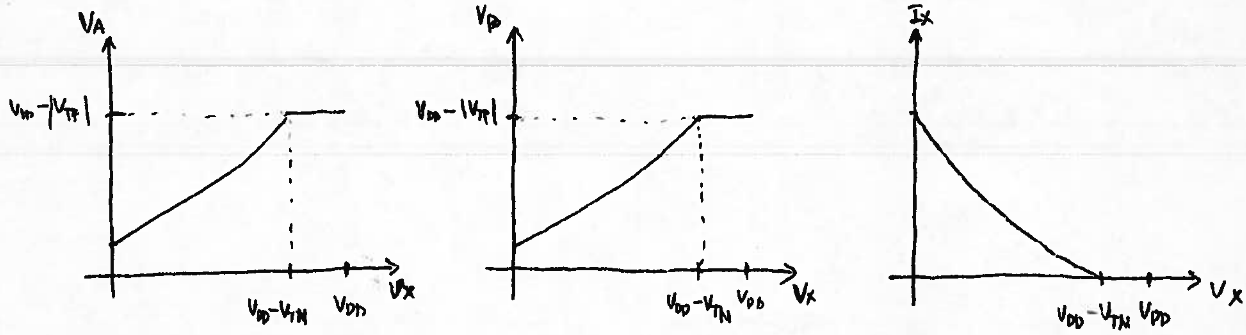
For simplicity, let's assume $\mu_n\left(\frac{W}{L}\right)_1 = \mu_p\left(\frac{W}{L}\right)_3$,

$V_A = V_{DD} - |V_{TP}| - (V_{DD} - V_{TN}) = \underline{V_{TN} - |V_{TP}|}$ (when $V_X =$ c

very small

So, M_1 is in the triode region. So as M_2

a)



when $V_X = V_{DD} - V_{TN}$, M_1 and M_2 are turned off, and V_A and V_B stay.

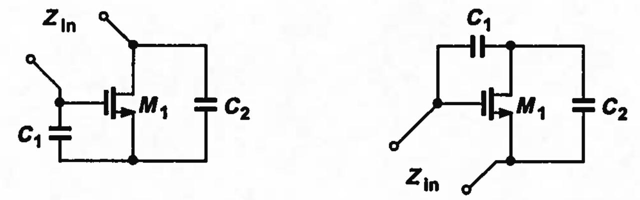
b) M_1 : triode region ($V_X = 0$)
M_3 : saturation region

$I_1 = I_3$.

$$\mu_1 C_{ox}\left(\frac{W}{L}\right)_1 \left\{ (V_{DD} - V_{TN})\cdot V_A - \frac{1}{2}V_A^2 \right\} = \frac{1}{2}\mu_p C_{ox}\left(\frac{W}{L}\right)_3 (V_{DD} - V_A - |V_{TP}|)^2 \quad \cdots ①$$

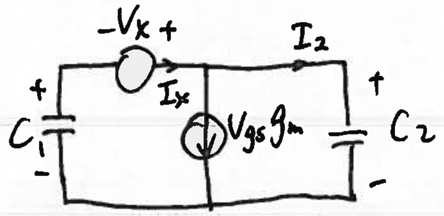
$V_A = V_B \quad \cdots ②$

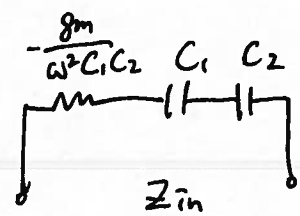
3. Neglecting channel-length modulation and body effect, determine Z_{in} for the circuits shown below. Assume M_1 is biased in saturation and neglect other capacitances. Can you express each Z_{in} as a series or parallel combination of resistors and capacitors?



(a) (b)

(A)



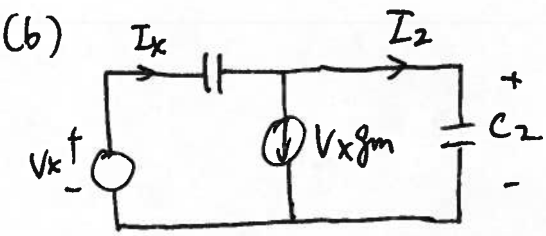


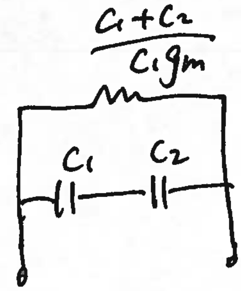
$$V_{gs} = V_{C_1} = -I_x \frac{1}{sC_1}$$

$$I_x = V_{gs} g_m + I_2 = -I_x \frac{g_m}{sC_1} + \left(V_x - I_x \frac{1}{sC_1}\right) sC_2$$

$$Z_{in} = \frac{V_x}{I_x} = \frac{g_m}{s^2 C_1 C_2} + \frac{1}{sC_1} + \frac{1}{sC_2} = -\frac{g_m}{\omega^2 C_1 C_2} + \frac{1}{sC_1} + \frac{1}{sC_2}$$

(b)



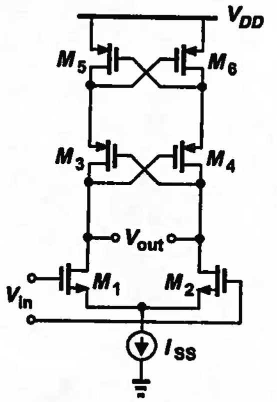


$$I_2 = \left(V_x - I_x \frac{1}{sC_1}\right) s C_2$$

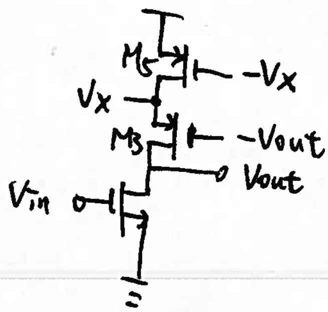
$$\Rightarrow I_x = V_x g_m + I_2 = V_x g_m + \left(V_x - I_x \frac{1}{sC_1}\right) s C_2$$

$$\Rightarrow Z_{in} = \frac{V_x}{I_x} = \frac{1 + \frac{sC_2}{sC_1}}{g_m + sC_2} = \frac{C_1 + C_2}{C_1 g_m + s C_1 C_2}$$

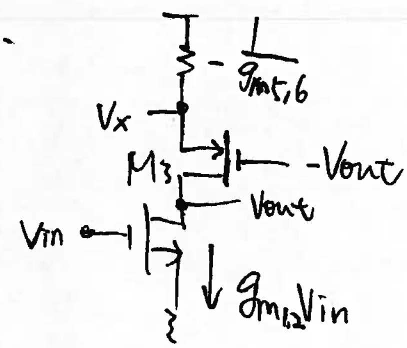
4. **Determine the differential voltage gain of the circuit shown below if channel-length modulation and body effect are negligible.**



When neglecting channel-length modulation and body effect, the half circuit:



Furthermore, M_5 can be equivalent to a resistor, whose resistence is $-\frac{1}{g_{m5,6}}$



$$g_{m1,2} \cdot V_{in} = V_x\, g_{m5,6} = (V_x + V_{out})\, g_{m3,4}$$

$$\Rightarrow V_{out} = \left(\frac{g_{m5,6}}{g_{m3,4}} - 1\right) V_x = \frac{g_{m5,6} - g_{m3,4}}{g_{m3,4}} \cdot V_x$$

$$g_{m1,2} V_{in} = \frac{g_{m3,4} \cdot g_{m5,6}}{g_{m5,6} - g_{m3,4}} V_{out}$$

$$\Rightarrow \frac{V_{out}}{V_{in}} = g_{m12} \cdot \frac{g_{m5,6} - g_{m3,4}}{g_{m3,4} \cdot g_{m5,6}}$$