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Problem 1

a) Eq. 2.116:

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$$NF = \frac{1}{4KTR_s} * \frac{\overline{V_{n,out}^2}}{A_0^2}$$

Noise figure of LNA w.r.t R_s ,

$$NF_1 = 1 + \frac{\overline{V_n^2}}{4KTR_s}$$

$$\overline{V_{n,out}^2} = 2 \cdot \alpha^2 A_v^2 4KTR_s + A_v^2 \overline{V_n^2}$$

$$NF = \frac{1}{4KTR_s} \cdot \frac{2 \cdot \alpha^2 A_v^2 4KTR_s + A_v^2 \overline{V_n^2}}{\alpha^2 A_v^2} = 2 + \frac{\overline{V_n^2}}{KTR_s}$$

$$NF = 4NF_1 - 2$$

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b) From Frii's equation we have,

$$NF_{tot} = 1 + (NF_1 - 1) + \frac{NF_2 - 1}{A_{p1}} + \dots$$

In this case the gain will be $\frac{1}{2}$,

$$NF_{tot} = 1 + (2 - 1) + \frac{2NF_1 - 1 - 1}{\frac{1}{2}}$$

$$NF_{tot} = 4NF_1 - 2$$

Problem 2

a) Using analytical model and equations W was found to be 38u,

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$$\mu_n = 310.235 \frac{cm^2}{V \cdot s}, \quad t_{ox} = 4.1 \text{ nm}, \quad C_{ox} = 0.8418 \frac{\mu F}{cm^2}$$

$$I_D = \frac{1}{2} \frac{W}{L} \mu_n C_{ox} (V_{gs} - V_{th})^2 = 5 * 0.6 \text{ mA} = 3 \text{ mA}$$

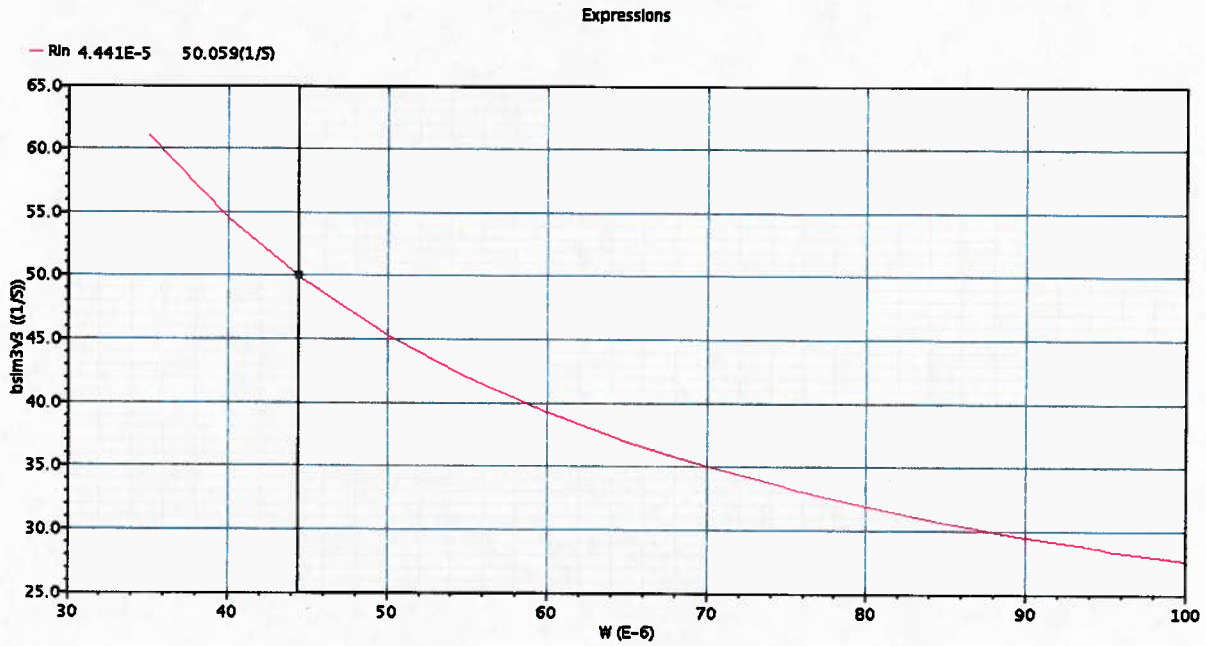
$$g_m = \frac{dI_D}{dV_{gs}} = \frac{W}{L} \mu_n C_{ox} (V_{gs} - V_{th}) = \frac{W}{L} \mu_n C_{ox} \times \sqrt{\frac{2I_D}{\frac{W}{L} \mu_n C_{ox}}} = \sqrt{2I_D \frac{W}{L} \mu_n C_{ox}}$$

Assuming $\eta=0.1$ and taking it into account,

$$(1 + \eta) \sqrt{2I_D \frac{W}{L} \mu_n C_{ox}} = \frac{1}{50} \Omega^{-1} \Rightarrow W = 38 \text{ uM}$$

However, simulation results gave an input resistance of 57 Ω when using 38u as the width of M1. By doing a parameter sweep on the size of the transistors the new width found to be 44.4u resulting in 50.059 Ω input resistance. The shortcoming in the calculations is due to rough approximations.

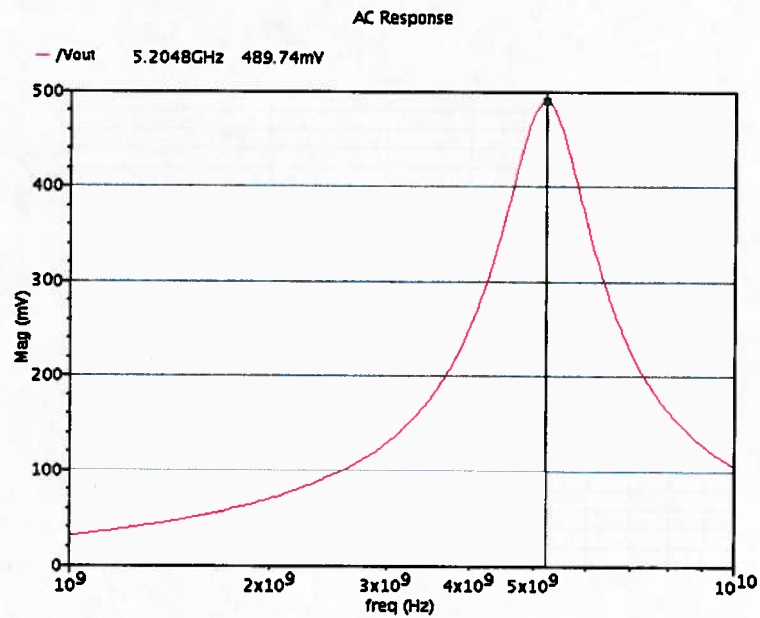
$$W = 44.4 \text{ uM}$$



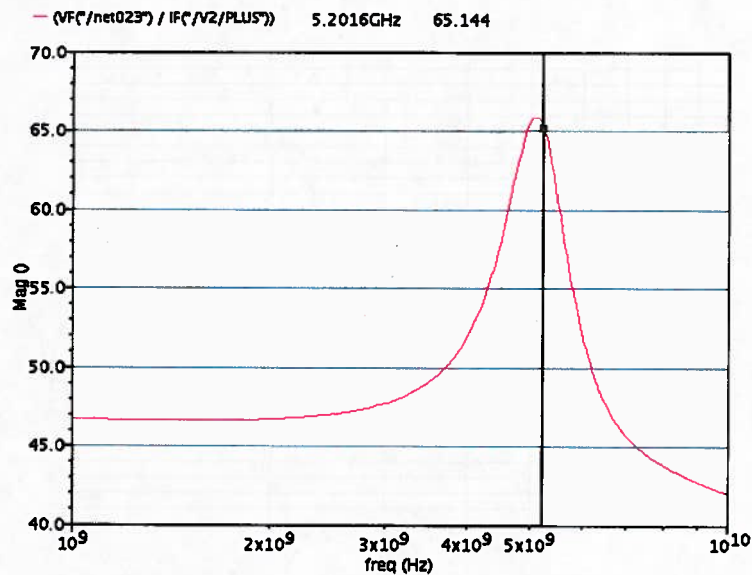
- b) Using simulation the estimated value for L_1 turned out to be 4.78 nH. This was achieved by sweeping the values of L_1 so we get a resonance frequency at 5.2 GHz.

$$Q = \frac{R_p}{L\omega} = 4$$

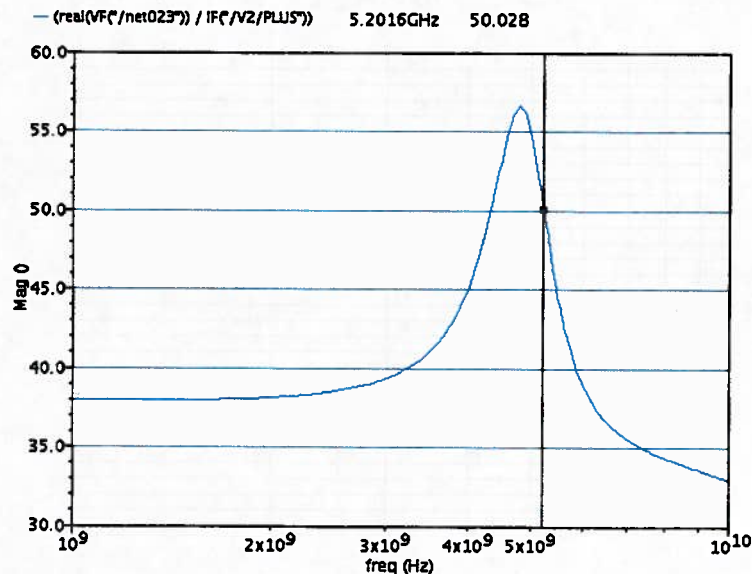
$$R_p = 41.6\pi \times L \text{ G}\Omega, \quad C_p = L \times 10^{-5} \text{ F}$$



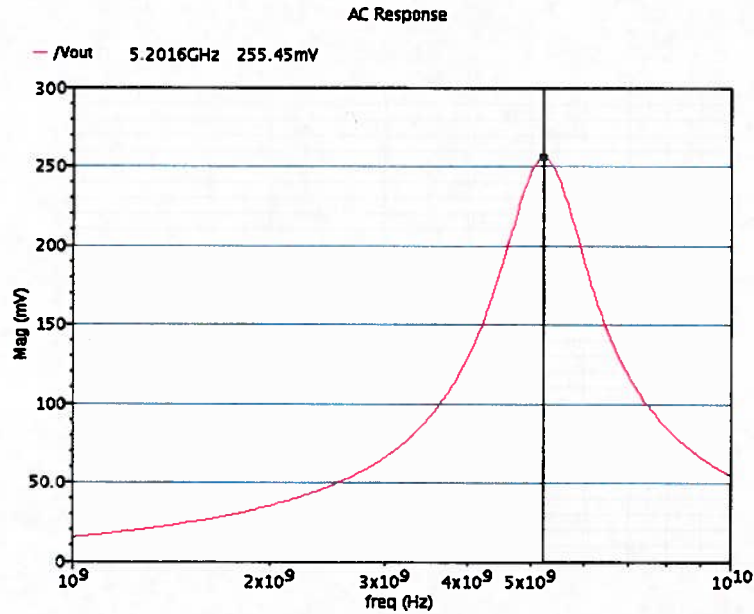
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c) Simulation shows an input resistance of **65.13 Ω** at 5.2 GHz which is not equal to 50 Ω . This is widely due to the non-idealities in the circuit especially the inductor because of its behavior at low and high frequencies. At higher frequencies it's more resistive hence increasing the input resistance



Now the width of M1 needs to be increased to decrease the input resistance. The New value for W is **58.05 μm** .

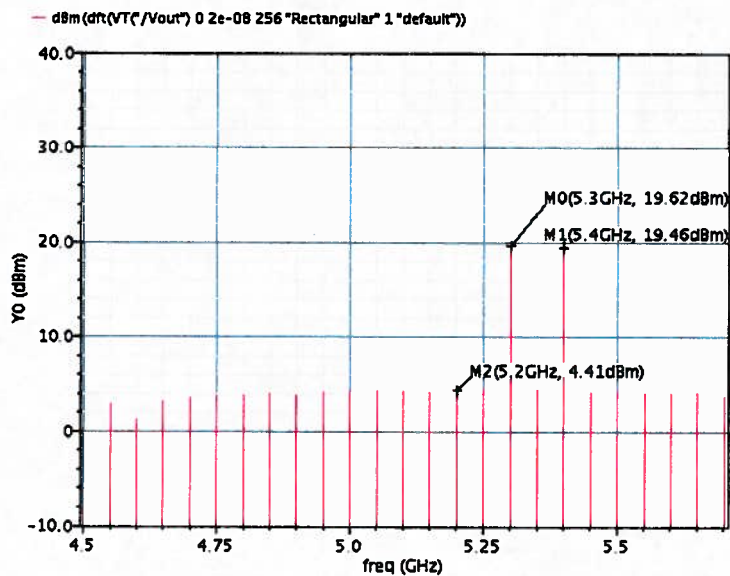


The resonance frequency is shifted as shown in the above figure. To shift back the resonance frequency to 5.2GHz we need to change L_1 . The new value for L_1 is **4.34 nH**.



d) Two tones one at 5.3 GHz and the other at 5.4 GHz were applied to the input and by performing FFT, the output spectrum was plotted as below,

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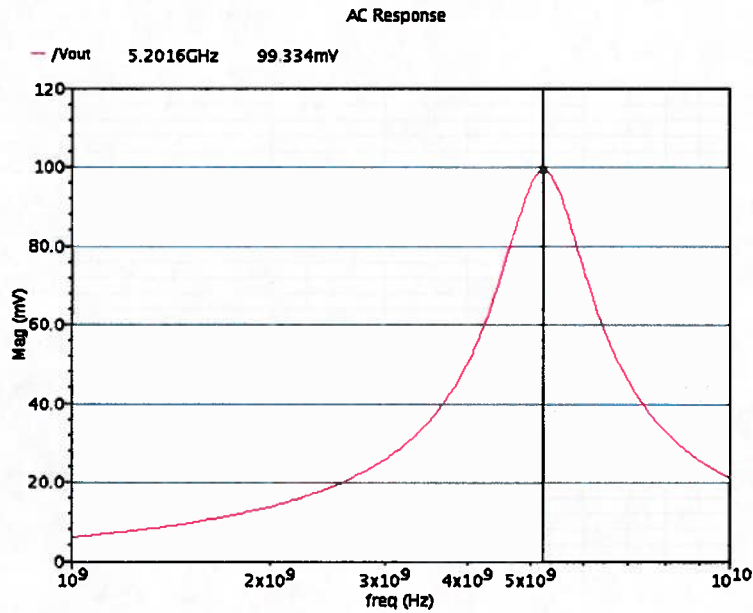
Now we can calculate IIP_3 using the shortcut method,

$$IIP_3|_{dBm} = \frac{\Delta P|_{dB}}{2} + P_{in}|_{dBm}$$

$$V_{in} = 20 \text{ mV}, \quad P_{in}|_{dBm} = -30 \text{ dBm}$$

$$IIP_3|_{dBm} = \frac{19.62 - 4.41}{2} - 30 = -22.395 \text{ dBm}$$

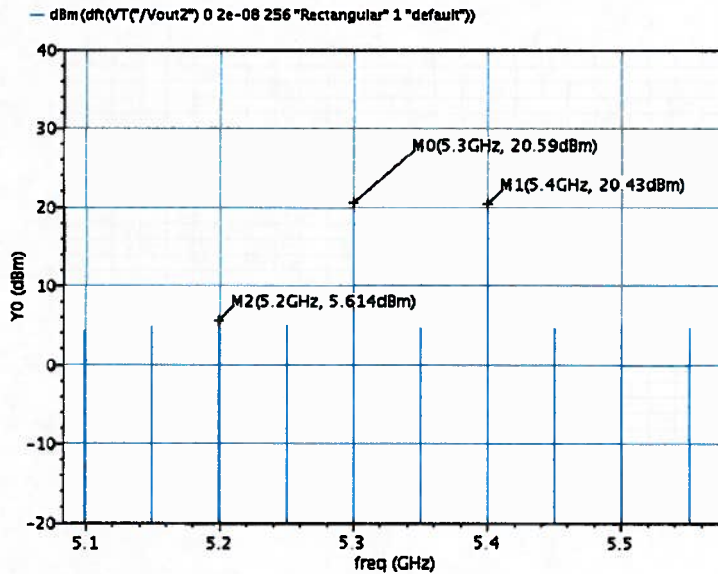
To find the voltage gain, we can look at the output voltage at the resonance frequency in the AC analysis,



$$\text{Voltage gain} = \frac{99.334}{20} = 4.967 \text{ or } 13.92 \text{ dB}$$

- e) By sweeping the value of L2 was found to be 7nH for the resonance frequency to be at 5.2 GHz. Two tones one at 5.3 GHz and the other at 5.4 GHz were applied to the input and by performing FFT, the output spectrum was plotted as below,

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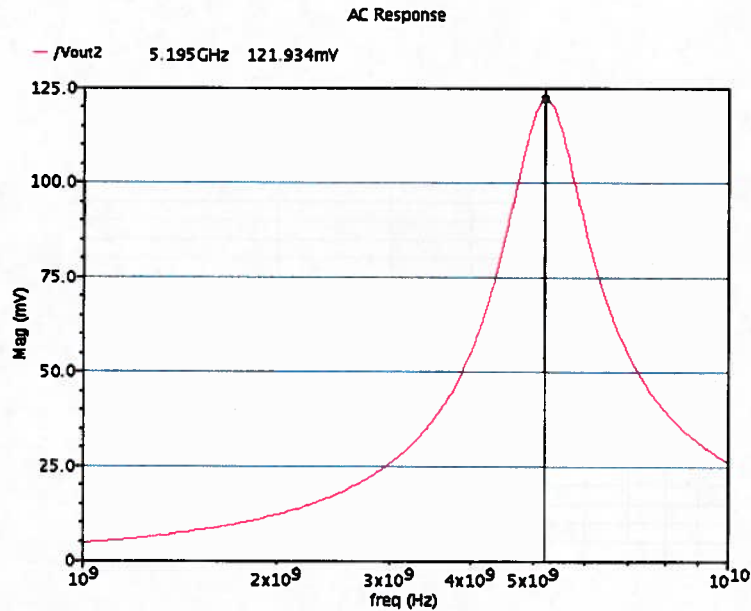
Similar to the last part we can find IIP_3 using the shortcut method,

$$IIP_3|_{dBm} = \frac{\Delta P|_{dB}}{2} + P_{in}|_{dBm}$$

$$V_{in} = 20 \text{ mV}, \quad P_{in}|_{dBm} = -30 \text{ dBm}$$

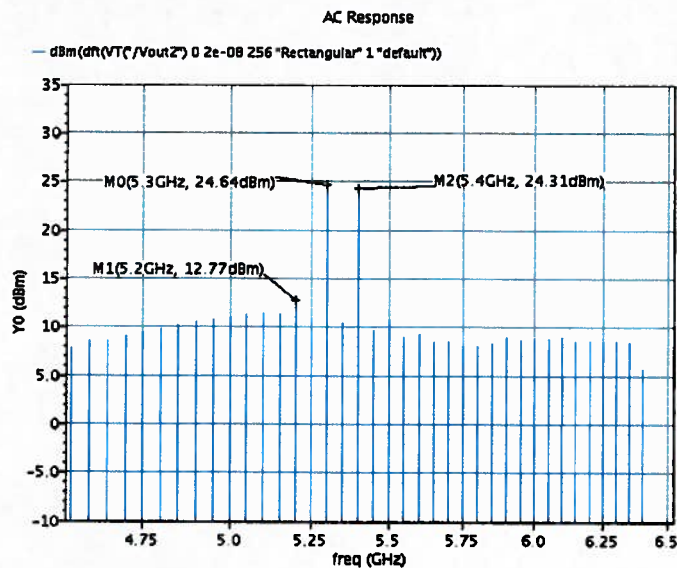
$$IIP_3|_{dBm} = \frac{20.59 - 5.614}{2} - 30 = -22.512 \text{ dBm}$$

And also the voltage gain as below,



$$\text{Voltage gain} = \frac{121.934}{20} = 6.0967 \text{ or } 15.70 \text{ dB}$$

- f) By replacing M2 with the second circuit we build the cascaded circuit. The value of L_1 had to be adjusted because the loading of the circuit has changed. The new Value for L_1 was found to be 3.05 nH. Similar to previous parts, Two tones one at 5.3 GHz and the other at 5.4 GHz were applied to the input and by performing FFT, the output spectrum was plotted as below,



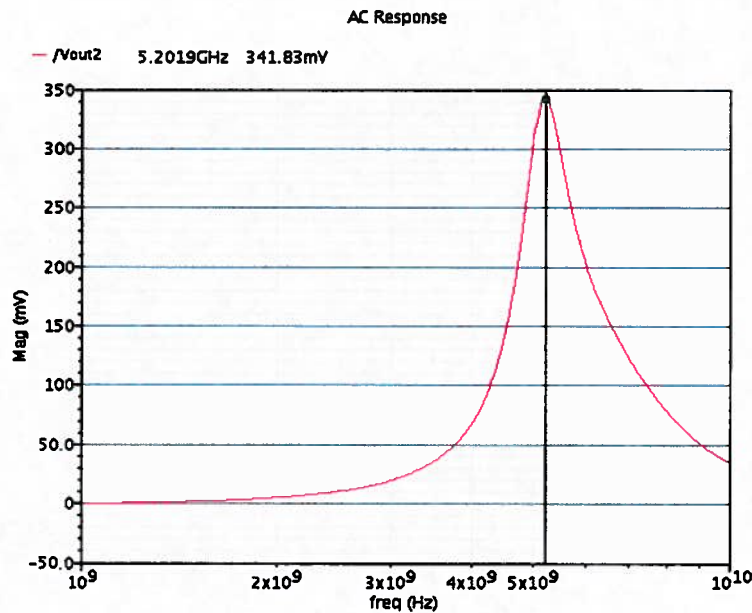
Just like last parts we can find IIP_3 using the shortcut method,

$$IIP_3|_{dBm} = \frac{\Delta P|_{dB}}{2} + P_{in}|_{dBm}$$

$$V_{in} = 20 \text{ mV}, \quad P_{in}|_{dBm} = -30 \text{ dBm}$$

$$IIP_3|_{dBm} = \frac{24.64 - 12.77}{2} - 30 = -24.065 \text{ dBm}$$

And the voltage gain,



$$\text{Voltage gain} = \frac{341.83}{20} = 17.09 \text{ or } 24.66 \text{ dB}$$

We can observe that L_1 had to be adjusted to maintain the correct resonance frequency, and we can also observe that the gain of the cascaded circuit, as a whole, turned out to be 17.09, however, if we were to multiply the loaded gain of each stage we would get 30.28. The difference is somewhat significant, and this is due to different loading effect. As for IIP_3 , theoretically we can estimate the IIP_3 of a cascaded system as below,

$$\frac{1}{IIP_{3tot}^2} = \frac{1}{IIP_{31}^2} + \frac{A_{v1}^2}{IIP_{32}^2}$$

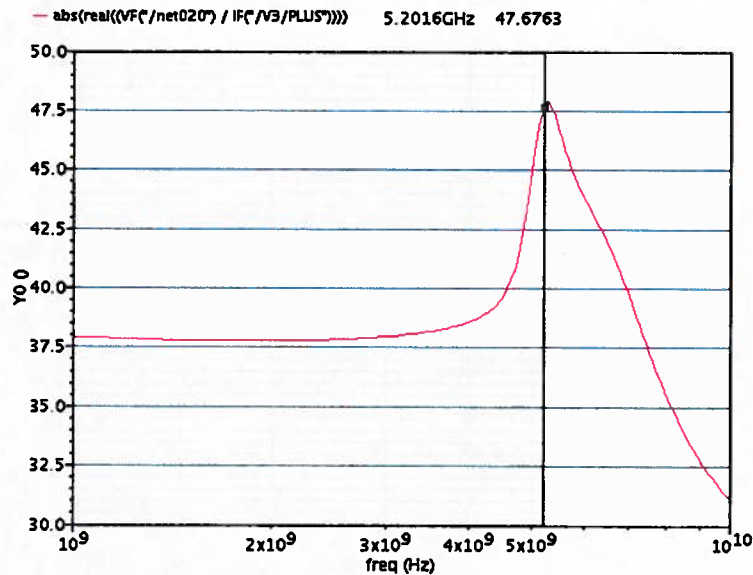
In our case the equation above would yield to,

$$\frac{1}{IIP_{3tot}^2} = \frac{1}{0.04797^2} + \frac{4.967^2}{0.047328^2}$$

$$IIP_{3tot} = -22.4$$

It's close to the simulation results.

- 5/5 g) Yes, the input resistance changes when the second stage is added. The new input resistance found to be $47.67\ \Omega$. The reason for this is first the change in the inductor value to correct the resonance frequency back to 5.2 GHz when the second stage was added, and second the increased capacitance at output of the first stage, resulting in lower impedance at the drain of D1. This would reduce the input resistance



- 3/5 h) The second stage has larger gain compared to the first stage hence it degrades and limits the IIP_3 . Comparing the analytical results from last part shows the second stage is dominant.