Problem 1   Jieqiong Du

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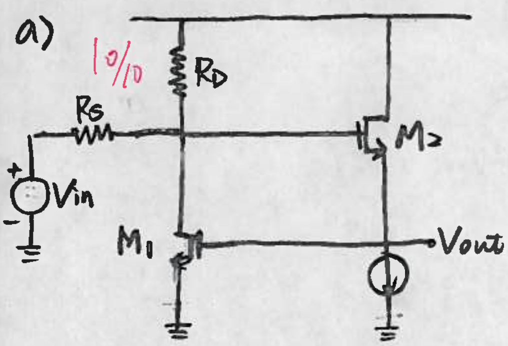
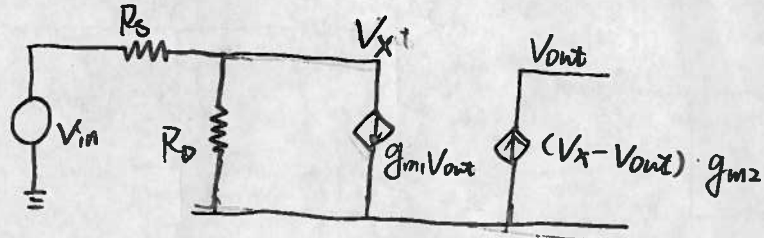
a) 10/10



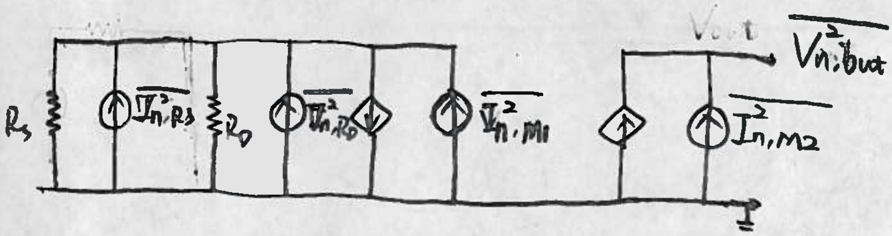


small-signal equivalent.

$$V_x = V_{out} \quad ①$$

$$\frac{V_{in}}{R_S} = \frac{V_x}{R_S} + g_{m1} V_{out} \quad ②$$

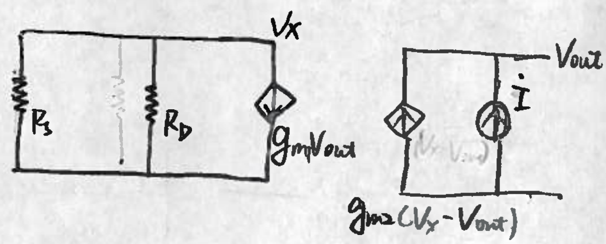
$$\Big\} \Rightarrow \frac{V_{out}}{V_{in}} = \frac{R_D}{g_{m1} R_D \cdot R_S + R_S + R_D}$$



$$\overline{V^2_{n,out}} = \overline{V^2_{n,out,R_S}} + \overline{V^2_{n,out,R_D}} + \overline{V^2_{n,out,M1}} + \overline{V^2_{n,out,M2}}$$

$$\overline{V^2_{n,out,R_S}} = 4KTR_S \cdot A^2 \; ; \; \overline{V^2_{n,out,R_D}} = \frac{4KTR_S^2}{R_{D1}} A^2 \; ; \; \overline{V^2_{n,out,RM1}} = 4KT\gamma g_{m1} R_S^2 \cdot A^2$$

$$\overline{V^2_{n,out,M2}} = 4KT\gamma g_{m2} \left|\frac{V_{out}}{I}\right|^2$$



$$\left|\frac{V_{out}}{I}\right|^2 = \left(\frac{R_S + R_D}{g_{m2}(R_S + R_D) + g_{m1} g_{m2} \cdot R_S R_D}\right)^2$$

$$\overline{V^2_{n,out}} = \left(4KTR_S + \frac{4KTR_S^2}{R_{D1}} + 4KT\gamma g_{m1} R_S^2\right) A^2 + 4KT\gamma g_{m2} \left|\frac{V_{out}}{I}\right|^2$$

$$NF = \frac{\overline{V^2_{n,out}}}{4KTR_S \cdot A^2} = 1 + \frac{R_S}{R_D} + \gamma g_{m1} R_S + \gamma \cdot \frac{(R_S + R_D)^2}{g_{m2} R_D^2 R_S}$$

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If $R_D \to \infty$, $NF = 1 + \gamma g_{m1} R_S + \frac{\gamma}{g_{m2} R_S}$. This is because even though $\overline{V^2_{n,RD}} \propto R_D$, $A^2 \propto R_D^2$, thereby, NF is smaller. 3/3

b) If $g_{m1} \to 0$, $NF = 1 + \frac{R_S}{R_D} + \gamma \frac{(R_S + R_D)^2}{g_{m2} R_D^2}$. The noise figure becomes smaller. 2/2

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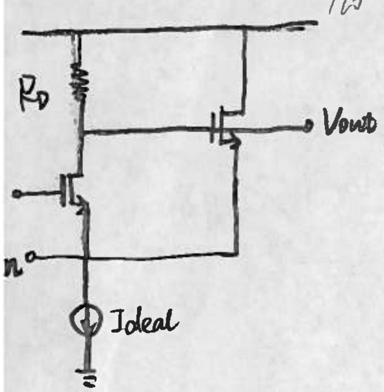
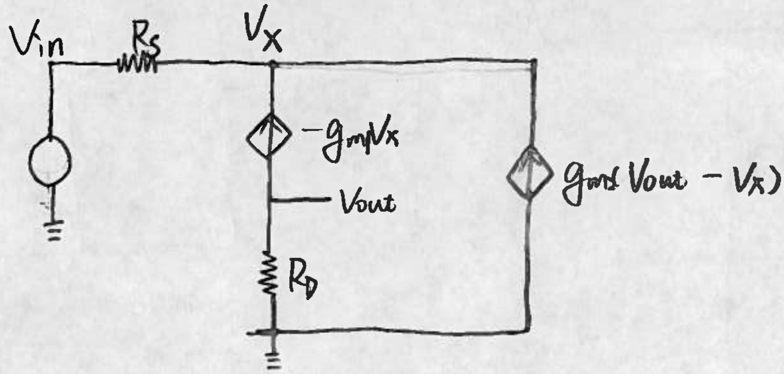
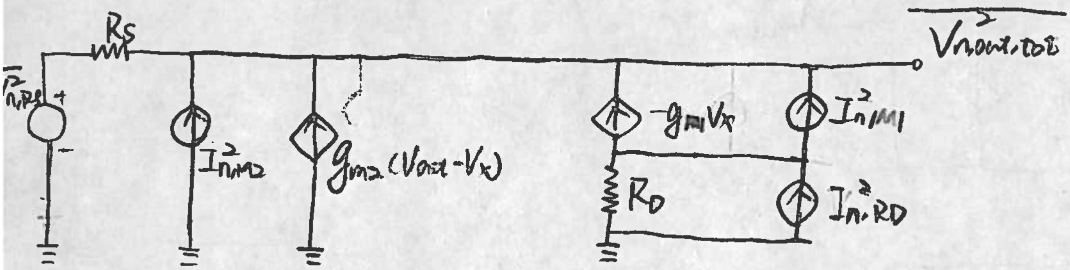




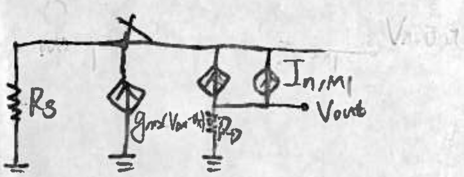
$$\frac{V_x - V_{in}}{R_s} = -g_{m1}V_x + g_{m2}(V_{out} - V_x) \quad ①$$

$$+g_{m1}V_x = V_{out}/R_D \quad ②$$

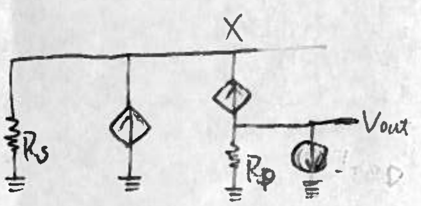
$$A = \frac{V_{out}}{V_{in}} = \frac{g_{m1}R_D}{1 + g_{m1}R_s - g_{m2}g_{m1}R_sR_D + g_{\cdots}}$$



$$\overline{V^2_{n,out\to R_s}} = 4KTR_s \cdot A^2 \ ; \quad \overline{V^2_{n,out\to M2}} = 4KT\gamma g_{m2} R_s^2 A^2$$



$$\left|\frac{V_{n,out}}{I_{n,M1}}\right| = \frac{(g_{m2}R_s + 1)R_D}{1 + g_{m1}R_s + g_{m2}R_s - g_{m1}g_{m2}R_sR_D}$$



$$\left|\frac{V_{n,out}}{I_{n,R_s}}\right| = \frac{(g_{m2}R_s + g_{m1}R_s + 1)R_D}{1 + g_{m1}R_s + g_{m2}R_s - g_{m1}g_{m2}R_sR_D}$$

$$\overline{V^2_{n,out,tot}} = 4KTR_s \cdot A^2 + 4KT\gamma g_{m2}R_s^2 \cdot A^2 + 4KT\gamma g_{m1}\left|\frac{V_{n,out}}{I_{n,M1}}\right|^2 + \frac{4KT}{R_D}\left|\frac{V_{n,out}}{I_{n,R_s}}\right|^2$$

$$NF = \frac{\overline{V^2_{n,out,tot}}}{4KTR_sA^2} = 1 + \gamma g_{m2}R_s + \frac{\gamma(g_{m2}R_s+1)^2}{g_{m1}R_s} + \frac{(g_{m1}R_s + g_{m2}R_s + 1)^2}{g_{m1}^2 R_s R_D}$$

If $R_D \to \infty$, $NF = 1 + \gamma g_{m2}R_s + \dfrac{\gamma(g_{m2}R_s+1)^2}{g_{m1}R_s}$

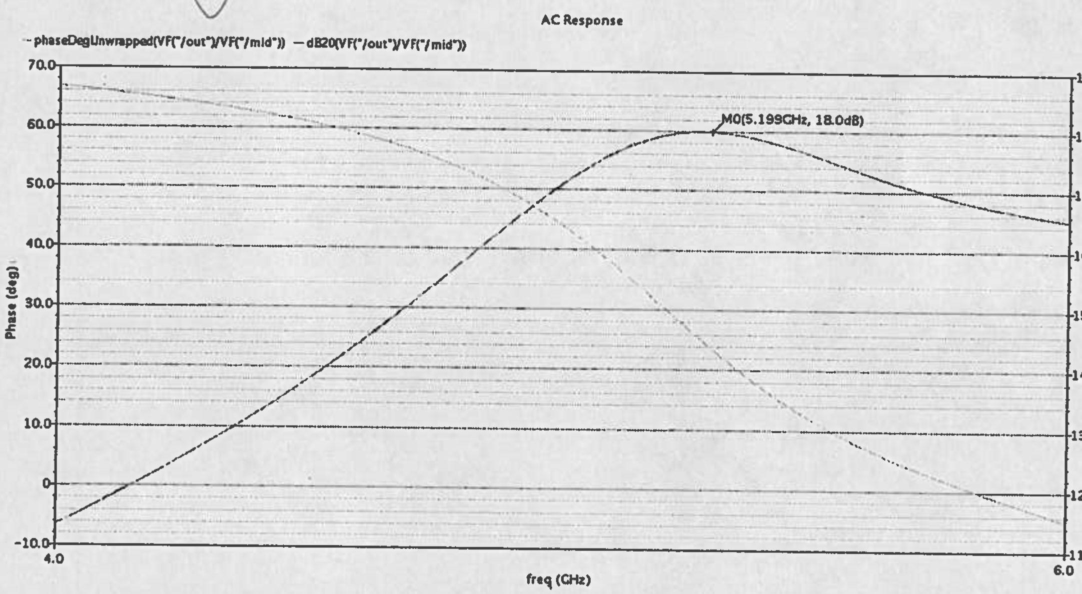
If $g_{m2} \to 0$, $NF = 1 + \dfrac{\gamma}{g_{m1}R_s} + \dfrac{(g_{m1}R_s+1)^2}{g_{m1}^2 R_s R_D}$

As R_D gets infinite, $\overline{V^2_{n,RD}} \propto R_D$, but, the ~~gain~~ signal power increased by R_D^2, so NF gets better.

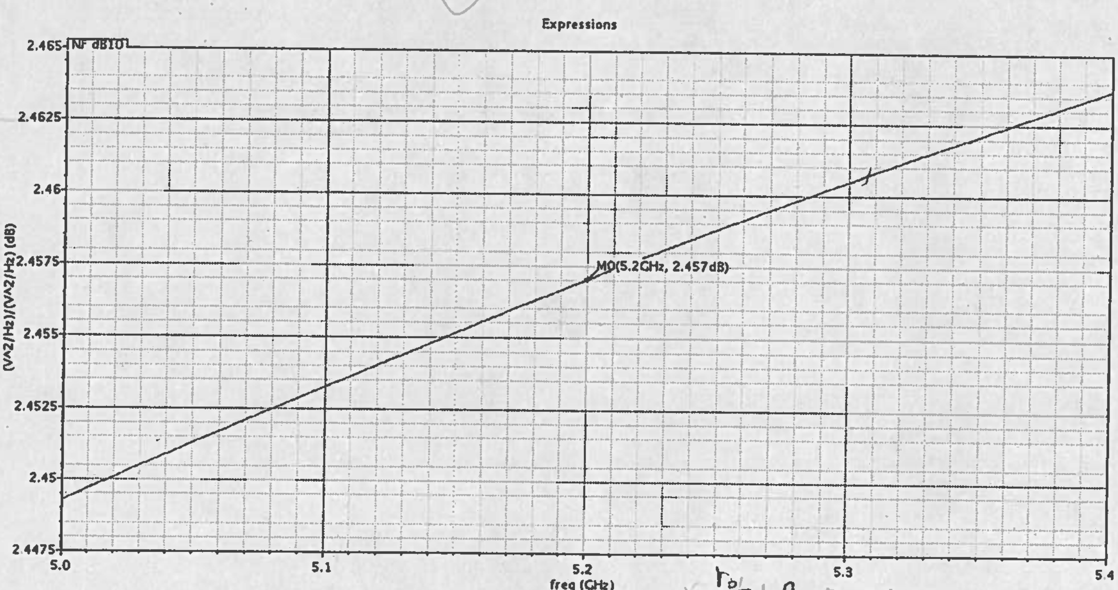
Problem 2

a) $L_2 = 8.5\,nH$

$W_2 = 4.1\,uM$



b) Measured NF = 2.457 dB



$g_{m1} = 21.66\,mS$ $\quad g_{mb} = 3.92\,mS$

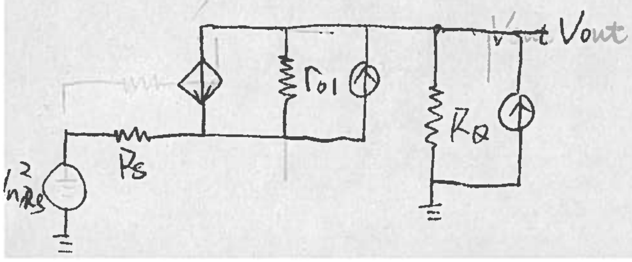
$r_{o1} = \dfrac{1}{g_{ds}} = 1003.5\,\Omega$

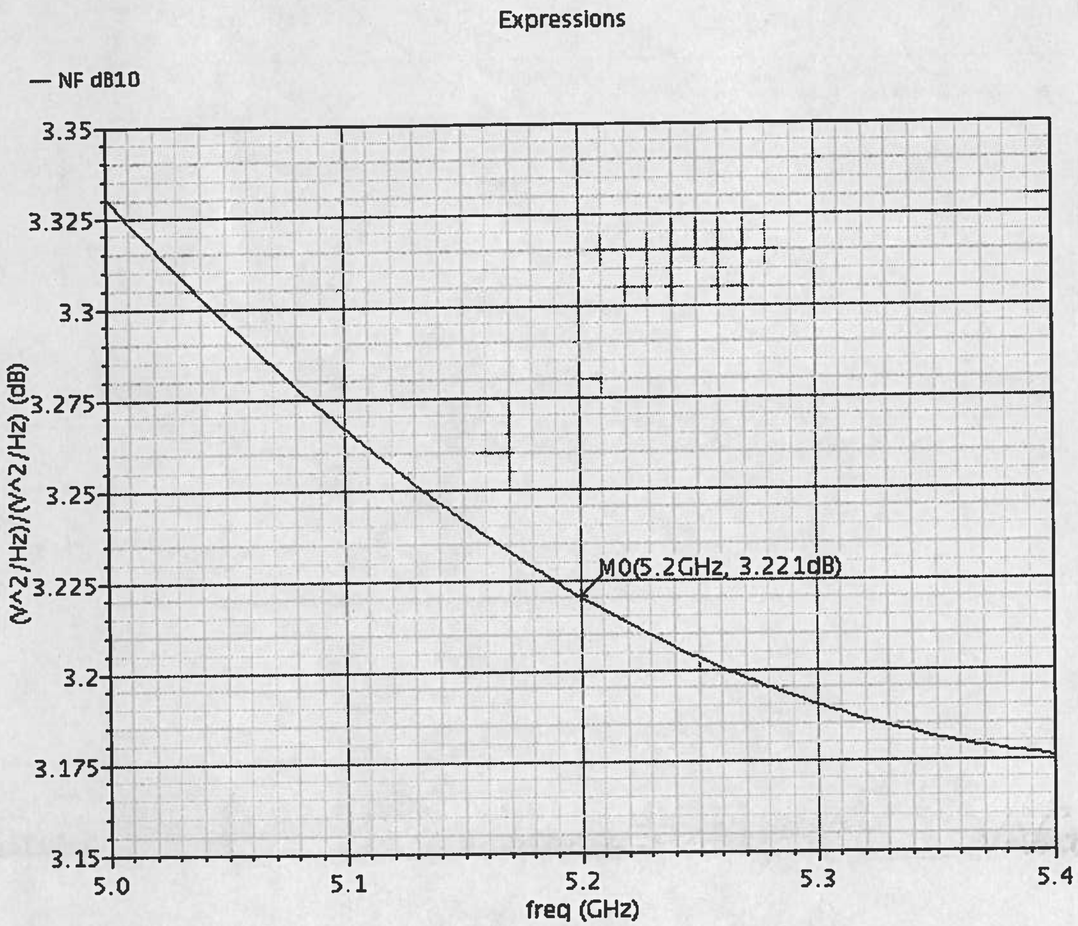
$$A = \frac{V_{out}}{V_{in}} = \frac{\frac{r_{o1}}{R_L} + g_{m1} r_{o1} + 1}{\left(1 + \frac{R_S}{R_Q}\left(\frac{r_{o1}}{R_L} + g_{m1} r_{o1} + 1\right)\right)} = 5.4$$

$$NF = 1 + \frac{(R_Q \| r_o)^2 / R_Q}{R_S A_0^2} + \frac{\gamma g_{m1}}{R_S A_0^2}\left(\frac{1}{1 + R_S g_m}\right)^2 \times (R_Q \| r_o)^2$$

$= 1 + 0.197 + \gamma \cdot 0.76 \quad (\gamma = 0.66)$

$= 1.6986$





$NF_1 = 1.76$ $NF_{tot} = 2.10$

$\Delta NF = 0.34$

The contribution of the second stage to NF is very small.