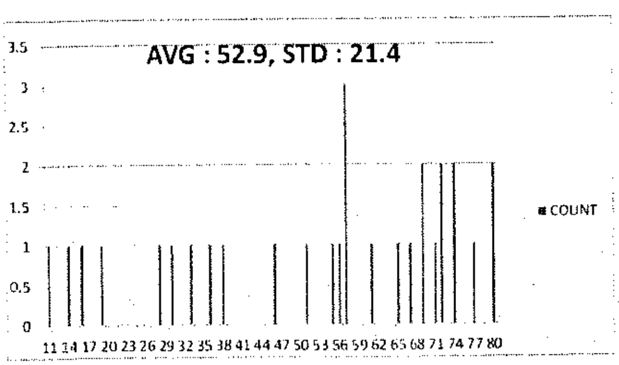
EE215C

Midterm Exam
Winter 2013

Name: Solutions

Time Limit: 2 Hours

Open Book, Open Notes, Calculators are allowed.



1. 20
2. 20
3. 20
4. 20

Total: 80

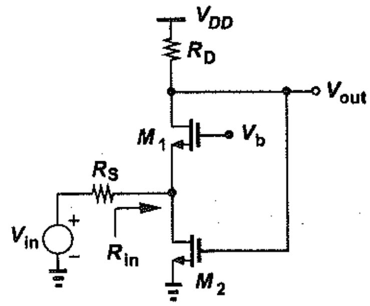
1. Consider the amplifier shown below. The transistors operate in saturation and channel-length modulation and body effect are neglected.

(a) Compute the input resistance, R_{in}.

(b) Compute the noise figure of the circuit with respect to a source impedance R_S.

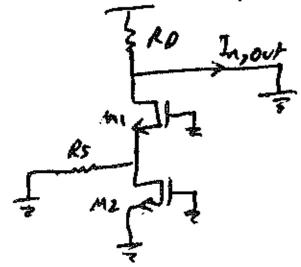
(c) Simplify the NF expression if $R_{in} = R_S$. Can you compare this result with the NF of a simple CG stage?

(d) Suppose a mixer following this circuit produces an LO voltage waveform $V_0 \cos\omega_0 t$ at the drain of M_1. Determine the LO leakge to the antenna.



a) $R_{in} = \frac{1}{g_{m_1}} \| \frac{1}{g_{m_1} R_D g_{m2}} = \frac{1}{g_{m_1}(1+g_{m2}R_D)}$

b)



$$\overline{I^2_{n,out,R_S}} = \frac{4kT}{R_S}\left(\frac{R_S}{R_S+\frac{1}{g_{m_1}}}\right)^2 \qquad \overline{I^2_{n,out,R_D}} = \frac{4kT}{R_D}$$

$$\overline{I^2_{n,out,M_2}} = 4kT\gamma g_{m_2}\left(\frac{R_S}{R_S+\frac{1}{g_{m_1}}}\right)^2 \qquad \overline{I^2_{n,out,M_1}} = 4kT\gamma g_{m_1}\left(\frac{\frac{1}{g_{m_1}}}{R_S+\frac{1}{g_{m_1}}}\right)^2$$

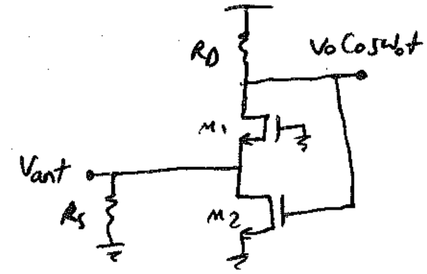
$$NF = \frac{\overline{I^2_{n,out,tot}}}{\overline{I^2_{n,out,R_S}}} = 1 + \gamma g_{m2} R_S + \frac{\gamma}{g_{m_1} R_S} + \frac{\left(R_S + \frac{1}{g_{m_1}}\right)^2}{R_D R_S}$$

c) $NF = 1 + \gamma\left[1 + g_{m2}(R_D + R_S)\right] + \frac{\left(R_S + \frac{1}{g_{m_1}}\right)^2}{R_D R_S}$ $\qquad \frac{1}{g_{m_1}} = R_S(1+g_{m2}R_D) > R_S$

$\Rightarrow \frac{\left(R_S+\frac{1}{g_{m_1}}\right)^2}{R_D R_S} > \frac{4R_S}{R_D}$, Also $\gamma\left[1+g_{m2}(R_D+R_S)\right] > \gamma \Rightarrow NF > NF_{CG}$

CG: $NF_{CG} = 1 + \gamma + \frac{4R_S}{R_D}$

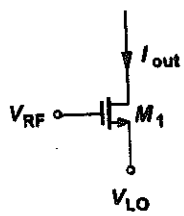
d) $V_{ant} = g_{m2} V_o \cos\omega_0 t \left(\frac{\frac{1}{g_{m_1}}}{R_S + \frac{1}{g_{m_1}}}\right) \times R_S$



2. A single MOSFET can serve as a "poor man's" mixer. Suppose the device is in saturation and the output of interest is I_{out}. Neglect channel-length modulation and body effect. Assume the device is characterized by $I_D = K(V_{RF} - V_{LO})^2$, where $V_{LO} = B\cos\omega_{LO}t$. The LO amplitude is not large enough to cause complete switching.

(a) Suppose $V_{RF} = A\cos\omega_1 t + V_{GS0}$, where V_{GS0} is a bias value to ensure the device is on and has the required transconductance. Determine the voltage conversion gain of the mixer.

(b) Suppose $V_{RF} = A\cos\omega_1 t + A\cos\omega_2 t + V_{GS0}$. Determine the IIP_2 of the mixer.



(a)

$$I_{out} = K(V_{RF} - V_{LO})^2$$

$$= K(A\cos W_1 t + V_{GS0} - B\cos W_{LO}t)^2$$

$$= K\big(A^2\cos^2 W_1 t + V_{GS0}^2 + B^2\cos^2 W_{LO}t$$

$$-2AB\cos W_1 t\cos W_{LO}t + 2AV_{GS0}\cos W_1 t$$

$$-2BV_{GS0}\cos W_{LO}t\big)$$

$$= K\Big(A^2\cdot\frac{1+\cos 2W_1 t}{2} + V_{GS0}^2 + B^2\frac{1+\cos 2W_{LO}t}{2}$$

$$-2AB\frac{\cos(W_1+W_{LO})t + \cos(W_1-W_{LO})t}{2} + 2AV_{GS0}\cos W_1 t$$

$$-2BV_{GS0}\cos W_{LO}t\Big)$$

$$\text{Conversion gain} = \frac{-ABK}{A} = -BK$$

(b)

$$I_{out} = K(A\cos W_1 t + A\cos W_2 t + V_{GS0} - B\cos W_{LO}t)^2$$

$$= K(2A^2\cos W_1 t\cos W_2 t - 2AB\cos W_1 t\cos W_{LO}t + \cdots)$$

$$= K\{A^2[\cos(W_1+W_2)t + \cos(W_1-W_2)t] - AB[\cos(W_1-W_{LO})t + \cos(W_1+W_{LO})t + \cdots\}$$

$$A_{IIP_2}^2 = A_{IIP_2}\cdot B$$

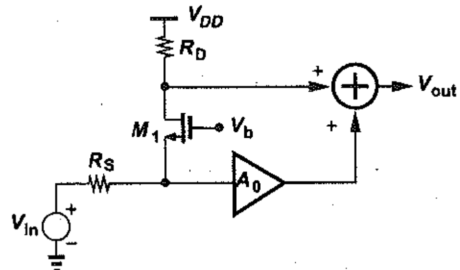
$$\Rightarrow A_{IIP_2} = B$$

3. Consider the common-gate LNA shown below, where an auxiliary amplifier is added to cancel the noise of M_1. Neglect channel-length modulation and body effect.

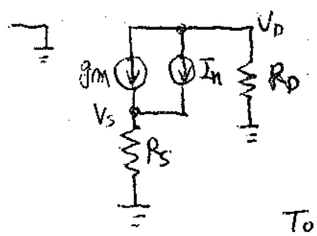
(a) Determine A_0 so that the noise of M_1 is cancelled.

(b) Determine the overall voltage gain of the circuit, V_{out}/V_{in}.

(b) Determine the overall noise figure if the auxiliary amplifier exhibits an input-referred noise voltage of $\overline{V_n^2}$.



a)



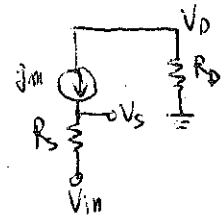
$$I_n = g_m V_s + \frac{V_s}{R_S} \qquad \therefore \frac{R_S}{1+g_m R_S} I_n = V_s$$

$$I_n - g_m V_s + \frac{V_D}{R_D} = 0 \qquad \therefore \frac{-R_D}{1+g_m R_S} I_n = V_D$$

To cancel I_n,

$$A_0 \cdot \frac{R_S}{1+g_m R_S} I_n - \frac{R_D}{1+g_m R_S} I_n = 0 \qquad \therefore A_0 = \frac{R_D}{R_S}$$

b)



$$V_s = \frac{\frac{1}{g_m}}{\frac{1}{g_m} + R_S} V_{in} \qquad V_D = g_m R_D V_s = \frac{g_m R_D}{1+g_m R_S} V_{in}$$

$$V_{out} = A_0 V_s + V_D = \left(\frac{A_0}{1+g_m R_S} + \frac{g_m R_D}{1+g_m R_S}\right) V_{in}$$

$$\therefore \frac{V_{out}}{V_{in}} = \frac{R_D}{R_S}$$

c) $$\overline{V_{n,out}^2} = 4kTR_S\left(\frac{R_D}{R_S}\right)^2 + \overline{V_n^2}\left(\frac{R_D}{R_S}\right)^2 + 4kTR_D$$

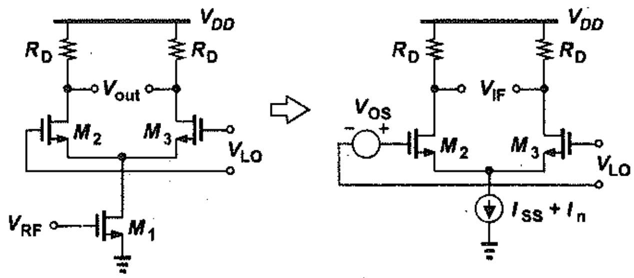
$$\overline{V_{n,in}^2} = 4kTR_S + \overline{V_n^2} + 4kT\cdot\frac{R_S^2}{R_D}$$

$$NF = \frac{\overline{V_{n,in}^2}}{4kTR_S} = 1 + \frac{R_S}{R_D} + \frac{\overline{V_n^2}}{4kTR_S}$$

4. We know that the input to a mixer can feed to the output without frequency translation if the circuit has asymmetries. We wish to study the feedthrough of the flicker noise of M_1 to the output in the presence of an offset voltage, V_{OS}.

(a) Modeling M_1 as shown on the right, where I_n represents its flicker noise, determine the total output flicker noise voltage.

(b) Can you roughly compare this result with the flicker noise contribution of M_2 as derived in class?



(a) For zero crossing due to V_{OS},

$$V_{cm} + V_{p,LO}\sin\omega_{LO}t + V_{OS} = V_{CM} - V_{p,LO}\sin\omega_{LO}t$$

$$2V_{p,LO}\sin\omega_{LO}t = -V_{OS}$$

For $t \to 0$

$$2V_{p,LO}\omega_{LO}t \approx -V_{OS}$$

$$|\Delta T| = \frac{|V_{OS}|}{2V_{p,LO}\omega_{LO}} = \frac{V_{OS}}{S_{LO}} \quad (\text{where } S_{LO} = 2V_{p,LO}\omega_{LO})$$

$$I_{out}(t) = \sum_{k=-\infty}^{\infty} \frac{2(I_{SS}+I_n)V_{OS}}{S_{LO}}\,\delta\!\left(t - k\frac{T_{LO}}{2}\right)$$

$$\Rightarrow I_{out}(f) = \frac{4(I_{SS}+I_n)}{T_{LO}S_{LO}}\,V_{OS}\sum_{k=-\infty}^{\infty}\delta(f-2kf_{LO})$$

For baseband component, $k=0$,

$$I_{out}(f)\big|_{k=0} = \frac{I_{SS}+I_n}{\pi V_{p,LO}}\cdot V_{OS} \Rightarrow V_{out}(f)\big|_{k=0} = \underbrace{\frac{I_{SS}\cdot V_{OS}R_D}{\pi V_{p,LO}}}_{\text{output offset}} + \frac{I_nV_{OS}R_D}{\pi V_{p,LO}}$$

$$\boxed{\therefore V_{n1,out}(f) = \frac{I_nV_{OS}R_D}{\pi V_{p,LO}}}$$

(b) the flicker noise contribution of M_2: $\frac{I_{SS}R_D}{\pi V_{p,LO}}V_{n2}(f)$ $(= V_{n2,out})$

$$\frac{V_{n1,out}}{V_{n2,out}} = \frac{I_nV_{OS}}{I_{SS}V_{n2}} = \frac{g_{m1}V_{n1}V_{OS}}{I_{SS}V_{n2}} = \frac{2V_{OS}}{(V_{GS}-V_{th})_1}\cdot\frac{V_{n1}}{V_{n2}} \quad \left(g_{m1} = \frac{2I_{SS}}{(V_{GS}-V_{th})_1}\right)$$

Generally, $(V_{GS}-V_{th})_1$ is larger than V_{OS} and hence $V_{n1,out}$ is less than $V_{n2,out}$.