EE215C

Midterm Exam

Winter 2010

Name: Solutions

Time Limit: 2 Hours

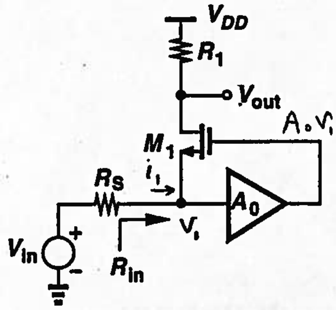
Open Book, Open Notes, Calculators are allowed.

1. 15
2. 15
3. 10
4. 10

Total: 50

1. Consider the amplifier shown below, where A_0 is an ideal voltage amplifier, the transistor operates in saturation, and channel-length modulation and body effect are neglected. Neglect all device capacitances.

(a) Compute the input resistance, R_{in}.
(b) Compute the noise figure of the circuit with respect to a source impedance R_S. Assume A_0 has an input-referred noise voltage of $\overline{V_n^2}$.
(c) Simplify the NF expression if $R_{in} = R_S$.



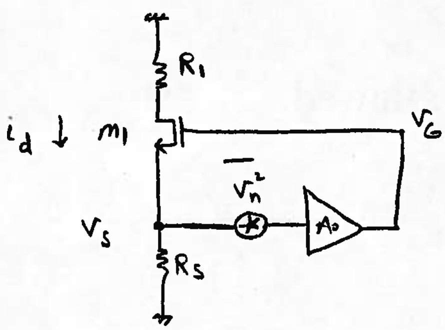
(a) $R_{in} = \dfrac{V_1}{i_1}$

$i_1 = -g_{m1}(A_0 - 1)V_1 \Rightarrow$

$R_{in} = \dfrac{-1}{g_{m1}(A_0 - 1)}$ $\boxed{3}$

(b) $\overline{V^2_{n,out1}}\Big|_{R_1} = 4KTR_1$ $\boxed{2}$

$\overline{V^2_{n,out2}}\Big|_{R_S} = \dfrac{4KT}{R_S}\cdot\left(\dfrac{R_S}{R_S + R_{in}}\cdot R_1\right)^2$ $\boxed{2}$



$V_S = i_d \cdot R_S \quad V_G = (i_d R_S + V_n)A_0$

$\Rightarrow i_d = g_m V_{gs} = g_m\left[A_0(V_n + i_d R_S) - i_d R_S\right]$

$\Rightarrow i_d = \dfrac{g_m A_0 V_n}{1 + g_m R_S - g_m A_0 R_S}$

$\Rightarrow \overline{V^2_{n,out3}}\Big|_{A_0} = \overline{V_n^2}\,\dfrac{g_m^2 A_0^2 R_1^2}{\left[1 + g_m R_S(1 - A_0)\right]^2}$ $\boxed{2}$

$\overline{V^2_{n,out4}}\Big|_{M_1} = \dfrac{4KT\gamma g_{m1}}{g_{m1}^2}\cdot\dfrac{1}{A_0^2}\cdot\dfrac{g_m^2 A_0^2 R_1^2}{\left[1 + g_m R_S(1 - A_0)\right]^2} = \dfrac{4KT\gamma g_m \cdot R_1^2}{\left[1 + g_m R_S(1 - A_0)\right]^2}$ $\boxed{2}$

$\overline{V^2_{n,tot}} = \sum \overline{V^2_{n,out\,i}} \Rightarrow NF = \dfrac{\overline{V^2_{n,tot}}}{\overline{V^2_{n,R_S}}}$

$$\Rightarrow NF = 1 + \frac{(R_S + R_{in})^2}{R_S \cdot R_1} + \frac{\overline{V_n^2}\,\dfrac{g_m^2 A_o^2 R_1^2}{[1 + g_m R_S (1 - A_o)]^2}}{\dfrac{4KT}{R_S}\left(\dfrac{R_S R_1}{R_S + R_{in}}\right)^2} + \frac{\gamma g_m \cdot (R_S + R_{in})^2}{R_S\,[1 + g_m R_S (1 - A_o)]^2} \qquad \boxed{1}$$

(C) if $R_{in} = R_S = \dfrac{-1}{g_m (A_o - 1)} \Rightarrow$

$\boxed{13}$

$$NF = 1 + \frac{4R_S}{R_1} + \overline{V_n^2}\,\frac{g_m^2 A_o^2 R_S}{4KT} + \gamma g_m R_S$$

2. For the circuit shown below, we express the input-output characteristic as

$$I_{out} - I_0 = \alpha_1(V_{in} - V_0) + \alpha_2(V_{in} - V_0)^2 + \cdots, \tag{1}$$

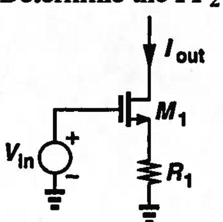
where I_0 and V_0 denote the bias values, i.e., the values in the absence of signals. We note that $\partial I_{out}/\partial V_{in} = \alpha_1$ at $V_{in} = V_0$ (or $I_{out} = I_0$). Similarly, $\partial I_{out}^2/\partial^2 V_{in} = 2\alpha_2$ at $V_{in} = V_0$ (or $I_{out} = I_0$).

(a) Write a KVL around the input network in terms of V_{in} and I_{out} (with no V_{GS}). Differentiate both sides *implicitly* with respect to V_{in}. You will need this equation in part (b). Noting that $2\sqrt{KI_0} = g_m$, where $K = \mu_n C_{ox} W/L$, find $\partial I_{out}/\partial V_{in}$ and hence α_1.

$\sqrt{2kI_0}$

(b) Differentiate the equation obtained in part (a) with respect to V_{in} once more and compute α_2 in terms of I_0 and g_m.

(c) Determine the IP_2 of the circuit.



(a) $$\frac{1}{2}k(V_{in} - I_{out}R_1 - V_{th})^2 = I_{out}$$

$$V_{in} = R_1 I_{out} + V_{th} + \sqrt{\frac{2I_{out}}{k}}$$

$$\frac{\partial V_{in}}{\partial V_{in}} = R_1\frac{\partial I_{out}}{\partial V_{in}} + \frac{1}{2}\left(\frac{2I_{out}}{k}\right)^{-\frac{1}{2}}\cdot\frac{2}{k}\frac{\partial I_{out}}{\partial V_{in}}$$

$$1 = \left(R_1 + \sqrt{\frac{1}{2I_{out}k}}\right)\frac{\partial I_{out}}{\partial V_{in}} \quad \#$$

$$\alpha_1 = \frac{\partial I_{out}}{\partial V_{in}}\Big|_{V_{in}=V_0} = \frac{g_m}{1+g_mR_1} \quad \#$$

(b) $$\frac{1}{\partial V_{in}} = \frac{\partial I_{out}}{\partial V_{in}}\left[-\frac{1}{2}(2kI_{out})^{-\frac{3}{2}}\cdot 2k\frac{\partial I_{out}}{\partial V_{in}}\right] + \frac{1}{\alpha_1}\frac{\partial I^2_{out}}{\partial^2 V_{in}}$$

$$0 = \alpha_1\left[-k\alpha_1\frac{1}{g_m^3}\right] + \frac{1}{\alpha_1}\frac{\partial I^2_{out}}{\partial^2 V_{in}}$$

$$\alpha_2 = \frac{1}{2}\frac{\partial I^2_{out}}{\partial^2 V_{in}}\Big|_{V_{in}=V_0} = \frac{\alpha_1^3}{2g_m^3}\frac{g_m^2}{2I_0} = \frac{\alpha_1^3}{4I_0g_m} \quad \#$$

$$k = \frac{g_m^2}{2I}$$

$$\left[= \frac{k}{2(1+g_mR_1)^3}\right]$$

(c) $$(A_1\cos\omega_1 t + A_2\cos\omega_2 t)^2 = A_1\cos^2\omega_1 t + A_2^2\cos\omega^2 t + 2A_1A_2\cos\omega_1 t\cos\omega_2 t$$

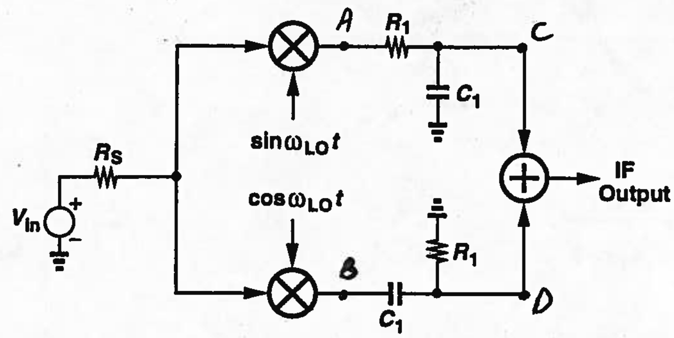
intermodulation term: $A_1A_2[\cos(\omega_1-\omega_2)t + \cos(\omega_1+\omega_2)t]$

$$\alpha_1\cdot IP2 = \alpha_2\cdot IP2^2$$

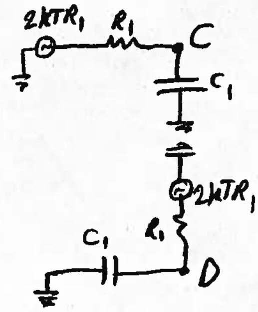
$$IP2 = \frac{\alpha_1}{\alpha_2} = \frac{4I_0g_m}{\alpha_1^2} = \frac{4I_0g_m}{\left(\frac{g_m}{1+g_mR_1}\right)^2} \quad \#$$

$$\left(\text{or } \frac{2g_m(1+g_mR_1)^2}{k}\right)$$

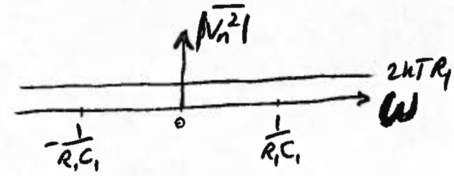
3. The simplified Hartley architecture shown below incorporates mixers having a voltage conversion gain of A_{mix} and an infinite input impedance. Taking into account only the noise of the two resistors, compute the noise figure of the receiver with respect to a source resistance of R_S at an IF of $1/(R_1C_1)$.



Assuming $R_1 \gg R_{out}|_{mixer}$:



Note that we have used Two-sided Noise spectrum.



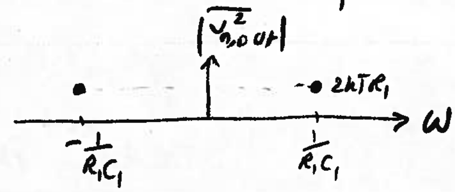
$$\overline{V_{n,C}^2} = 2kTR_1 \left|\frac{1}{1+R_1C_1s}\right|^2$$

$$\overline{V_{n,D}^2} = 2kTR_1 \left|\frac{1}{1+R_1C_1s}\right|^2$$

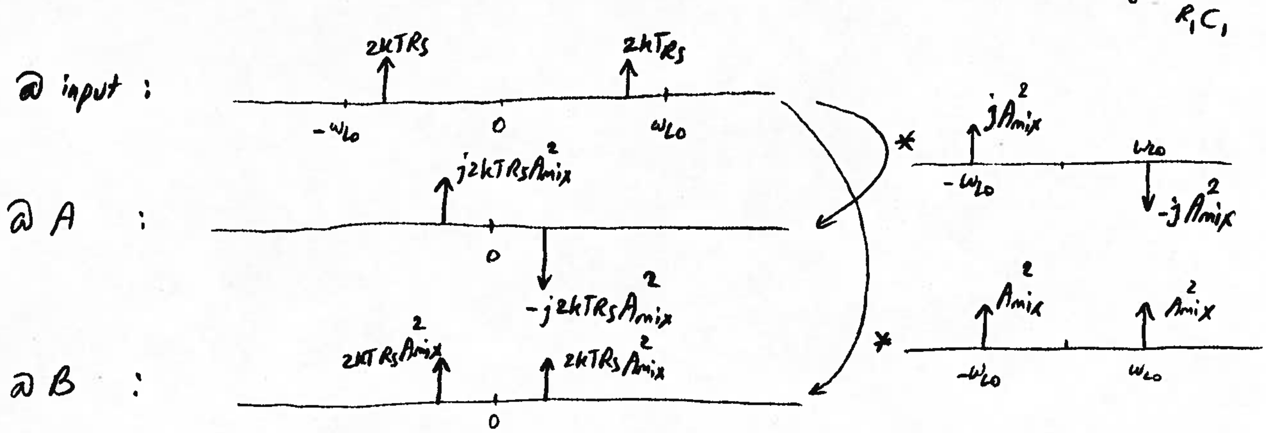
$s = \pm j\left(\frac{1}{R_1C_1}\right)$

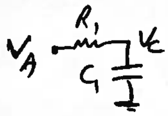
$$\overline{V_{n,C}^2} = \overline{V_{n,D}^2} = 2kTR_1\left|\frac{1}{1\pm j}\right|^2 = kTR_1 \Rightarrow \overline{V_{n,out}^2} = \overline{V_{n,C}^2} + \overline{V_{n,D}^2}$$

$$= 2kTR_1$$



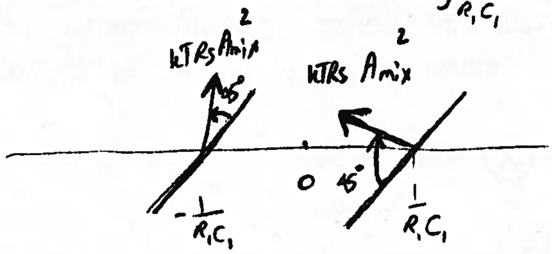
For noise of R_S, we should look into the noise at $|\omega| = \omega_{LO} - \frac{1}{R_1C_1}$.

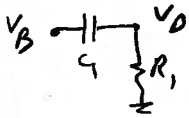




$$\frac{V_C}{V_A} = \frac{1}{R_1C_1s+1} \qquad \left.\frac{V_C}{V_A}\right|_{j\frac{1}{R_1C_1}} = \frac{1}{j+1} \qquad \left.\frac{V_C}{V_A}\right|_{-j\frac{1}{R_1C_1}} = \frac{1}{1-j}$$

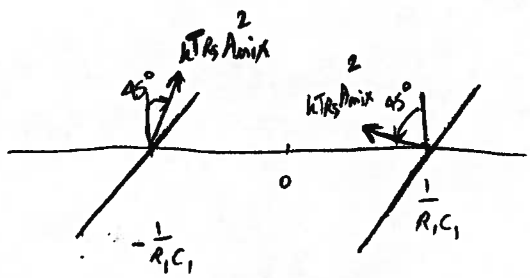
@ C :



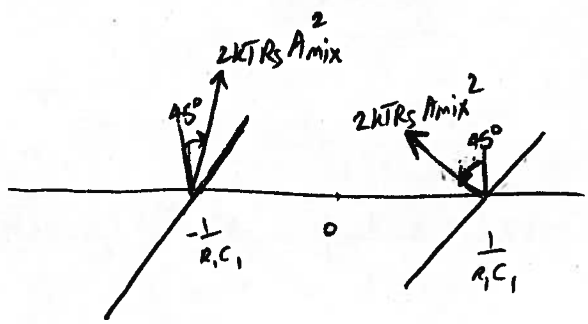


$$\frac{V_D}{V_B} = \frac{R_1C_1s}{R_1C_1s+1} \qquad \left.\frac{V_D}{V_B}\right|_{j\frac{1}{R_1C_1}} = \frac{j}{j+1} = \frac{1}{1-j} \qquad \left.\frac{V_D}{V_B}\right|_{-j\frac{1}{R_1C_1}} = \frac{-j}{-j+1} = \frac{1}{1+j}$$

@ D :



@ output

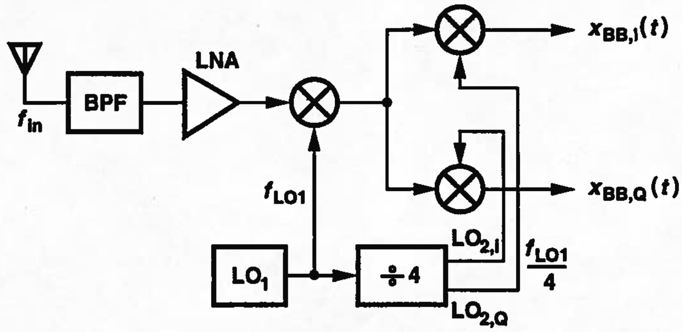


$$NF = \frac{2kTR_sA_{mix}^2 + 2kTR_1}{2kTR_sA_{mix}^2} = 1 + \frac{R_1}{R_sA_{mix}^2}$$

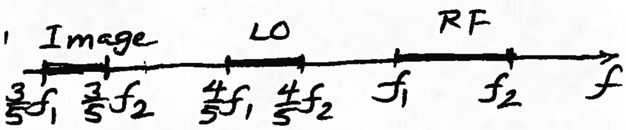
4. The sliding-IF architecture shown below is designed for the 11g band.

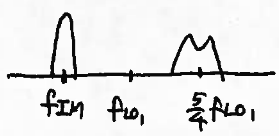
(a) Determine the image band.

(b) Determine interferer frequencies that can appear in the output baseband as a result of mixing with the third harmonic of the first LO or the third harmonic of the second LO.



a) $f_{in} - f_{LO1} - \frac{f_{LO1}}{4} = 0$ ∴ $f_{in} = \frac{5}{4} f_{LO1}$





$f_{IM} = f_{LO1} - \frac{f_{LO1}}{4} = \frac{3}{4} f_{LO1}$

Image Band = 1.44 GHz − 1.488 GHz

b) i) 3rd harmonic of the first LO

$f_{INT} - 3f_{LO1} - \frac{f_{LO1}}{4} = 0$ or $3f_{LO1} - f_{INT} - \frac{f_{LO1}}{4} = 0$

∴ $f_{INT} = \frac{11}{4} f_{LO1}$ or $\frac{13}{4} f_{LO1}$: 5.28 GHz , 6.24 GHz

ii) 3rd harmonic of the second LO

$f_{INT} - f_{LO1} - \frac{3}{4} f_{LO1} = 0$ or $f_{LO1} - f_{INT} - \frac{3}{4} f_{LO1} = 0$

∴ $f_{INT} = \frac{1}{4} f_{LO1}$ or $\frac{7}{4} f_{LO1}$: 0.48 GHz , 3.36 GHz

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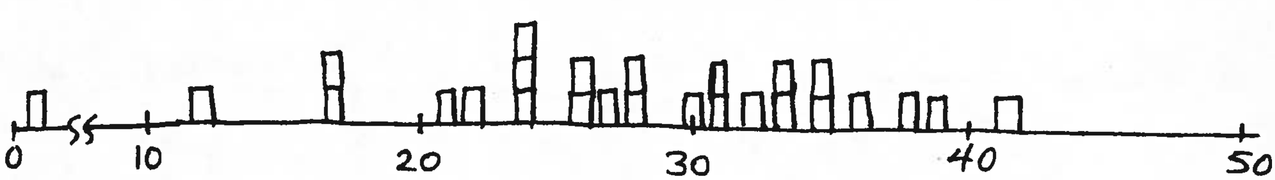
Total: 50



Avg. = 27/50

1. Consider the cascade of two nonlinear stages. The first stage is described by $y = \alpha_1 x + \alpha_3 x^3$ and the second by $y = \beta_1 x + \beta_3 x^3$.

5 (a) Determine the overall input 1-dB compression point of the cascade in terms of the 1-dB compression points of each stage.

5 (b) Determine the output 1-dB compression point of the cascade in terms of α_j and β_j.

(a)

$$A_{-1dB,1} = \sqrt{0.145 \left| \frac{\alpha_1}{\alpha_3} \right|} \qquad \text{(Stage 1)}$$

$\boxed{1}$

$$A_{-1dB,2} = \sqrt{0.145 \left| \frac{\beta_1}{\beta_3} \right|} \qquad \text{(Stage 2)}$$

After cascading: $y = \beta_1 (\alpha_1 x + \alpha_3 x^3) + \beta_3 (\alpha_1 x + \alpha_3 x^3)^3$

$$\Rightarrow y = (\beta_1 \alpha_1) x + (\beta_1 \alpha_3 + \beta_3 \alpha_1^3) x^3 + \cdots$$

$$\Rightarrow A_{-1dB,tot} = \sqrt{0.145 \left| \frac{\alpha_1 \beta_1}{\beta_1 \alpha_3 + \beta_3 \alpha_1^3} \right|} \Rightarrow \frac{1}{A_{-1dB,tot}^2} = \frac{1}{0.145} \cdot \left[\frac{\alpha_3}{\alpha_1} + \alpha_1^2 \frac{\beta_3}{\beta_1} \right] \quad \boxed{2}$$

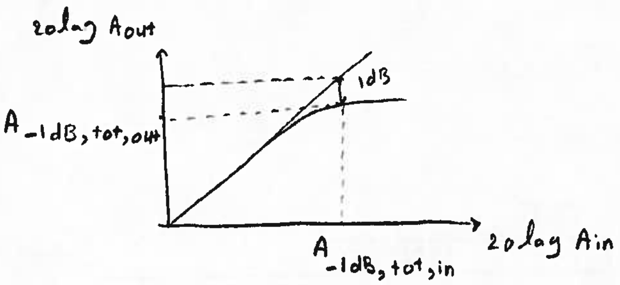
$\boxed{2}$

$$\Rightarrow \frac{1}{A_{-1dB,tot}^2} = \frac{1}{A_{-1dB,1}^2} + \frac{\alpha_1^2}{A_{-1dB,2}^2}$$

(b)

$$A_{-1dB,tot,in} = \sqrt{0.145 \left| \frac{\alpha_1 \beta_1}{\beta_1 \alpha_3 + \beta_3 \alpha_1^3} \right|} \quad \text{at the input} \quad \boxed{3}$$

ideal gain $= \alpha_1 \beta_1$



$$20 \log (\alpha_1 \beta_1 A_{-1dB,tot,in}) - 1\,dB = 20 \log (A_{-1dB,tot,out})$$

$$\Rightarrow 20 \log \left(\frac{A_{-1dB,tot,out}}{\alpha_1 \beta_1 A_{-1dB,tot,in}} \right) = -1\,dB \quad \boxed{2}$$

$$\Rightarrow A_{-1dB,tot,out} = 10^{-1/20} \cdot \alpha_1 \beta_1 \sqrt{0.145 \left| \frac{\alpha_1 \beta_1}{\beta_1 \alpha_3 + \beta_3 \alpha_1^3} \right|} \quad \text{at the output}$$

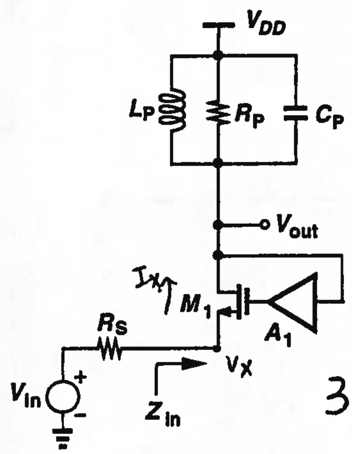
2. Consider the circuits shown below, where an ideal voltage amplifier with a gain of A_1 returns a fraction of the output to the gate of the input transistor. Neglect channel-length modulation, body effect, and other capacitances.

3 2

5 (a) Compute the input impedance. Can the real part become negative at any frequency? Explain in detail.

5 (b) Compute the noise figure with respect to a source resistance of R_S at the resonance frequency of the tank.

5 (c) Simplify the noise figure expression if the input is matched at the resonance frequency of the tank.



(a) $g_m(A_1 V_{out} - V_x) = -V_{out}\left(\frac{1}{sL_p} + \frac{1}{R_p} + sC_p\right)$

$$V_{out} = \frac{g_m V_x}{g_m A_1 + \frac{1}{sL_p} + \frac{1}{R_p} + sC_p}$$

$$I_x = V_{out}\left(\frac{1}{sL_p} + \frac{1}{R_p} + sC_p\right)$$

$$= \frac{g_m V_x}{g_m A_1 + \frac{1}{sL_p} + \frac{1}{R_p} + sC_p}\left(\frac{1}{sL_p} + \frac{1}{R_p} + sC_p\right)$$

3 $$Z_{in} = \frac{V_x}{I_x} = \frac{g_m A_1 + \frac{1}{sL_p} + \frac{1}{R_p} + sC_p}{g_m\left(\frac{1}{sL_p} + \frac{1}{R_p} + sC_p\right)} \quad \#$$

$$\text{real-}Z_{in} = \frac{1}{g_m}\,\frac{\left(g_m A_1 + \frac{1}{R_p}\right)\frac{1}{R_p} + \left(\omega C_p - \frac{1}{\omega L_p}\right)^2}{\left(\frac{1}{R_p}\right)^2 + \left(\omega C_p - \frac{1}{\omega L_p}\right)^2}$$

(if $A_1 > 0$)

2 $\because \left(\omega C_p - \frac{1}{\omega L_p}\right)^2 > 0$ at all frequencies, $\Rightarrow$ real-$Z_{in} > 0$

real-Z_{in} never becomes negative. #

5 b) Output noise from R_s: $4kTR_s A^2_{total}$

Output noise from R_p: $\frac{4kT}{R_p} R^2_{out}$

Output noise from M_1: $\frac{4kT\gamma}{g_m}\left(\frac{g_m}{1+g_m R_s}\right)^2 R^2_{out}$

$$\frac{R^2_{out}}{A^2_{total}} = \frac{1}{G_m^2} = \left(\frac{1+g_m R_s}{g_m}\right)^2$$

$$F = \frac{\text{output noise}/A^2_{total}}{4kTR_s} = 1 + \frac{1}{R_p R_s}\frac{R^2_{out}}{A^2_{total}} + \frac{\gamma}{g_m R_s}\left(\frac{g_m}{1+g_m R_s}\right)^2 \frac{R^2_{out}}{A^2_{total}}$$

$$= 1 + \frac{1}{R_p R_s}\left(\frac{1+g_m R_s}{g_m}\right)^2 + \frac{\gamma}{g_m R_s} = 1 + \frac{1}{R_p g_m^2 R_s} + \frac{R_s}{R_p} + \frac{\gamma}{g_m R_s} \quad \#$$

$NF = 10 \log(F)$ #

5 c) If $Z_{in} = R_s$ at resonate $f \Rightarrow Z_{in} = \frac{1}{g_m}(1 + g_m A_1 R_p) = R_s$ (2)

$$\Rightarrow F = 1 + \frac{1}{R_p g_m (1 + g_m A_1 R_p)} + \frac{1 + g_m A_1 R_p}{g_m R_p} + \frac{\gamma}{1 + g_m A_1 R_p} \quad \#$$

($\because$ NF is dominated by the term with γ in this case

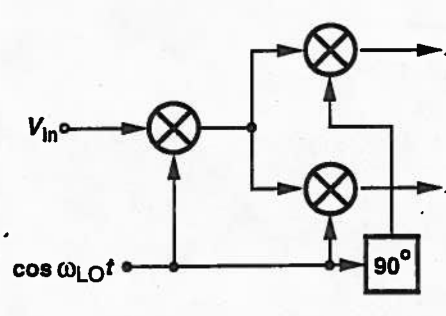
$\therefore A_1$ helps reduce the noise.)

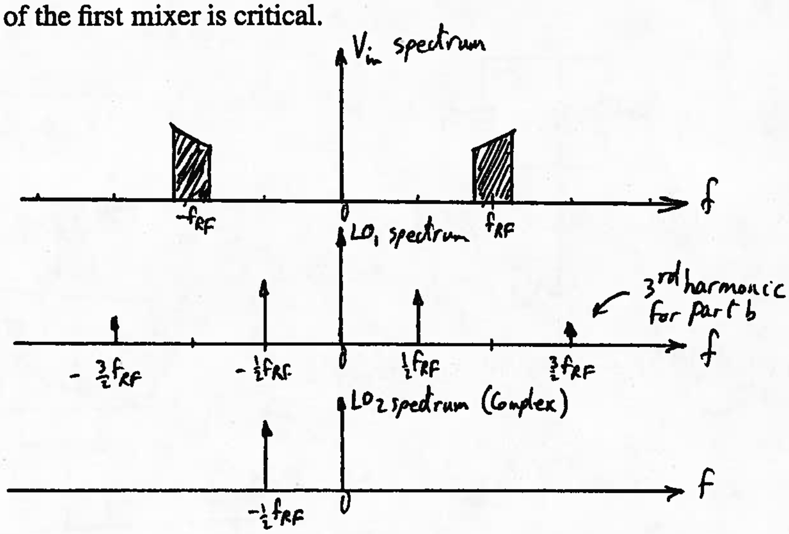
3. The receiver architecture shown below operates with an LO frequency equal to half of the input carrier frequency. Assume the input has an asymmetrically-modulated spectrum.

5 (a) Plot the IF and baseband spectra assuming ideal mixers.

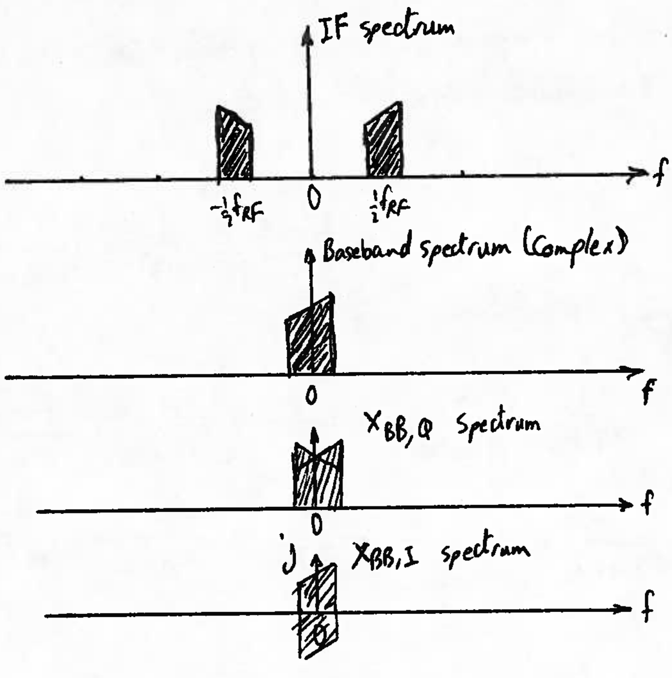
5 (b) Now suppose the first mixer experiences hard switching and introduces the third harmonic of the LO, i.e., it mixes the RF input with an LO of the form $\cos\omega_{LO}t + \alpha\cos(3\omega_{LO}t)$. Plot the IF spectrum and explain whether or not this architecture operates well with asymmetrically-modulated inputs.

5 (c) Explain why the flicker noise at the input of the first mixer is critical.

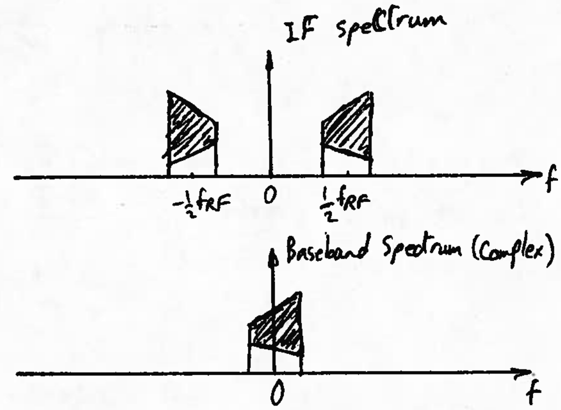




a)



b)



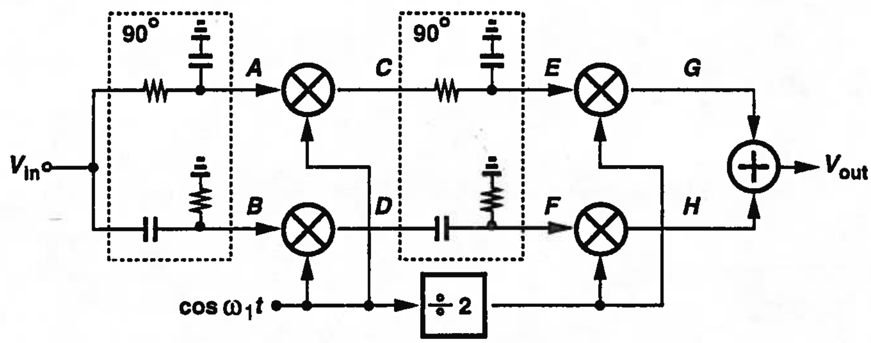
So this architectures will not work well with asymmetrically-modulated inputs in the presence of first LO third harmonic. This is because the third harmonic folds the signal on itself corrupting the information.

c) The flicker noise is present at DC which is of equal distance from the first LO as the signal. The first LO will thus upconvert it to IF and this corrupts the signal. Also, this is in the very beginning of the RF receiver where noise is so critical and gets amplified.

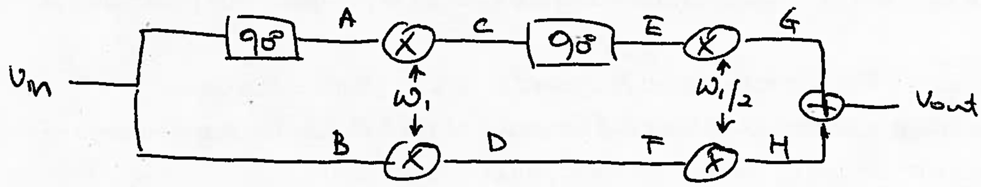
4. An engineer constructs the receiver shown below and chooses ω_1 such that the second IF is zero. (Only the I branch is shown for the sake of simplicity.) The RC-CR networks are inserted to perform a 90° phase shift at the frequency of interest.

5 (a) Does the receiver reject the image? Explain with the aid of spectra at various points in the chain.

5 (b) Does the receiver reject the image with respect to the third harmonic of the LO, i.e., the mirror image of ω_{in} with respect to $3\omega_1$? Explain with the aid of spectra at various points in the chain.



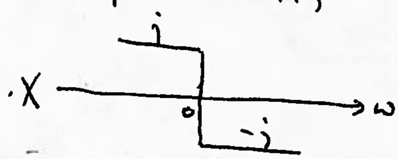
4. Simplified CKT Diagram



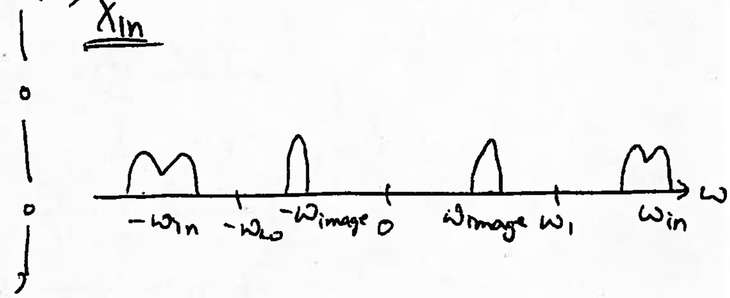
i) for zero IF,

$$\omega_{in} = \omega_1 + \frac{\omega_1}{2} = \frac{3}{2}\omega_1$$

ii) 90° phase shift,

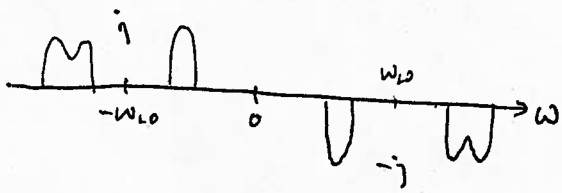


iii) X_{in}



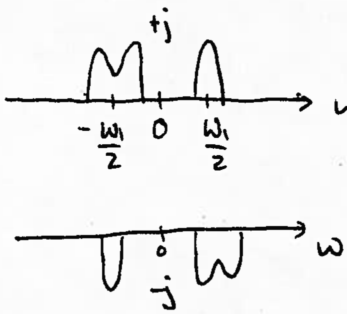
a)

X_A

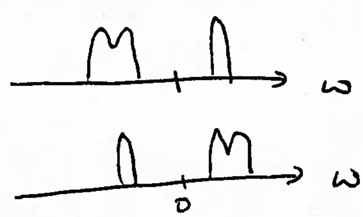


$X_B = X_{in}$

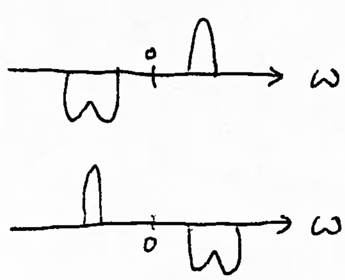
X_C



X_D

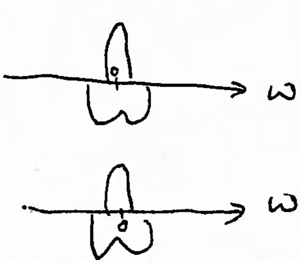


X_E

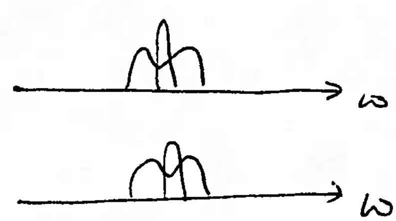


$X_F = X_D$

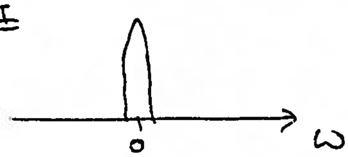
X_G



X_H

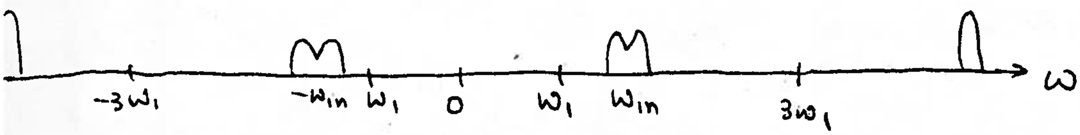


∴ X_{OUT}

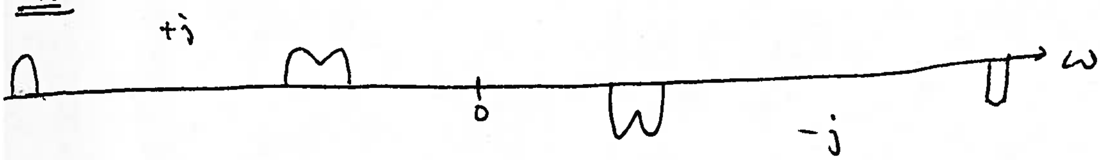


The signal will be rejected, However if $\omega_{in} = \frac{1}{2}\omega_1$,
the image will be rejected, but only to some extent, due to the magnitude mismatch between the upper / lower branch.
(RC−CR : 90° phase shift for all freq, same Amplitude response only at 1/RC

(b) X_{in}

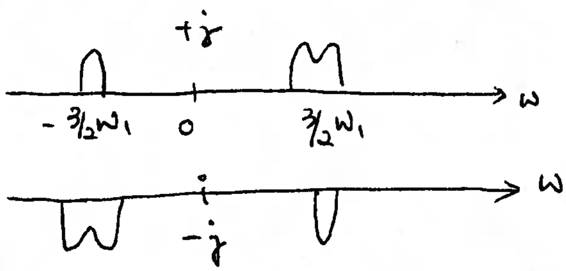


X_A

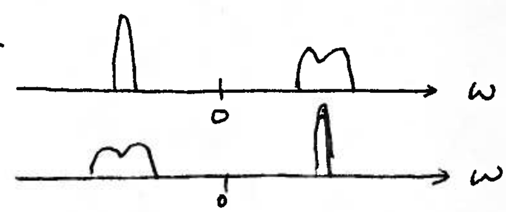


$X_B = X_{in}$,

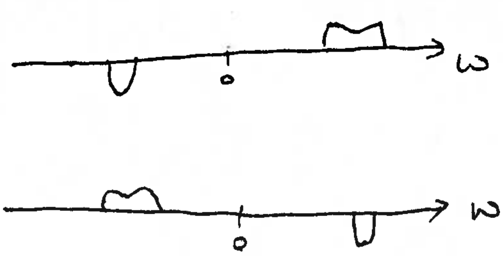
X_C



X_D

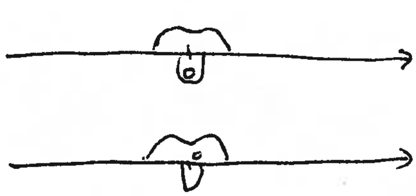


X_E

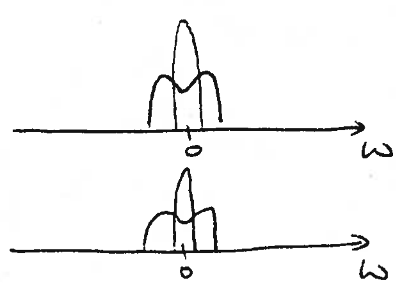


$X_F = X_D$

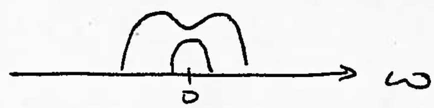
X_G



X_H



$\therefore$ $\underline{X_{out}}$



The image would be attenuated (not fully rejected) because of the frequency dependent magnitude mismatch between upper/lower branch.