Phase Noise (II)

Behzad Razavi

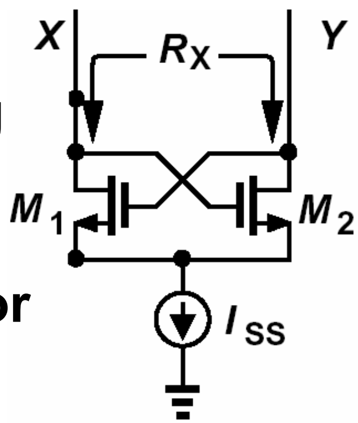
University of California, Los Angeles

Outline

- Analysis of Phase Noise
  - Approach I
  - Approach II
- Computation of Phase Noise Spec.
- GSM Example

Analysis of Phase Noise

- Tens of papers have been published on phase noise in oscillators. Many mechanisms result in phase noise.
- No single approach has been sufficient to give insight into all mechanisms.
- We follow two approaches here:
  - Approach I: based on time averages →
    (a) the average spectrum of noise of a device while the noise spectrum varies with time.
    (b) the "average resistance," defined as the "dc" term in the Fourier series of a periodically-varying resistance.

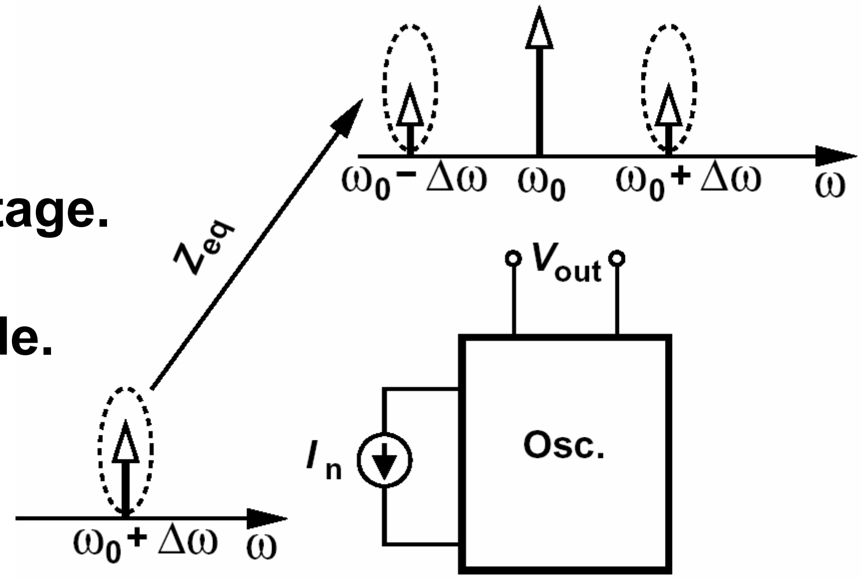


  - Approach II: based on phase response of an oscillator to an injected impulse in the time domain [Hajimiri & Lee, JSSC, Feb. 98].

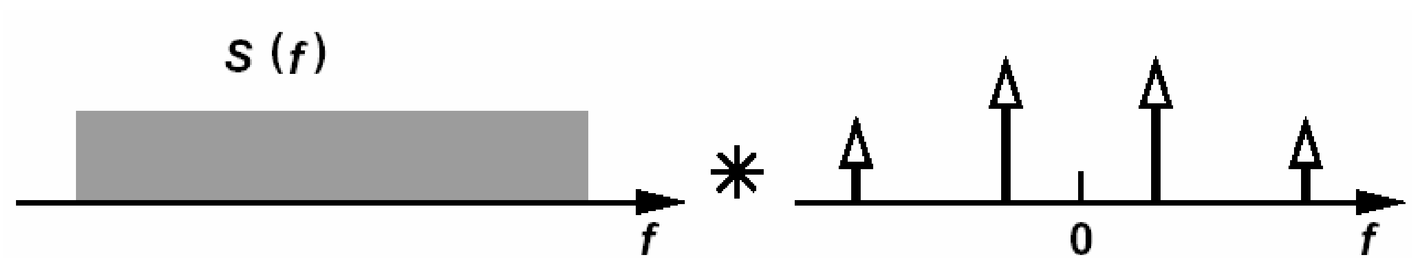
Phase Noise Analysis: Approach I

- Three steps:

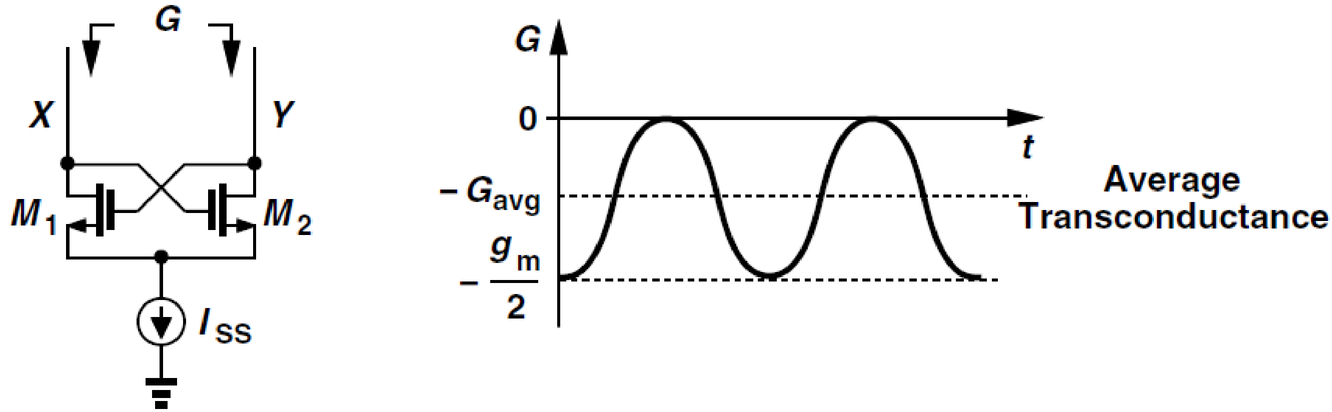
1. Determine the transfer function from injected noise to output voltage.
2. Multiply by device noise.
3. Normalize to oscillation amplitude.

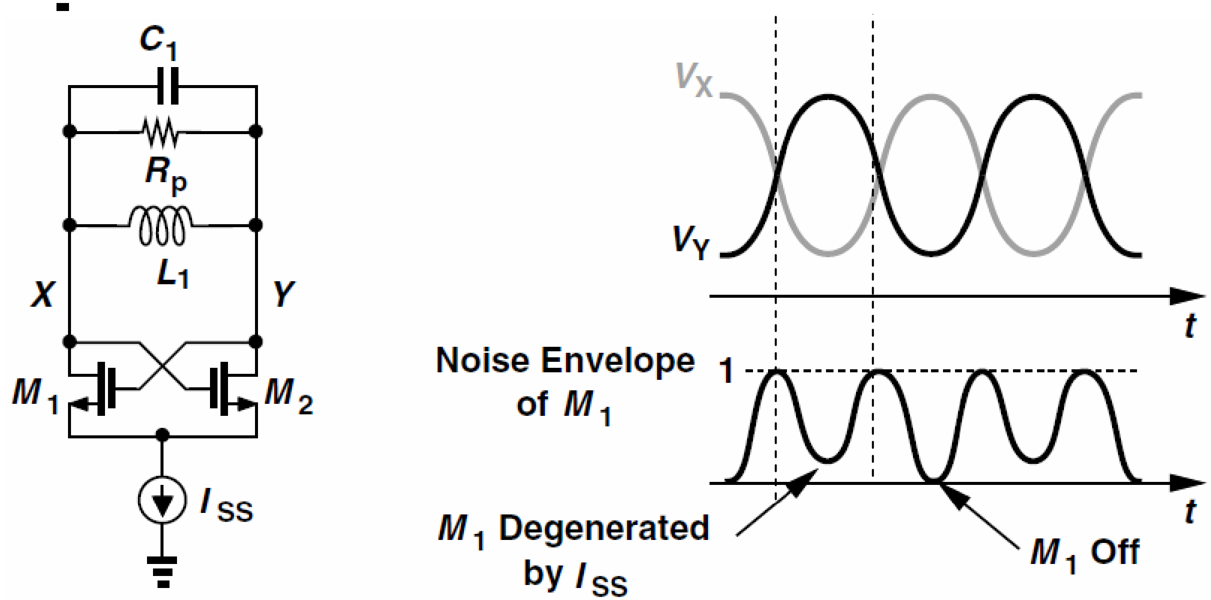


- Periodially-switched white noise is white:



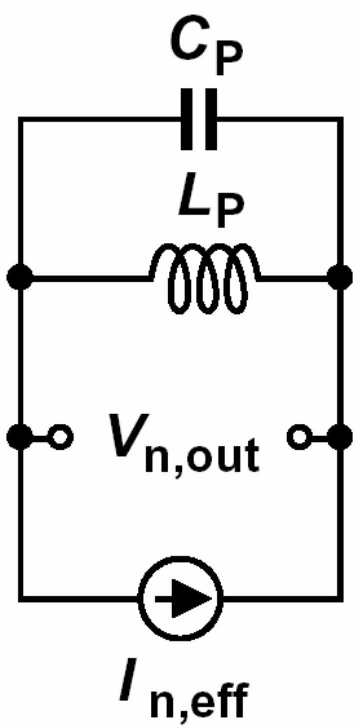
Example





- Average value of 1/G is equal to –Rp.
- Noise of M1 and M2 is modulated periodically.
- Maximum noise is injected at zero crossings.

Example of Phase Noise Calculation



$$\frac{\overline{V_{n,out}^2}}{\overline{I_{n,eff}^2}} = \frac{L_P^2\,\omega^2}{(1 - L_P C_P \omega^2)^2}$$

$$\omega = \omega_0 + \Delta\omega$$

$$= \frac{R_P^2}{Q^2(2L_P C_P \Delta\omega)^2}$$

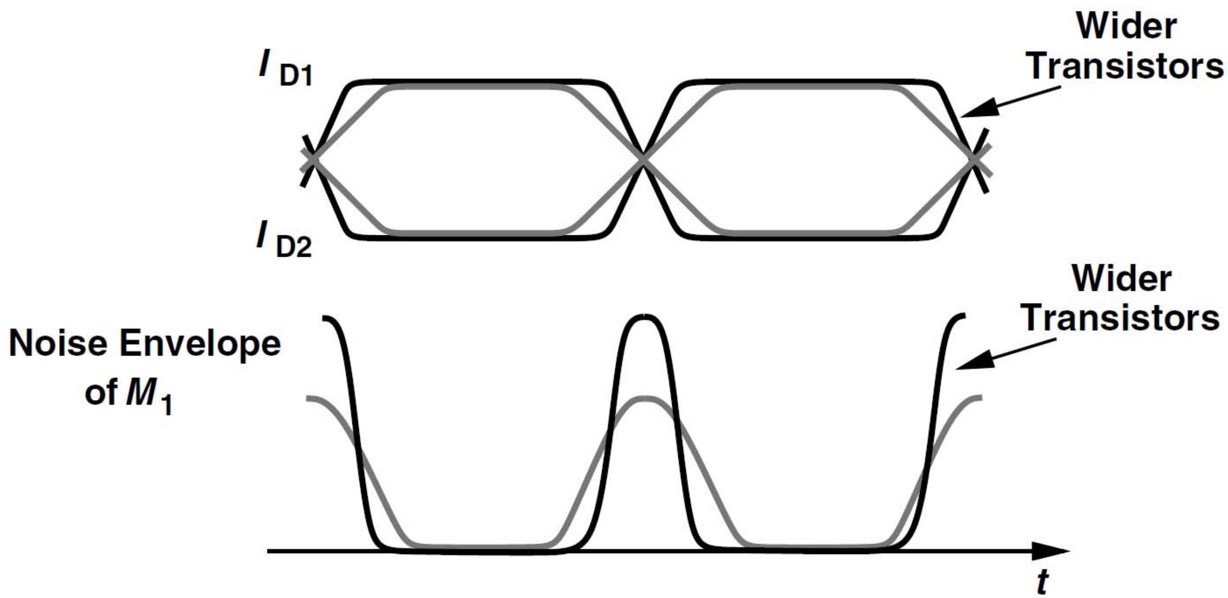
$$= \frac{R_P^2\,\omega_0^2}{4Q^2(\Delta\omega)^2}$$

- Need to multiply by spectrum of $I_{n,eff}$ and normalize to carrier power, $(2/\pi)^2 I_{SS}^2 R_p^2$.

Overall Result for X-Coupled Osc.

$$S(\Delta\omega) = \frac{\pi^2}{2}\frac{kT}{I_{SS}^2}\left(\frac{3}{8}\gamma g_m + \frac{2}{R_p}\right) \frac{\omega_0^2}{4Q^2\Delta\omega^2}$$

- But the in practice, phase noise is not a strong function of g_m:

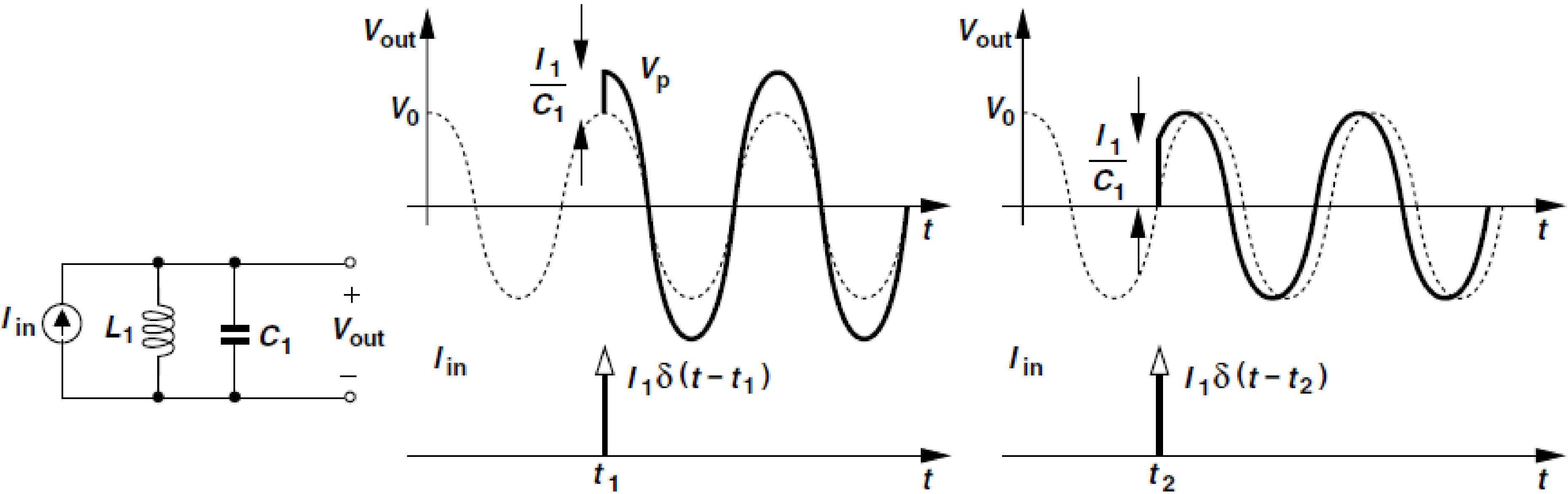


- Replace g_m with $2/R_p$:

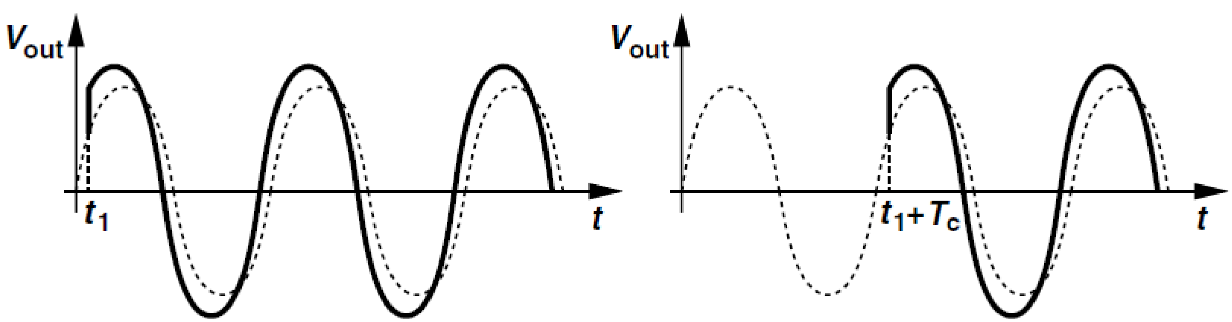
$$S(\Delta\omega) = \frac{\pi^2}{R_p}\frac{kT}{I_{SS}^2}\left(\frac{3}{8}\gamma + 1\right) \frac{\omega_0^2}{4Q^2\Delta\omega^2}$$

Phase Noise Analysis: Approach II

- An impulse of current changes the phase and/or amplitude depending on when it is injected.



- “Impulse response” (“impulse sensitivity function”) of oscillator is periodic:

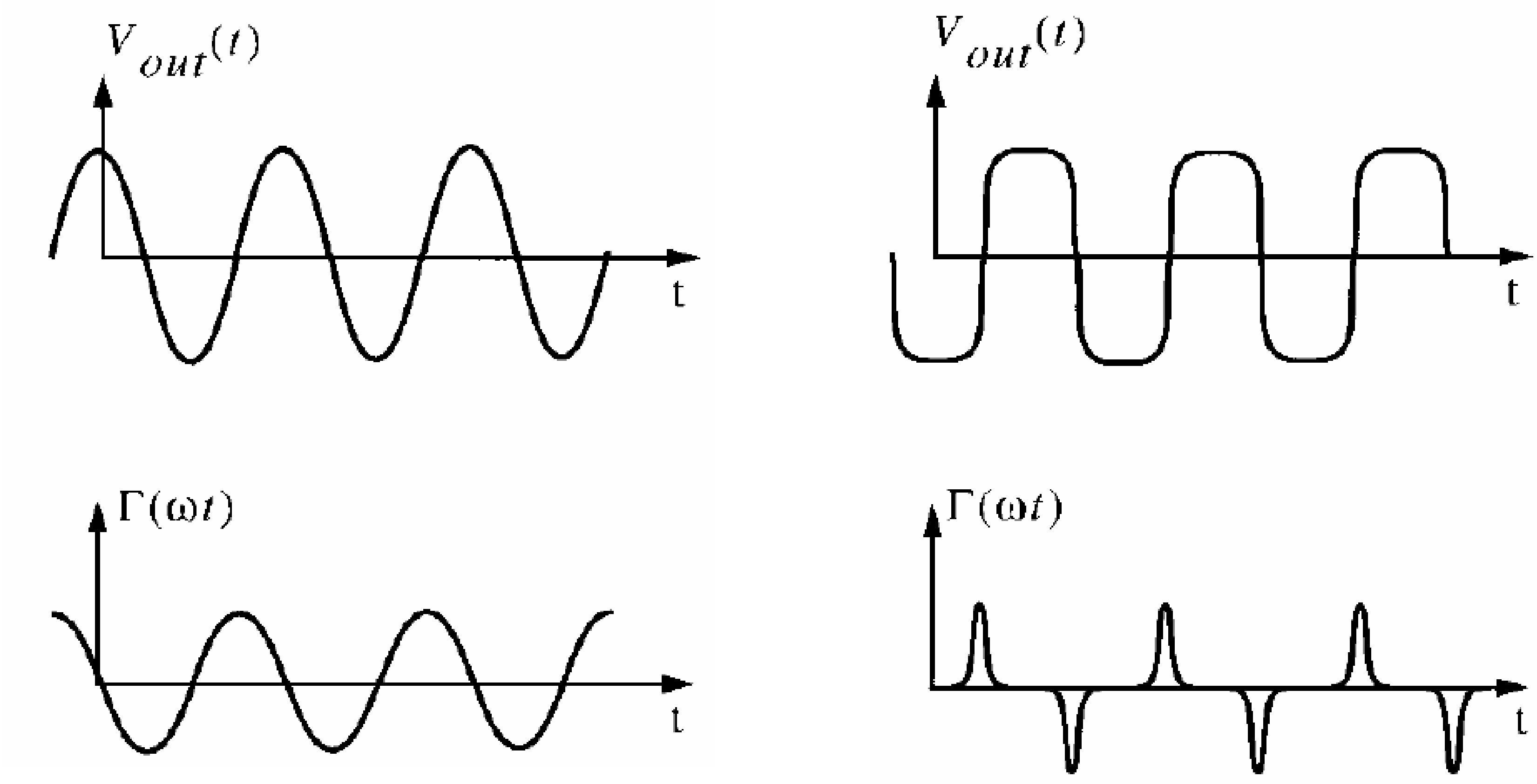


[Hajimiri & Lee, JSSC, Feb. 98]

Impulse Sensitivity Function

- **ISF is obtained by injecting an impulse whose arrival time slides along the period of oscillation.**

Sinusoidal Osc. **Ring Osc.**

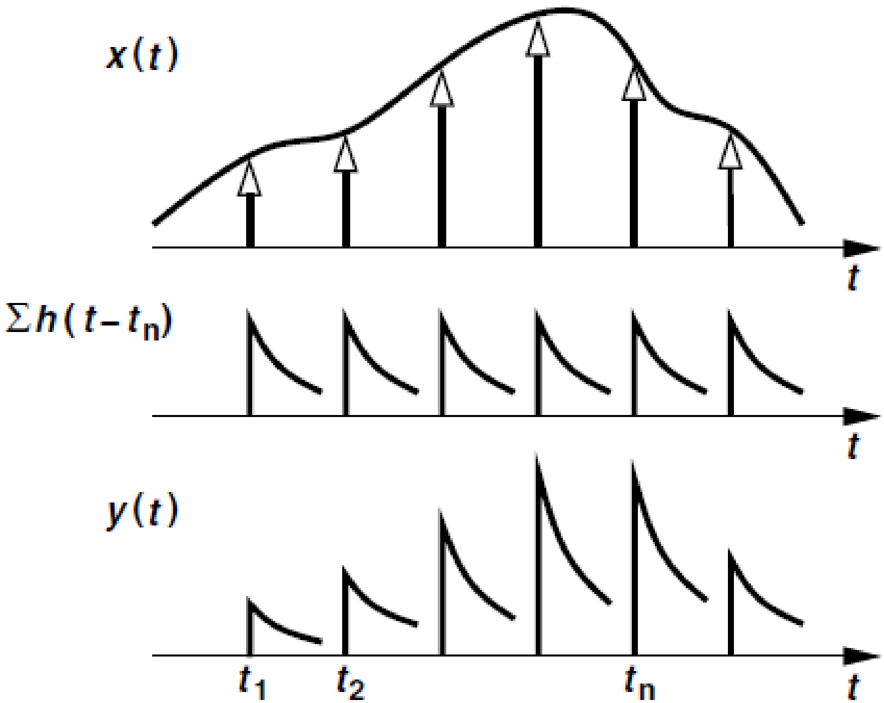


[Hajimiri & Lee, JSSC, Feb. 98]

How do we find the output phase?

$$\phi(t) = h(t, \tau) * n(t)$$

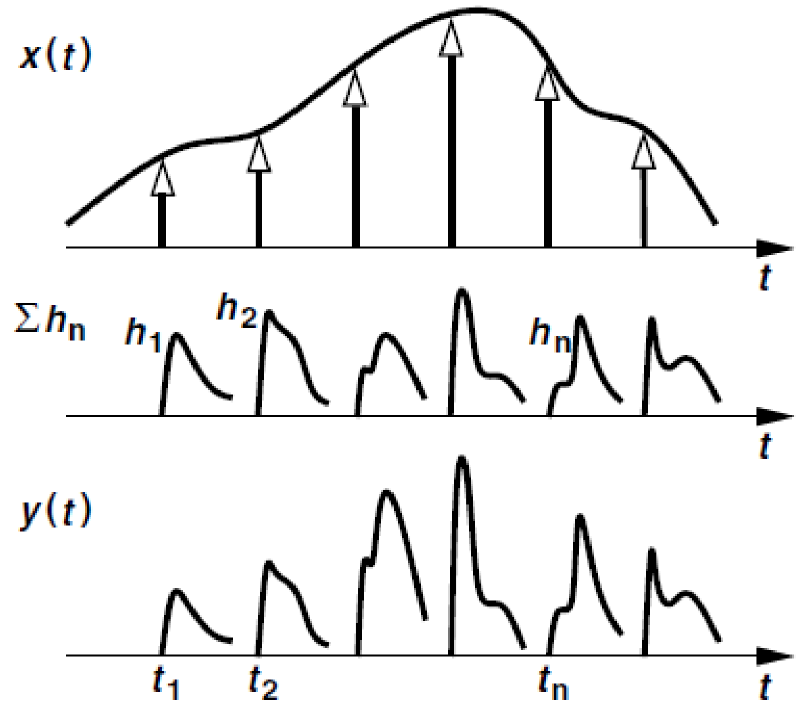
LTI



$$y(t) \approx \sum_{n=-\infty}^{+\infty} x(t_n)h(t - t_n)$$

$$= \int_{-\infty}^{+\infty} x(\tau)h(t - \tau)d\tau.$$

LTV



$$y(t) \approx \sum_{n=-\infty}^{+\infty} x(t_n)h_n(t)$$

$$y(t) = \int_{-\infty}^{+\infty} x(\tau)h(t, \tau)d\tau$$

Computation of Phase Noise

- Since ISF is periodic, it can be expanded as a Fourier series:

$$\Gamma(\omega_0\tau) = \frac{c_0}{2} + \sum_{n=1}^{\infty} c_n \cos(n\omega_0\tau + \theta_n)$$

- Phase modulation is obtained by convolving injected noise with ISF:

$$\phi_n(t) = \frac{1}{q_{\max}}\left[\frac{c_0}{2}\int_{-\infty}^{t} i(\tau)\,d\tau + \sum_{n=1}^{\infty} c_n \int_{-\infty}^{t} i(\tau)\cos(n\omega_0\tau)\,d\tau\right]$$

- Phase noise spectrum is obtained by noting that:

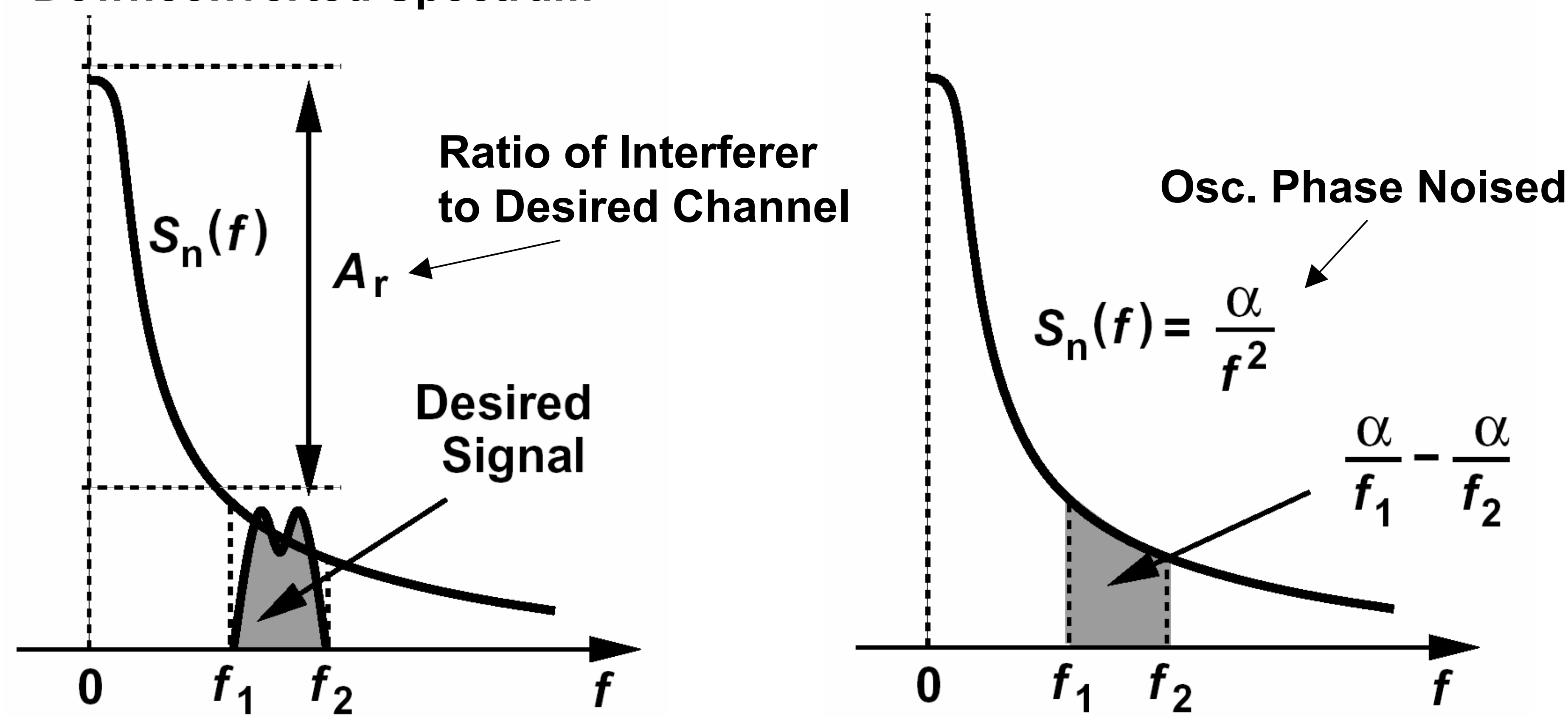
$$\begin{aligned} x(t) &= A\cos[\omega_c t + \phi_n(t)] \\ &\approx A\cos\omega_c t - A\phi_n(t)\sin\omega_c t \end{aligned}$$

[Hajimiri & Lee, JSSC, Feb. 98]

Computation of Phase Noise Requirements

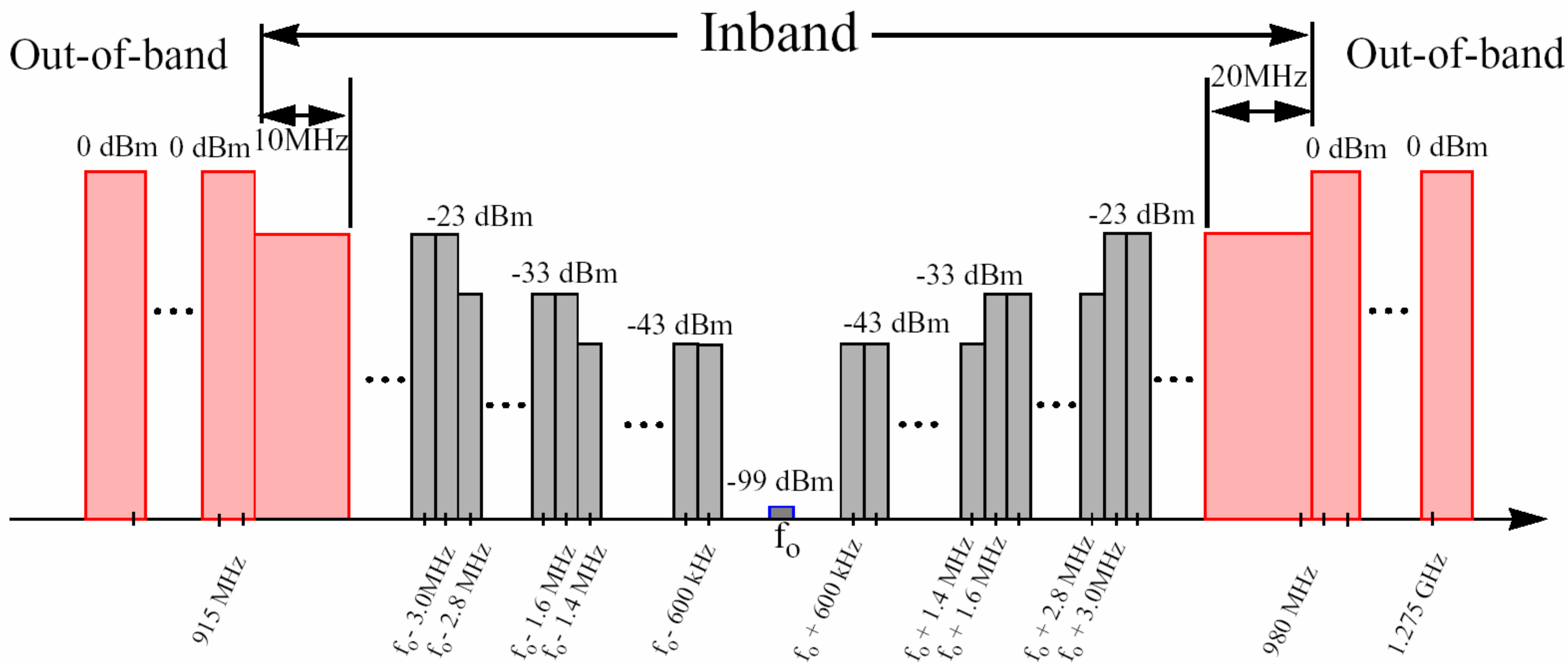
- **Assume a narrow-band interferer at zero frequency offset and a desired channel from f1 to f2:**

Downconverted Spectrum



$$\mathrm{SNR} = \frac{P_{sig}}{P_{carr}\left(\frac{\alpha}{f_1} - \frac{\alpha}{f_2}\right)} \Rightarrow \alpha = \frac{1}{A_r\left(\frac{1}{f_1} - \frac{1}{f_2}\right)\mathrm{SNR}}$$

GSM Example



Inband
Out-of-band
Out-of-band
10MHz
20MHz
0 dBm
0 dBm
0 dBm
0 dBm
-23 dBm
-33 dBm
-43 dBm
-99 dBm
-43 dBm
-33 dBm
-23 dBm
f_o
915 MHz
f_o - 3.0MHz
f_o - 2.8 MHz
f_o - 1.6 MHz
f_o - 1.4 MHz
f_o - 600 kHz
f_o + 600 kHz
f_o + 1.4 MHz
f_o + 1.6 MHz
f_o + 2.8 MHz
f_o + 3.0MHz
980 MHz
1.275 GHz

GSM RX Phase Noise Computation

- **Assume SNR=20 dB to leave margin for other imperfections.**