# *Integer-N and Fractional-N Synthesizers*

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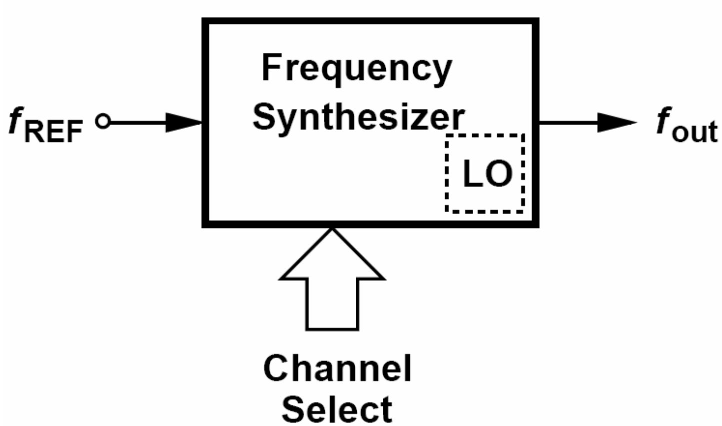
# Outline

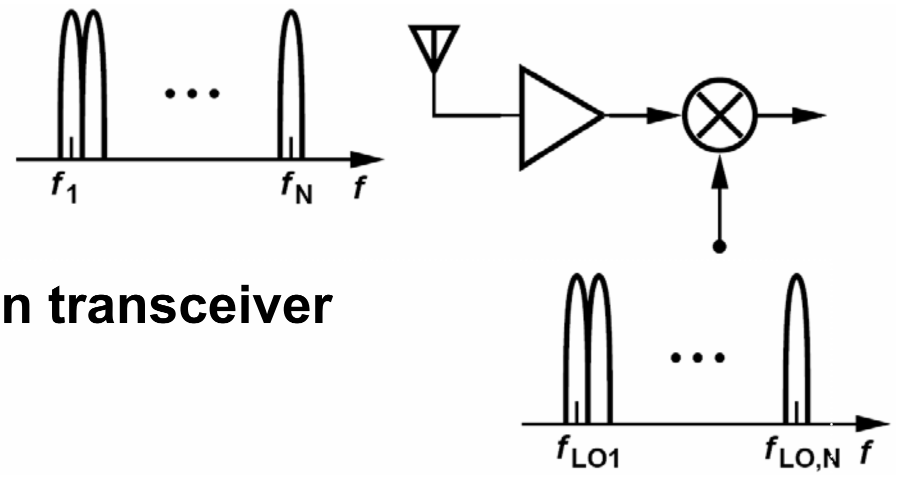
- General Synthesizer Requirements
- Integer-N Synthesizers
- Basic Fractional-N Synthesizer
- Randomization and Noise Shaping

# General Considerations

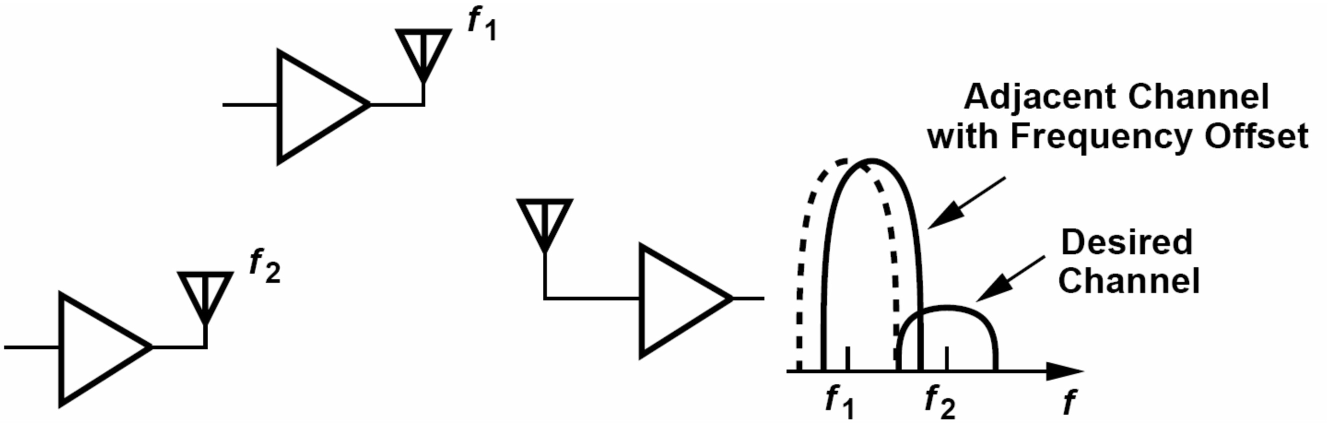



- Channel Spacing
- Frequency Accuracy
- Phase Noise
- Sidebands (Spurs)
- Lock Time
- Power Dissipation

# Channel Spacing and Frequency Accuracy



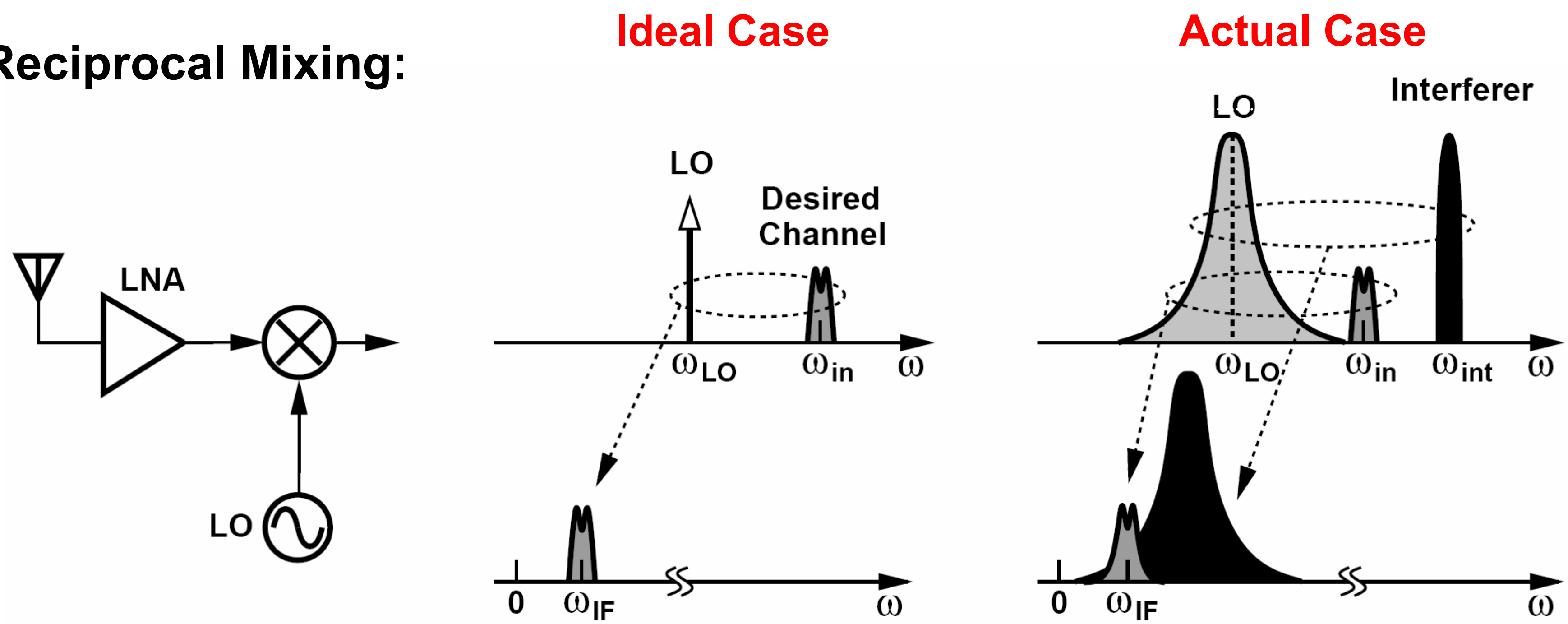

- Channel spacing depends on transceiver architecture.




- Slight shift leads to significant spillage of high-power interferer.

# Phase Noise

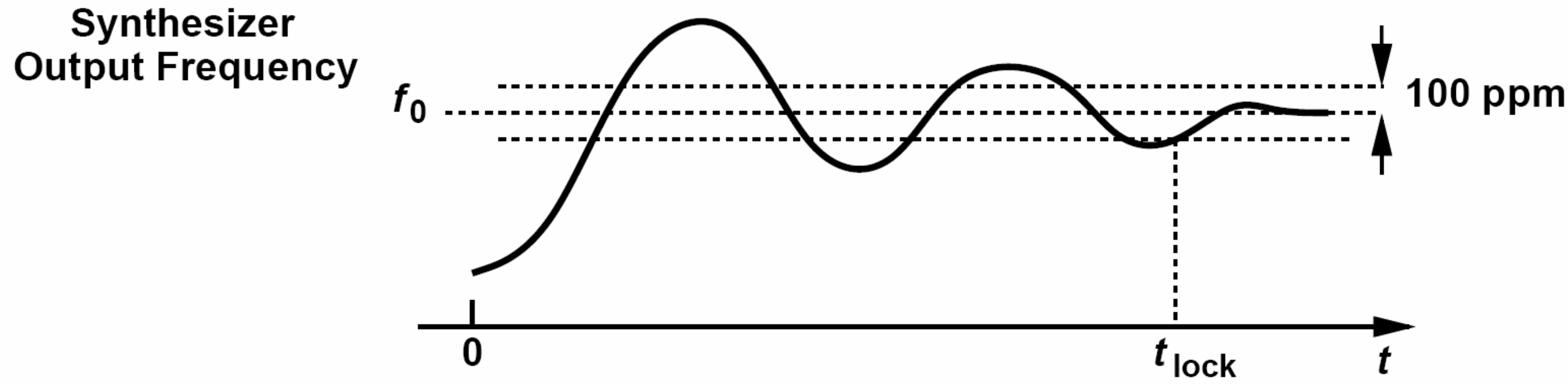
**Reciprocal Mixing:**




**Corruption of Signal:**

$$x_{QPSK}(t) = A\cos\left[\omega_c t + (2k+1)\frac{\pi}{4} + \phi_n(t)\right] \quad k = 0, \cdots, 3$$

# Lock Time



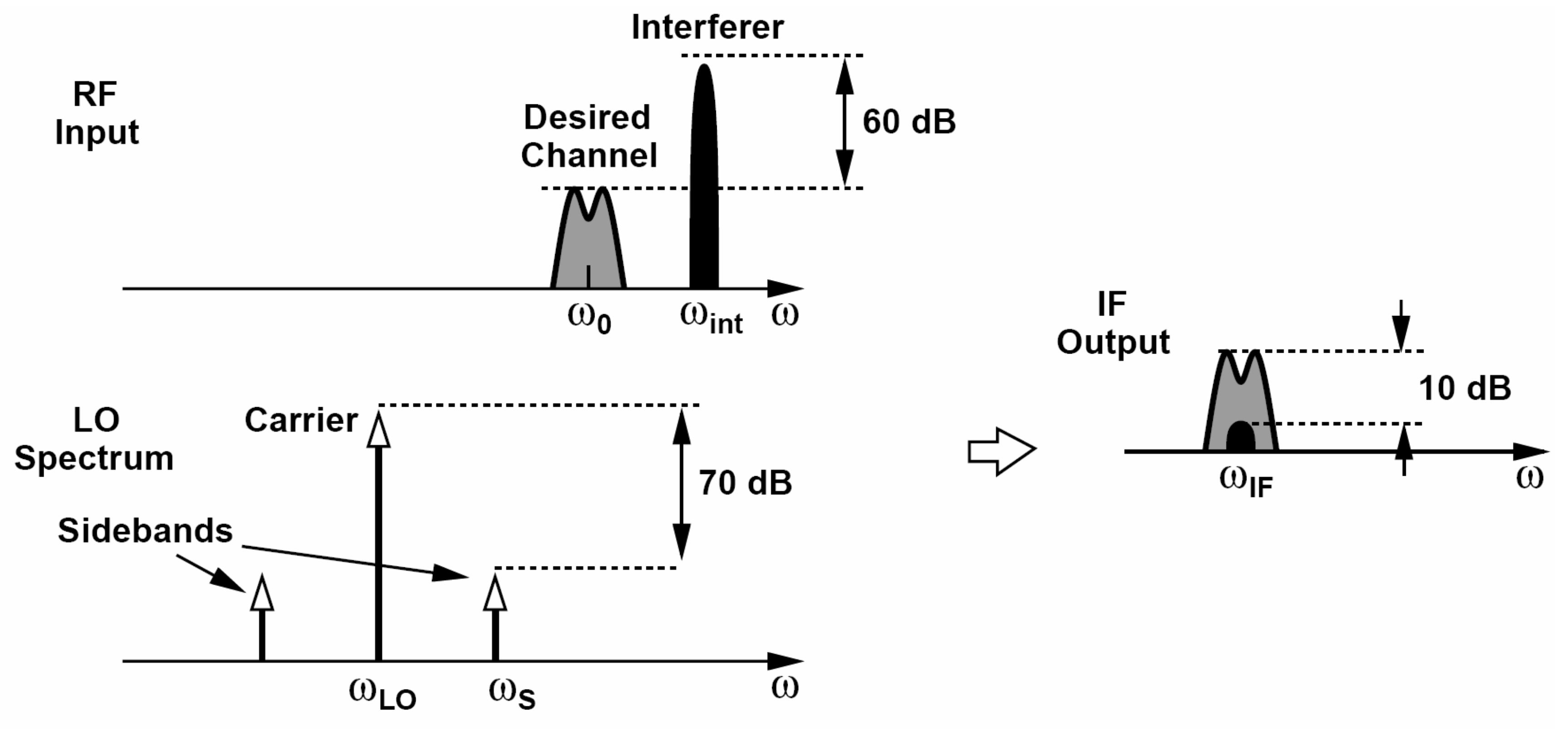

$$\frac{\Delta\omega_{out}}{\omega_{in}} = N\frac{s^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

- If damping factor is $\sqrt{2}/2$ then the settling time is given by

$$t_s = \frac{\sqrt{2}}{\omega_n}\ln\left|\sqrt{2}\left(1 - \frac{N_1}{N_2}\right)\frac{1}{\alpha}\right|$$

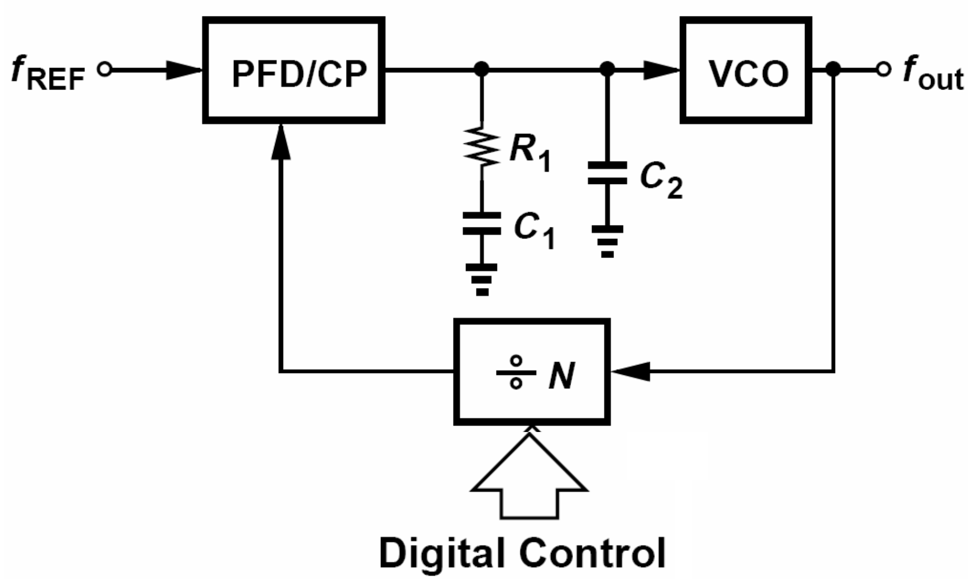
- Causes spillage of TX output power to other channels.
- A well-designed PLL settles in roughly 100 input cycles.

# Sidebands




- Manifests itself in blocking tests and adjacent channel tests.
- Trades with settling time.

# Basic Integer-N Synthesizer

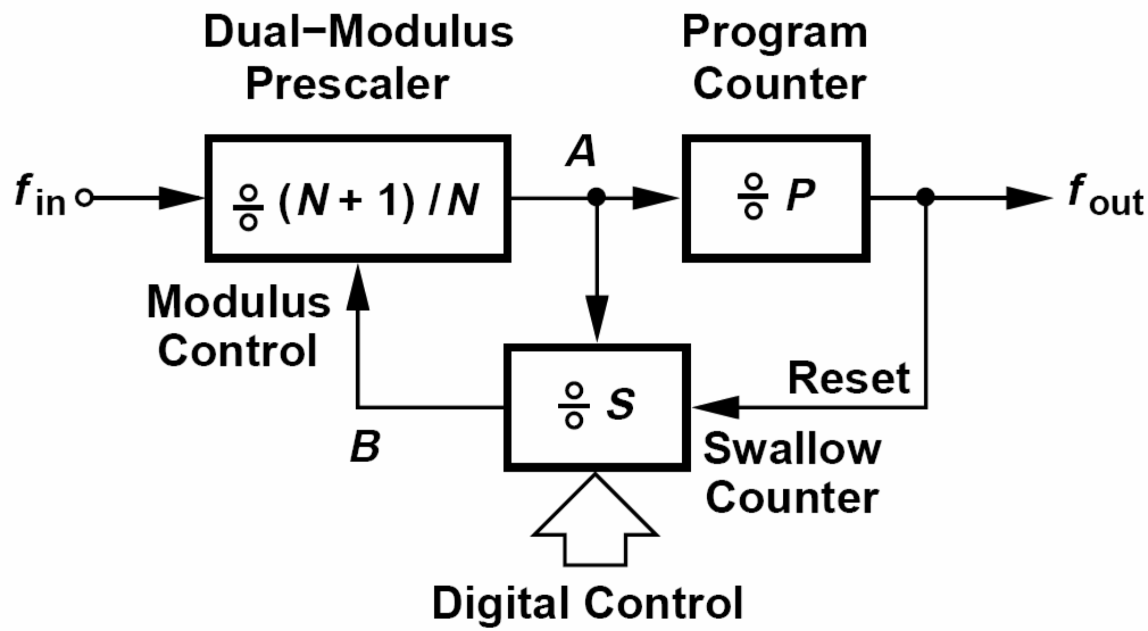



- Frequency channel is assigned by the base station at the beginning of communication.
- Output frequency step = reference frequency
- Example: Find the reference frequency for a Bluetooth receiver using sliding-IF conversion with $f_{LO} = (2/3)f_{RF}$.

# Integer-N Synthesizer Design

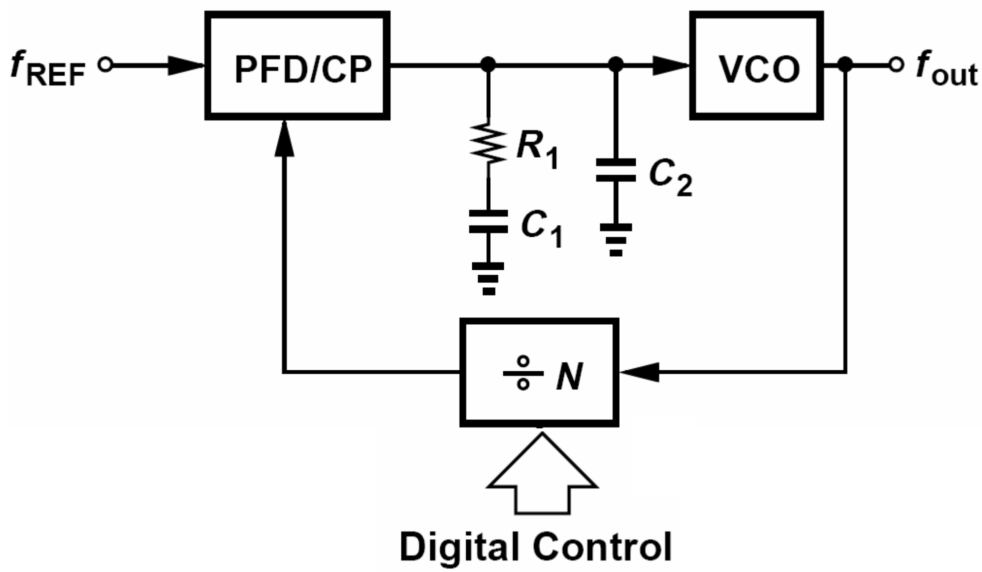
- VCO
- Dual-Modulus Divider
- PFD/CP
- Loop Filter
- Spur Reduction Techniques
  - Up/Down Skew Reduction
  - Up/Down Current Mismatch Reduction
  - Sampling Loop Filter

# Pulse Swallow Divider




- Prescaler begins with N+1 and counts until swallow counter fills up.
- Prescaler now divides by N until program counter fills up.

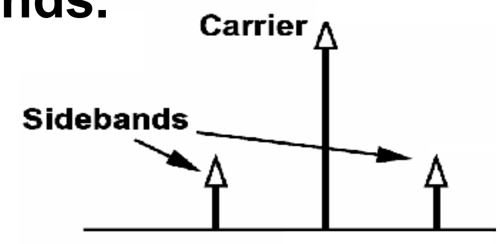
# Drawbacks of Integer-N Synthesizers



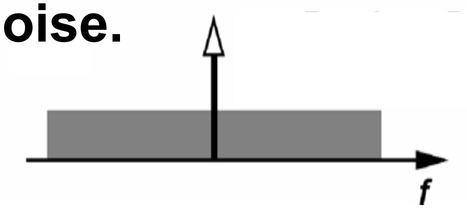

- Output frequency step = reference frequency
- Slow settling if channel spacing is small.
- Little phase noise suppression of VCO if channel spacing is small.
- High amplification of reference phase noise
- Difficult to operate with different crystal frequencies.

# Fractional-N Synthesizers: Preview

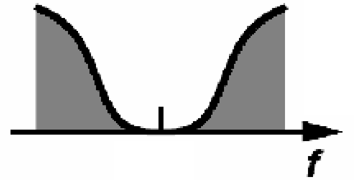
- Toggle the divide ratio between N and N+1 periodically to create an average value equal to N+$\alpha$.
- But this modulates the VCO frequency periodically, generating sidebands.



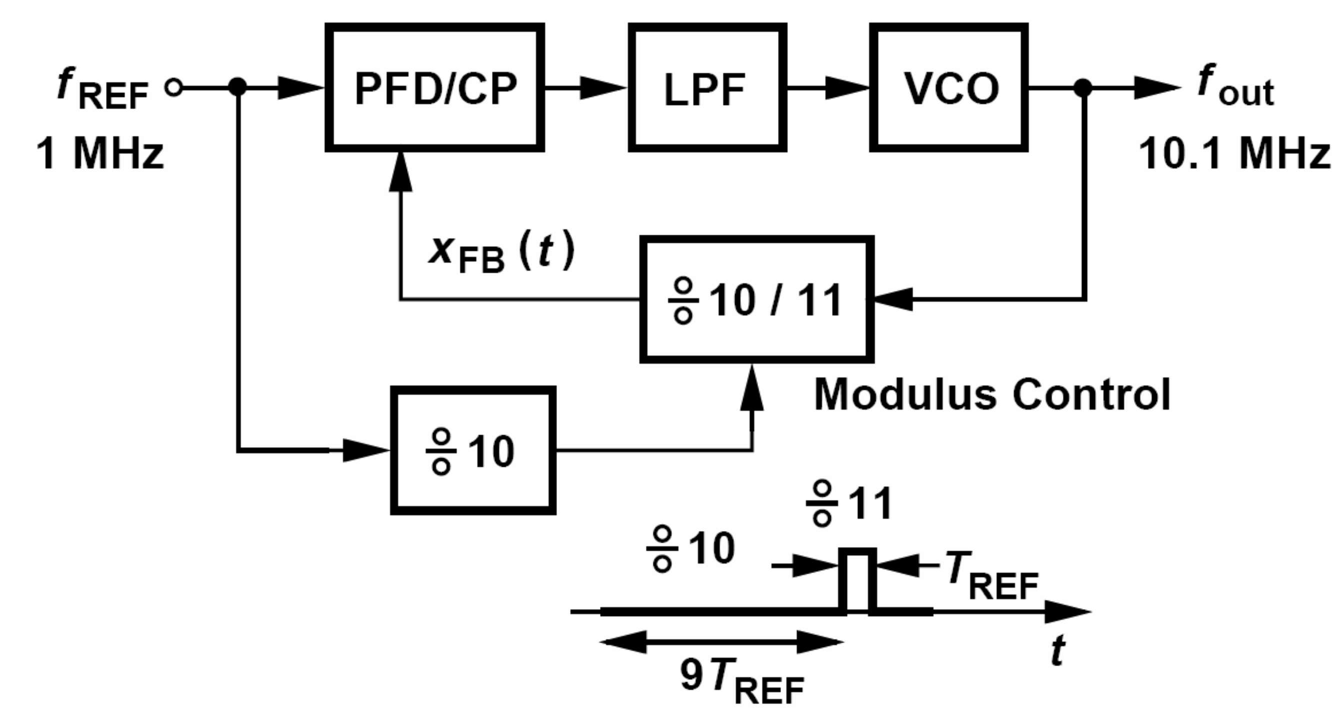

- Toggle the divide ratio between N and N+1 randomly to convert sidebands to noise.




- But the phase noise is now too high.
- "Shape" the spectrum of noise to move its energy to high frequencies, and let the PLL filter out the high-frequency noise.

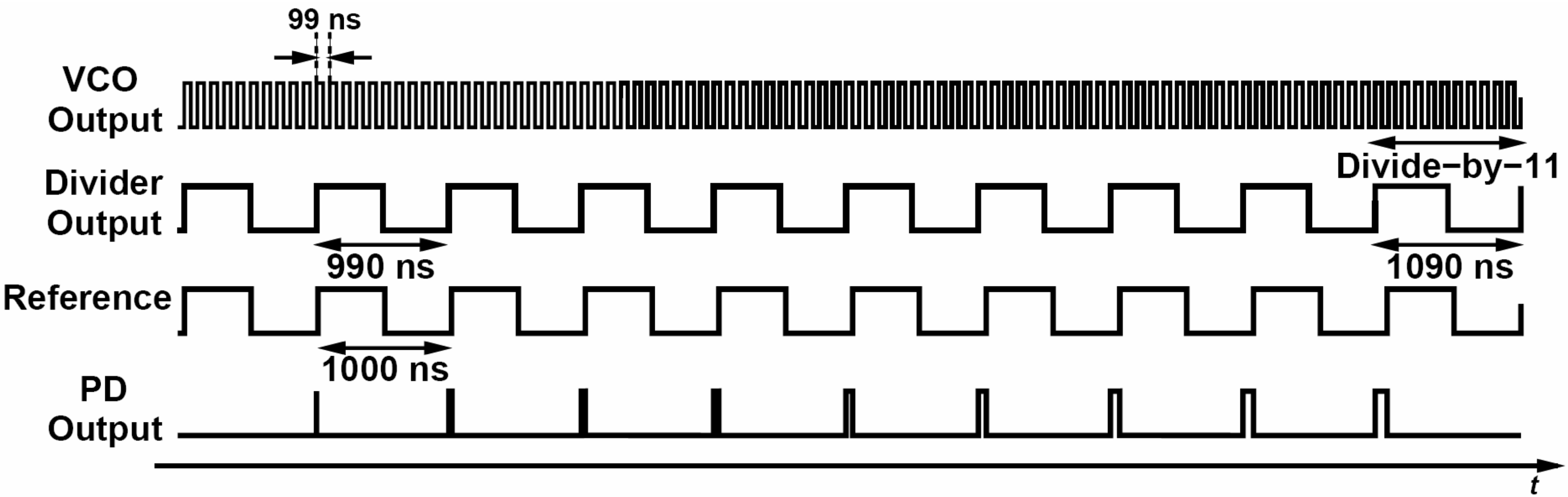


# How to create a fractional divide ratio?



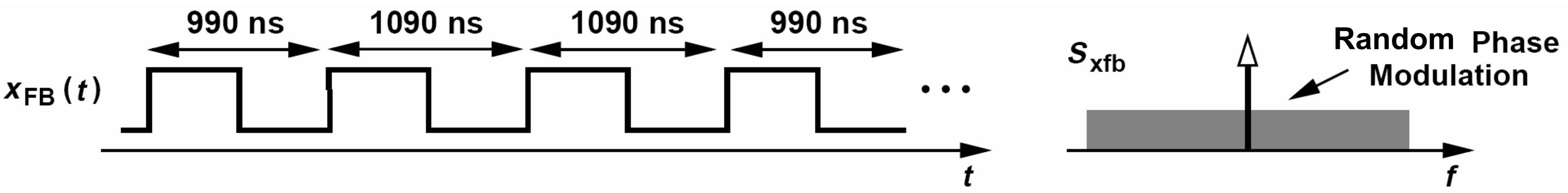

- Decouples output frequency step from the input reference frequency → Wider loop bandwidth →
  - Faster settling
  - Greater VCO phase noise suppression
  - Less amplification of reference phase noise

# Fractional Spurs




- VCO produces sidebands at ±0.1MHz×$n$ around 10.1MHz.

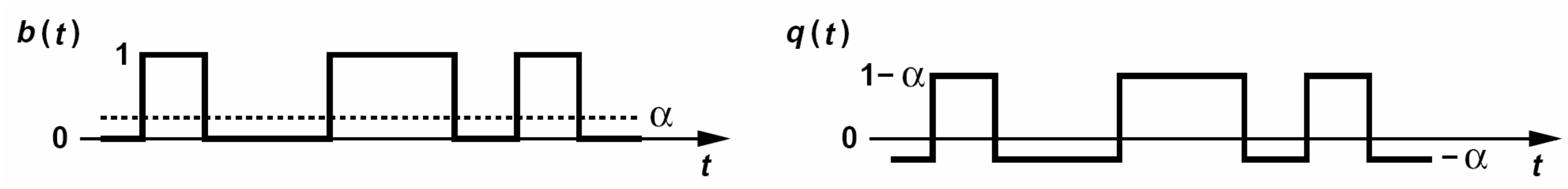
# Conversion of Spurs to Noise



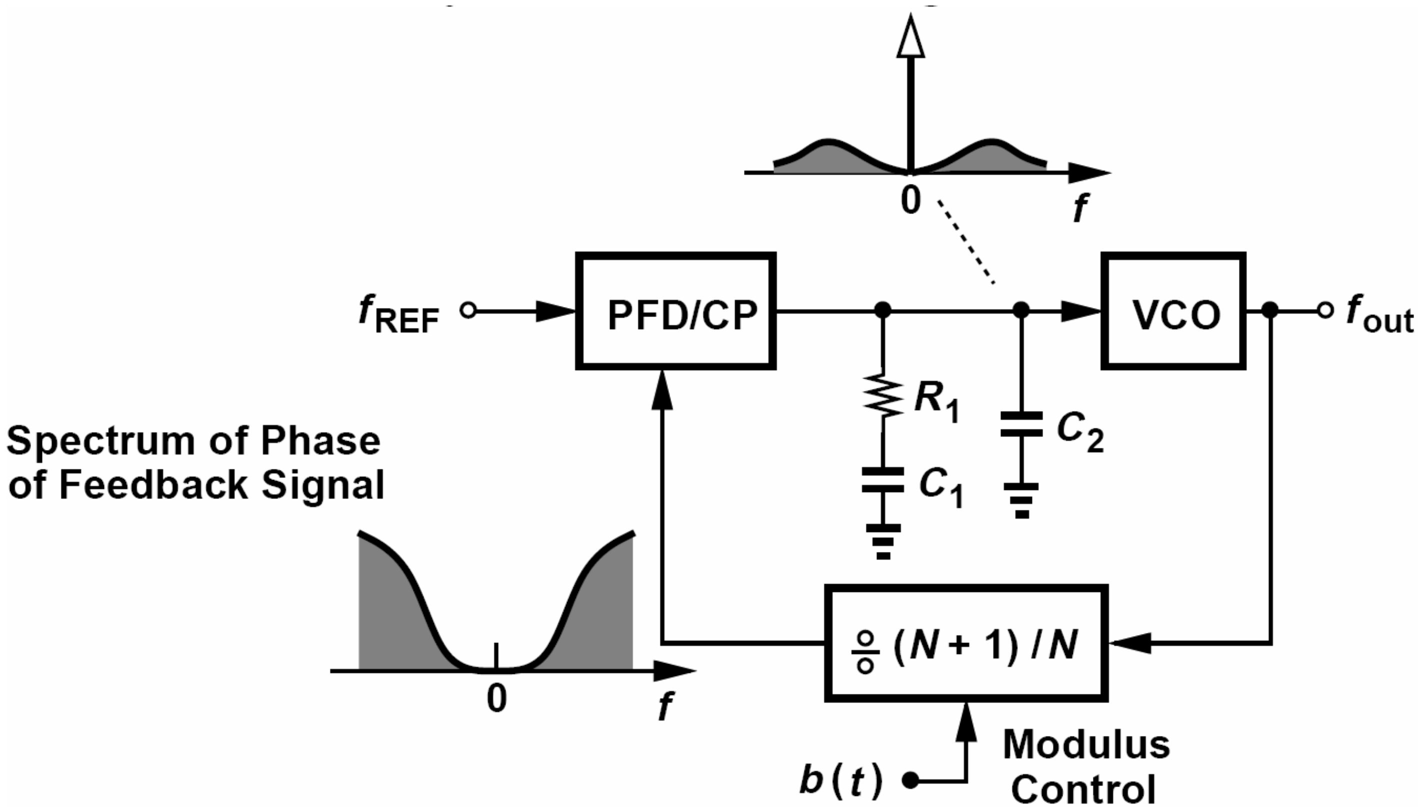

- **Instantaneous frequency of feedback signal:**

$$f_{FB}(t) = \frac{f_{out}}{N + b(t)}$$

**b(t) randomly toggles between 0 an 1 and has an average value of $\alpha$:**



# Basic Noise Shaping

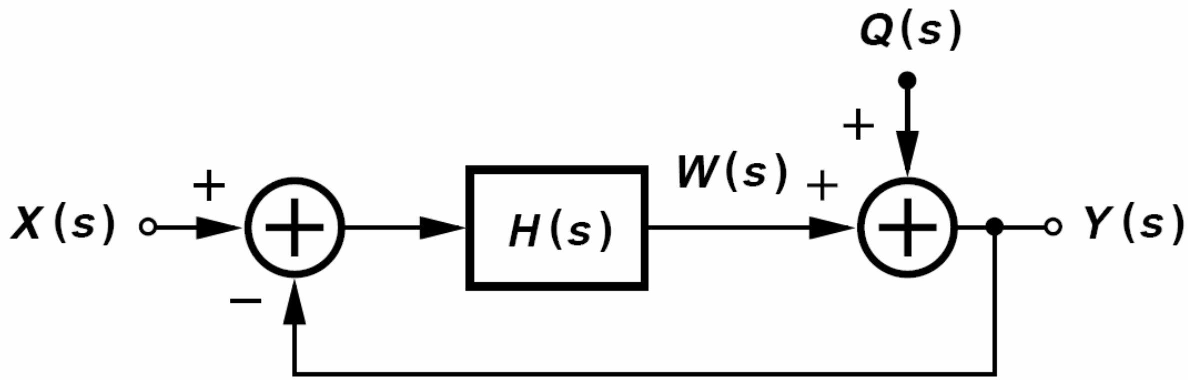



- Generate a random binary sequence, *b(t)*, that switches the divider modulus between *N* and *N+1* such that

(1) the average value of the sequence is $\alpha$.

(2) the noise of the sequence has a high-pass spectrum.

# Negative Feedback System as a High-Pass System

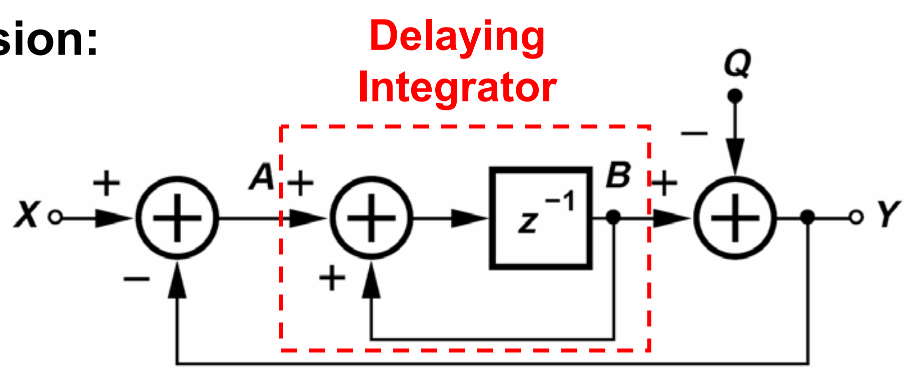


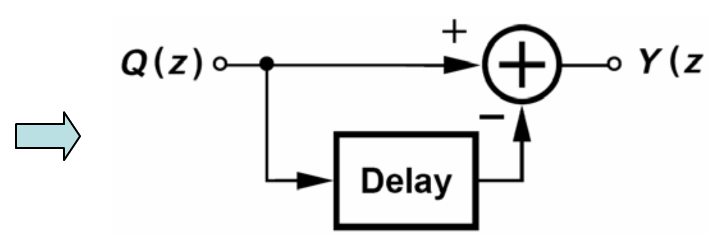

$$\frac{Y(s)}{Q(s)} = \frac{1}{1 + H(s)}$$

**If H is an integrator:**

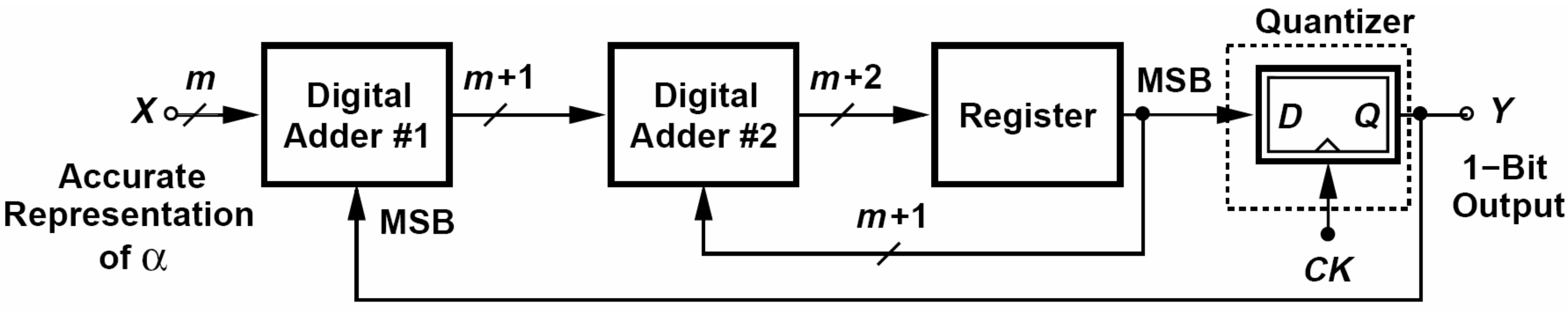
$$\frac{Y(s)}{Q(s)} = \frac{s}{s+1}$$

**Discrete-time version:**



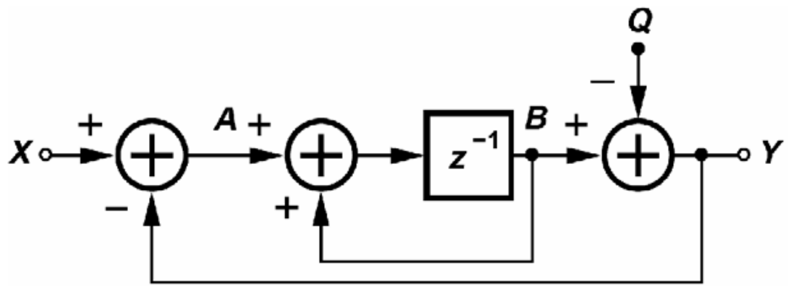



# Σ-Δ Modulator Example



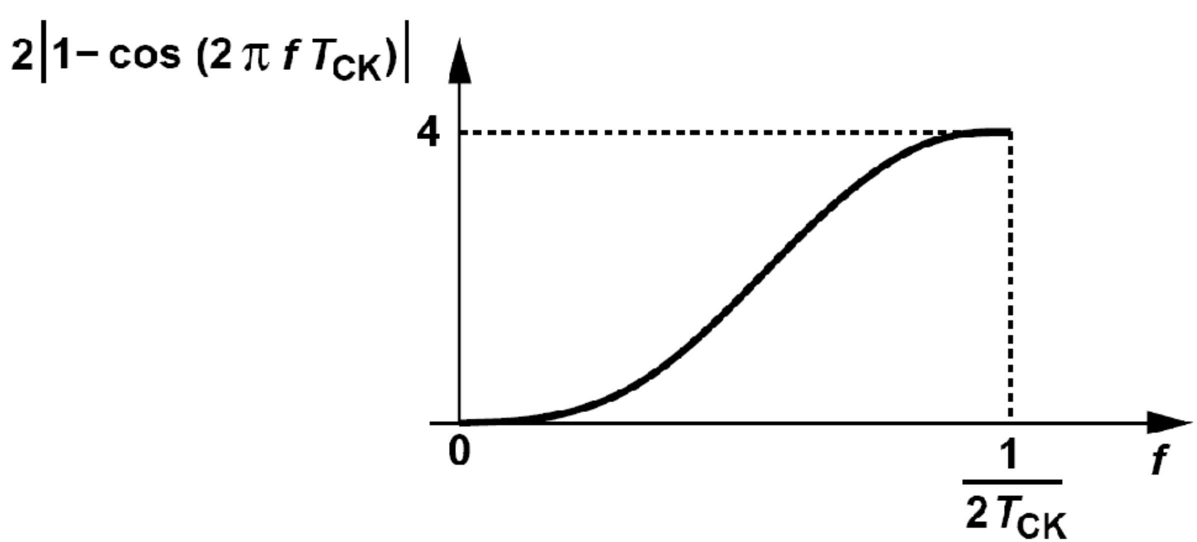

- Quantization from *m+2* bits to 1 bit introduces significant noise, but the feedback loop shapes this noise in proportion to $1-z^{-1}$.
- Choice of *m* is given by the accuracy with which the synthesizer output frequency must be defined.

# Noise Shaping in a ΣΔ Modulator

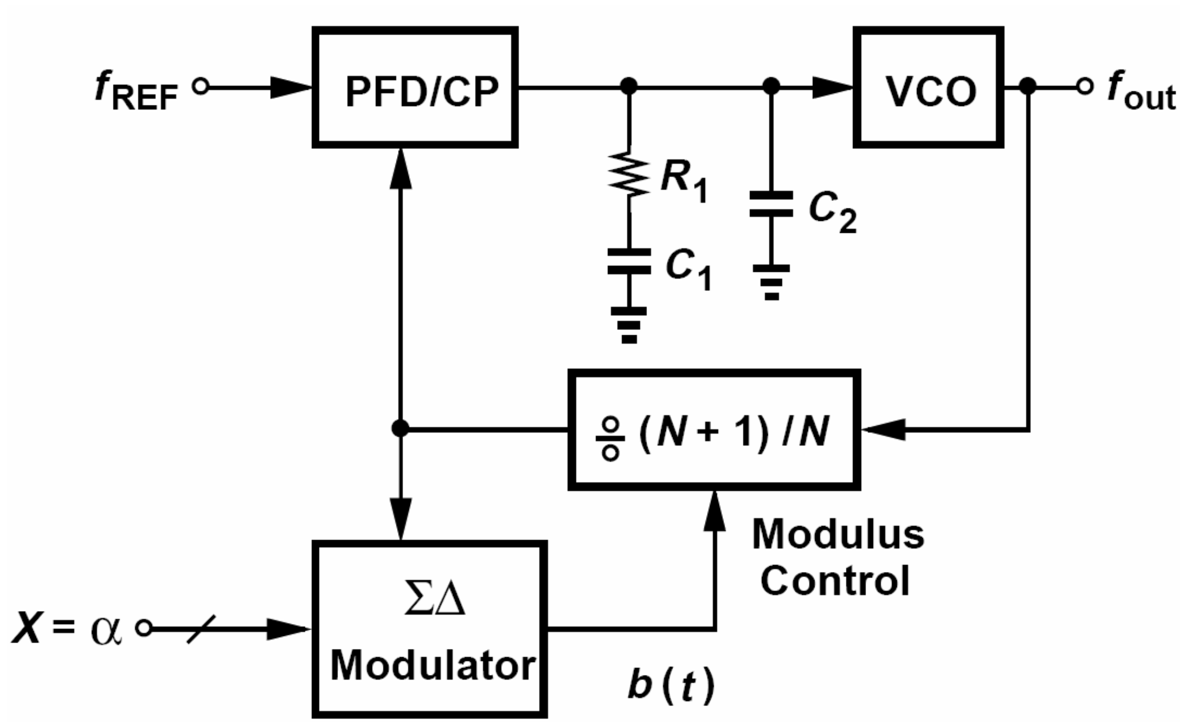



$$\frac{Y}{Q}(z) = 1 - z^{-1}$$
$$= e^{-j\pi f T_{CK}}\left(e^{j\pi f T_{CK}} - e^{-j\pi f T_{CK}}\right)$$
$$= 2je^{-j\pi f T_{CK}}\sin(\pi f T_{CK}).$$

**Quantization Noise in Output Frequency**



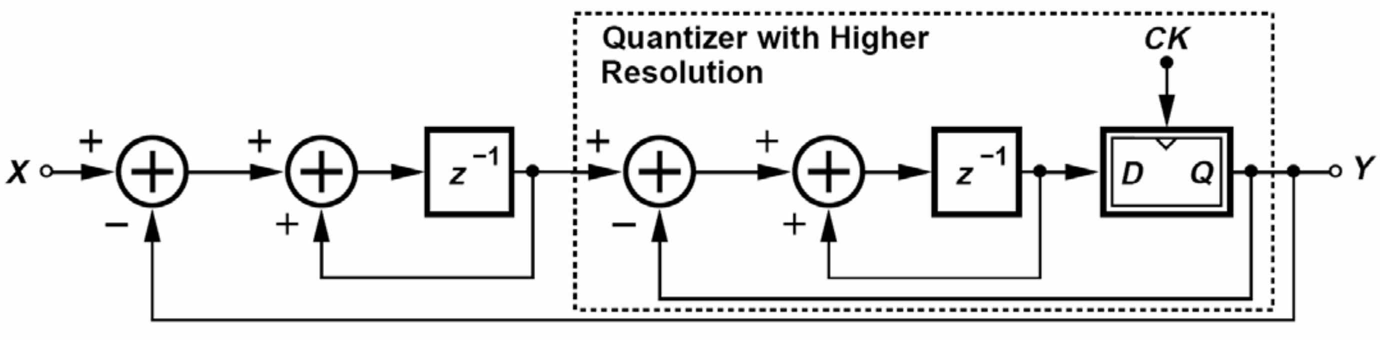
# Basic ΣΔ Fractional-N Synthesizer




- Σ-Δ modulator toggles divide ratio between *N* and *N+1* so that the average is equal to $N+\alpha$.
- Quantization noise in divide ratio is high-pass shaped.

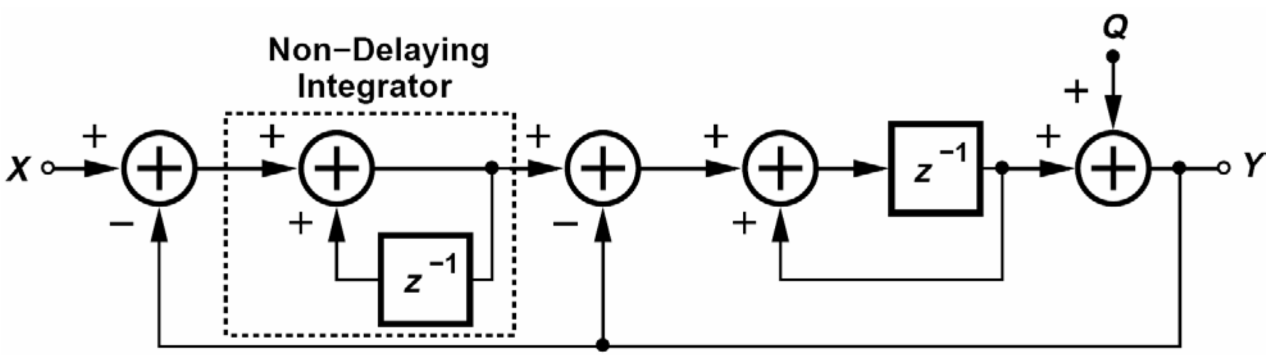
# Higher-Order Noise Shaping

- **High-Order Loop: Replace 1-bit quantizer with a finer quantizer:**



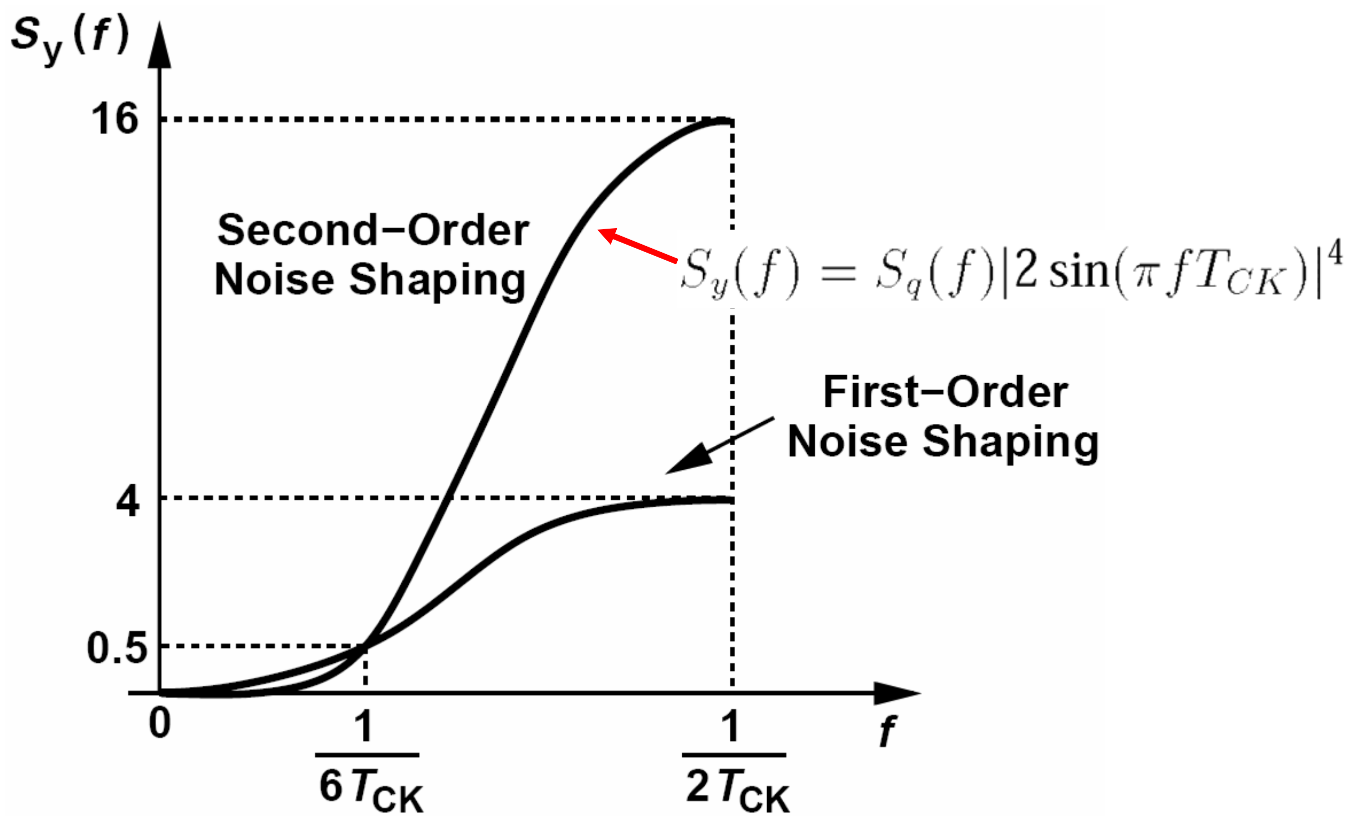

$$\frac{Y}{Q}(z) = \frac{(1 - z^{-1})^2}{z^{-2} - z^{-1} + 1}$$

- **Replace delaying integrator with non-delaying integrator:**




$$\frac{Y}{Q}(z) = (1 - z^{-1})^2.$$

# Noise Shaping in First- and Second-Order Modulators



# Problem of Out-of-Band Noise

- Transfer function from quantization noise to frequency noise:

$$Y(z) = (1 - z^{-1})^2 Q(z).$$

$$\Phi(z) = (1 - z^{-1}) Q(z).$$

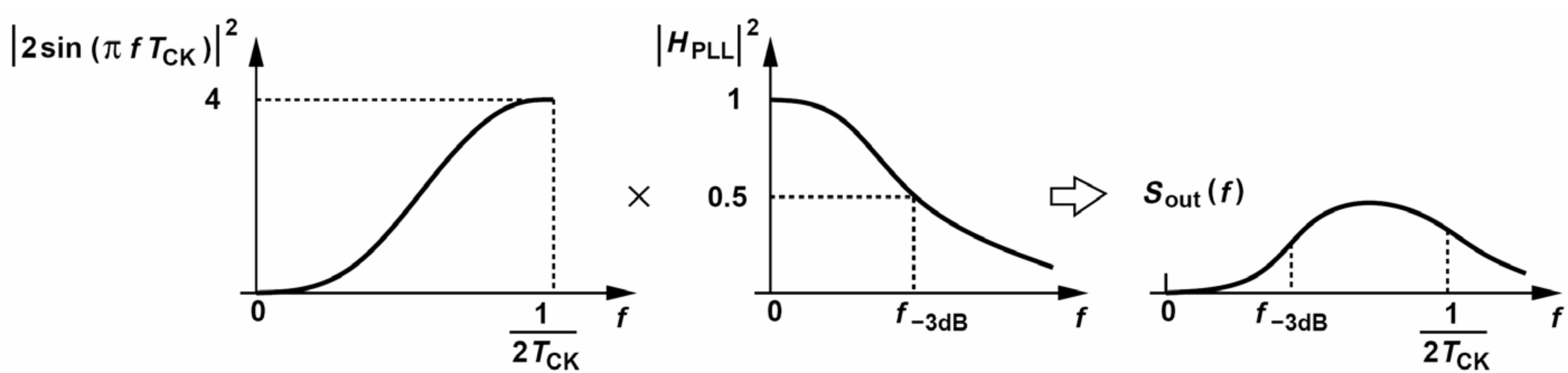
Second-Order Shaping

- Spectrum of ΣΔ phase noise:

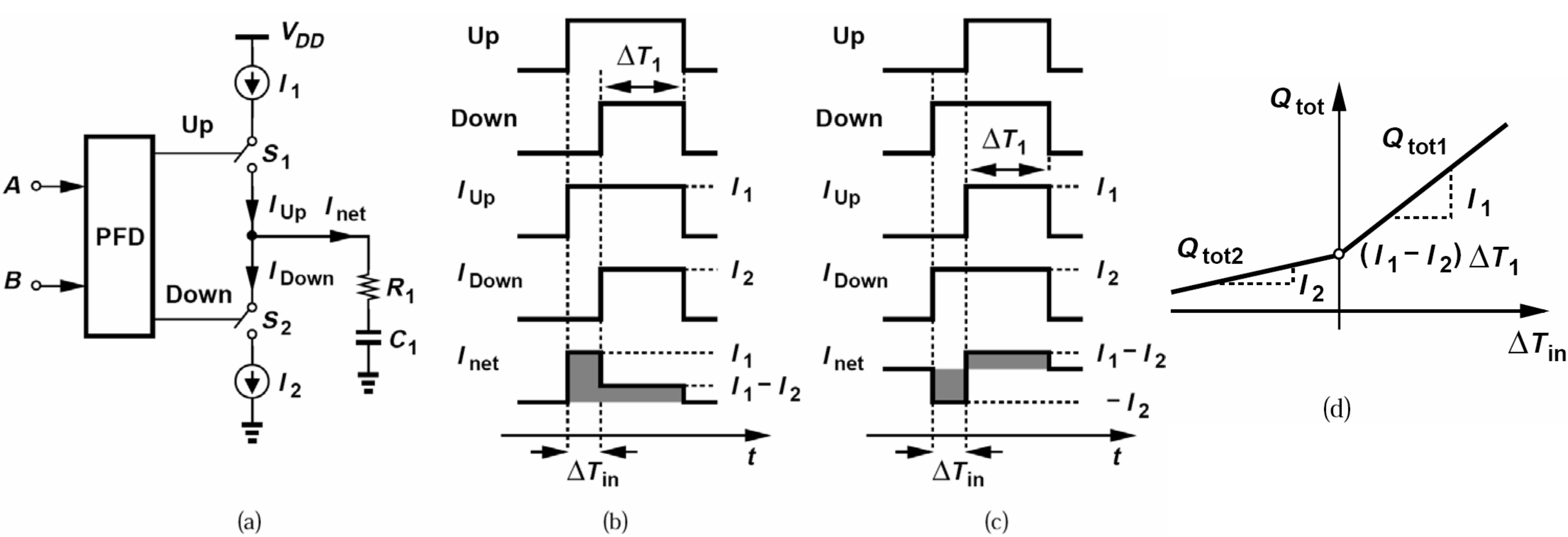
$$\begin{aligned} S_\Phi(f) &= |1 - z^{-1}|^2 S_q(f) \\ &= |2\sin(\pi f T_{CK})|^2 S_q(f). \end{aligned}$$

- Spectrum of PLL output phase noise:

$$S_{out}(f) = |2\sin(\pi f T_{CK})|^2 S_q(f) N^2 \frac{4\zeta^2\omega_n^2\omega^2 + \omega_n^4}{(\omega^2 - \omega_n^2)^2 + 4\zeta^2\omega_n^2\omega^2},$$



# Nonlinearity Due to Charge Pump Mismatch




(a) (b) (c) (d)

($\Delta T_{in}$ is proportional to quantization noise)

- **Total charge delivered to the loop filter in (b) is equal to:**

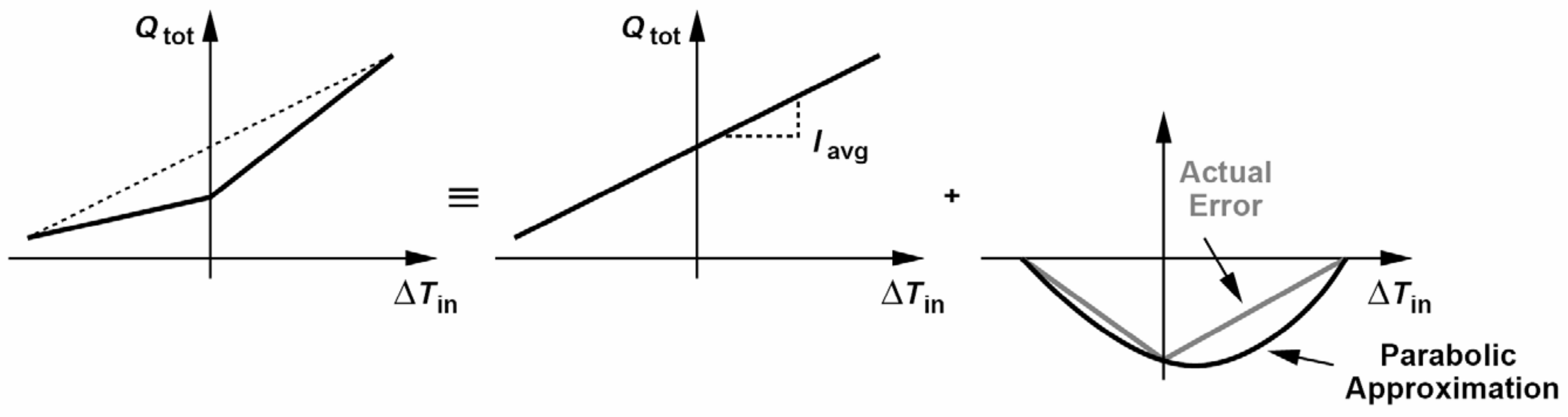
$$Q_{tot1} = I_1 \cdot \Delta T_{in} + (I_1 - I_2) \cdot \Delta T_1.$$

- **Now reverse the polarity of the input phase difference:**

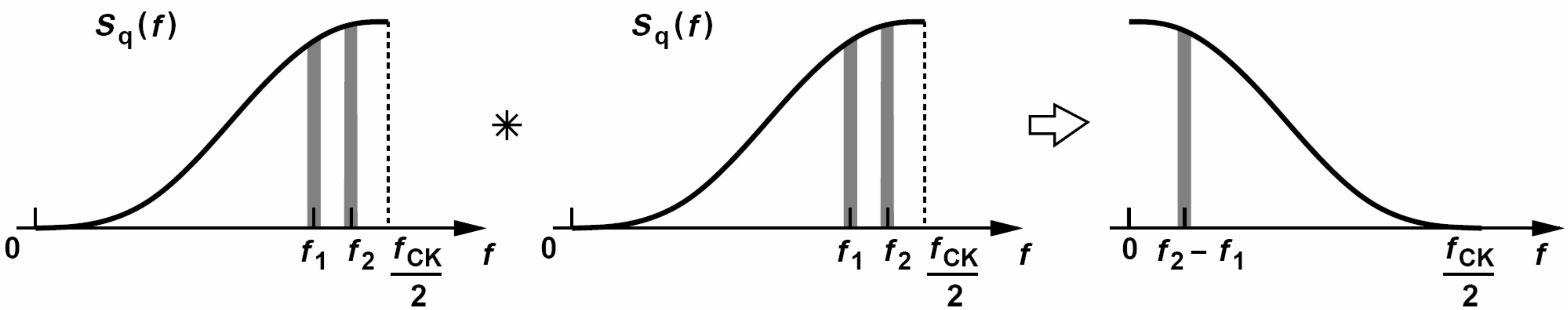
$$Q_{tot2} = I_2 \cdot \Delta T_{in} + (I_1 - I_2)\Delta T_1.$$

**$\Delta T_{in}$ is negative here**

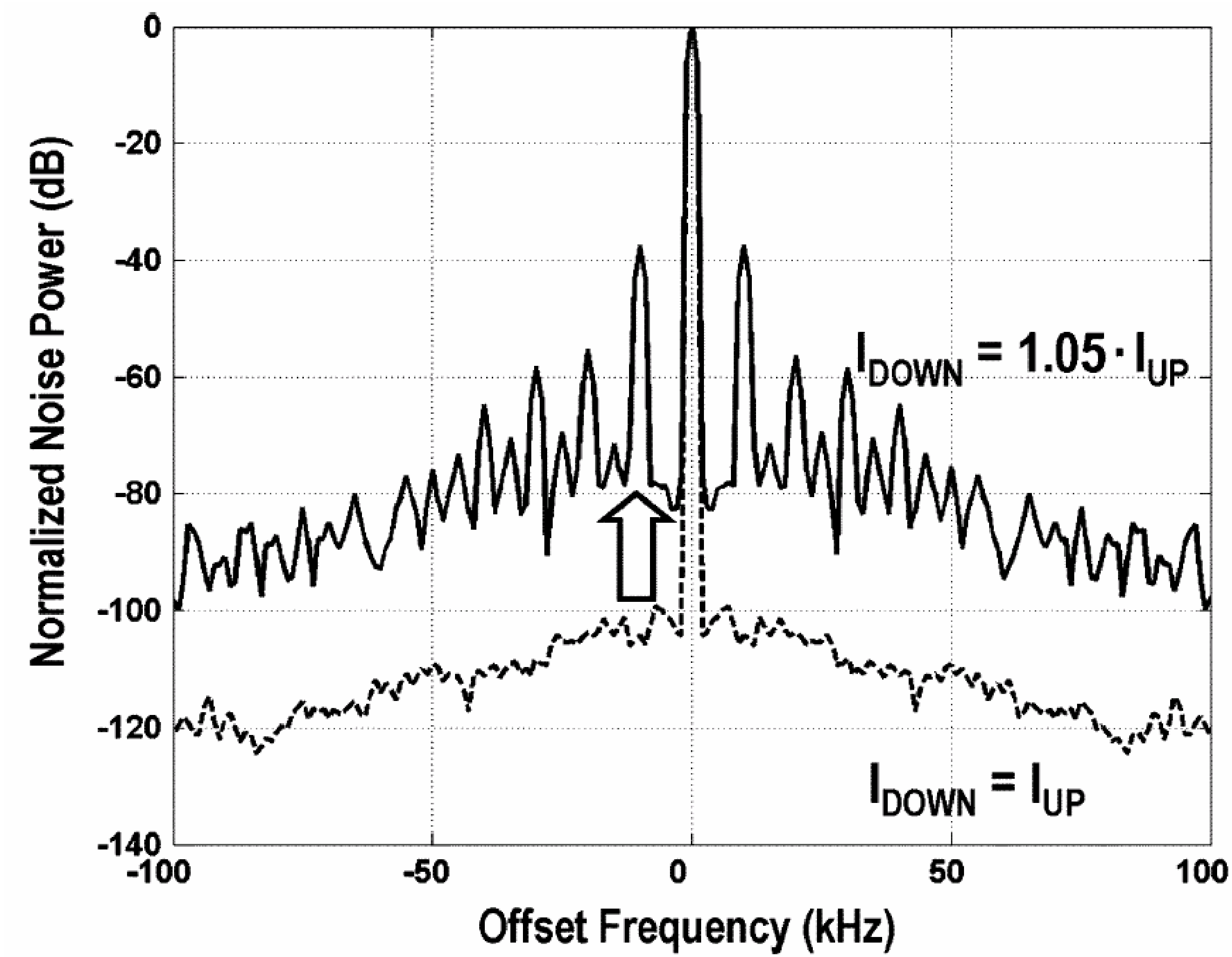
# Effect of Charge Pump Nonlinearity




- Approximate the error by a parabola, $\alpha\Delta T^2_{in} - b$, and write $Q_{tot} \approx I_{avg}\Delta T_{in} + \alpha\Delta T^2_{in} - b$
- The multiplication of $\Delta T_{in}$ by itself is a mixing effect and causes convolution:



# Effect of Charge Pump Nonlinearity




**[Huh, JSSC, Nov. 05]**