EE215C

Final Exam

Winter 2009

Name: ....Solutions....

Time Limit: 3 Hours

Open Book, Open Notes

1. 20

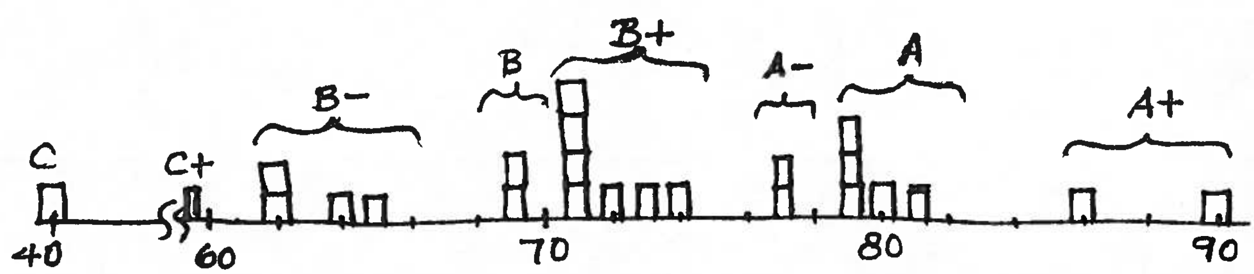
2. 10

3. 10

4. 10

5. 15

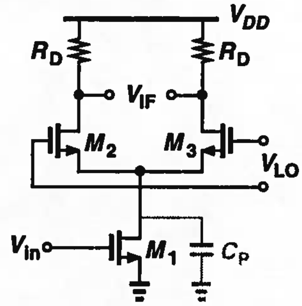
Total: 65



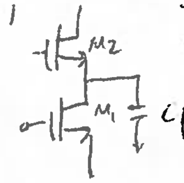
1. Consider the active mixer shown below, where all other capacitances are neglected and no transistor enters the triode region. Also, channel-length modulation and body effect are negligible.

(a) Suppose M_2 is on and M_3 is off. Derive an expression for the small-signal drain current of M_2 due to the thermal noise of M_2. Plot the spectrum of the drain current. Neglect all other sources of noise.

(b) Now suppose the impedance of C_P is much larger than $1/g_{m2}$ at the frequencies of interest, and the LO is applied with 50% duty cycle. If M_2 and M_3 switch abruptly, and only the first and third harmonics of the LO are considered, explain in detail how the thermal noise of M_2 appears at IF. Sketch the IF noise spectrum due to the first and third harmonics of the LO. Neglect all other sources of noise.



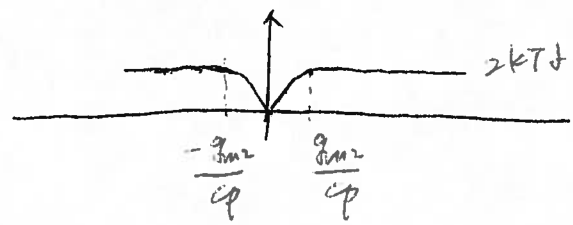
(a)



$$\overline{i_{dn}^2} = 4kT\gamma g_{m2} \left| \frac{g_{m2}}{\frac{1}{g_{m2}} + \frac{1}{sC_p}} \right|^2$$

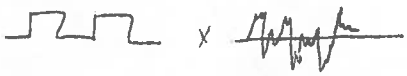
$$= 4kT\gamma g_{m2} \frac{\omega^2 C_p^2}{\omega^2 C_p^2 + g_{m2}^2} \quad (6) \quad \#$$

spectrum:

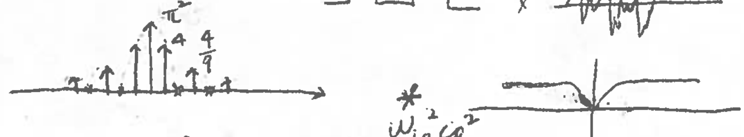


(4) #

b) time domain: square wave × noise

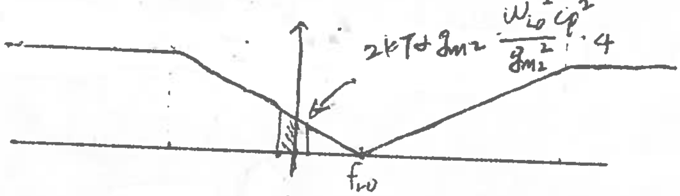


⇒ frequency domain:



$*$

for $+f_{LO}$

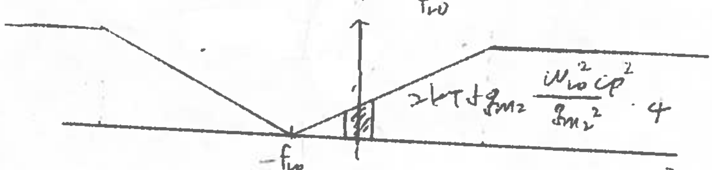


$\because \frac{1}{\omega C_p} \gg \frac{1}{g_{m2}}$

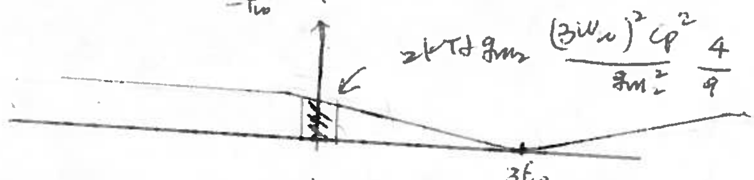
$\Rightarrow \omega C_p \ll g_{m2}$

$\therefore \overline{i_{d2}^2} = 4kT\gamma g_m \frac{(\omega C_p)^2}{g_{m2}^2}$

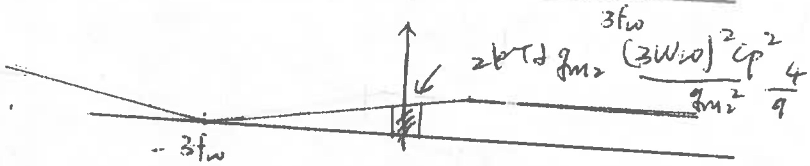
for $-f_{LO}$



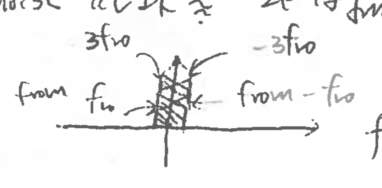
for $3f_{LO}$



for $-3f_{LO}$.



$\therefore$ the total noise at IF $\approx 2kT\gamma g_{m2} \frac{\omega_{LO}^2 C_p^2}{g_{m2}^2} \cdot 16$ #

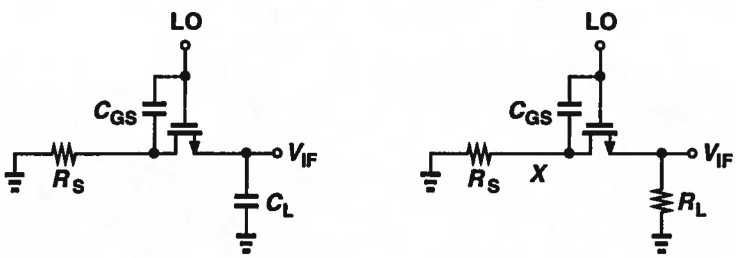


the 3rd harmonic contribut
the same noise as the 1s

2. A student considers the arrangement shown on the left, where C_{GS} introduces LO leakge at the input and hence a dc component at the output. (Note that the capacitor does not carry a dc current.) The student then decides that the arrangement on the right is *free* from dc offsets, reasoning that a dc voltage, V_{ds}, at the output would require a dc current, V_{dc}/R_L, through R_L and hence an equal current through R_S. But this is impossible because a dc current through R_S gives rise to a negative voltage at node X.

5 (a) Explain in detail whether the student's conjecture is correct and why.

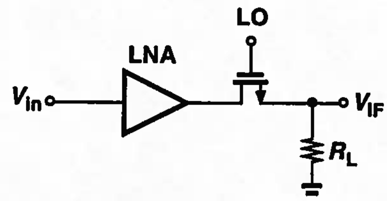
5 (b) Neglecting C_{GS}, calculate the voltage conversion gain of the circuit on the right. Assume a square-wave LO with 50% duty cycle. Also, assume the switch has an on-resistance of R_{on}.



(a) The flaw in the student's reasoning is that a dc current at the output requires a dc current at the input. In other words, KCL only holds for <u>all</u> of the frequency components summed together - not for each individual component. Thus, both circuits suffer from dc offsets.

(b) The mixer operates as an ideal return-to-zero topology, except that R_{on} and R_L attenuate the signal when the switch is on. That is, the input signal is multiplied by a square wave toggling between 0 and $\frac{R_L}{R_L + R_{on} + R_S}$. The conversion gain is therefore equal to $\frac{1}{\pi}\frac{R_L}{R_L + R_{on} + R_S}$.

3. Shown below is part of a direct-conversion receiver. Suppose the LNA input-output characteristic can be expressed as $y(t) = \alpha_1 x(t) + \alpha_2 x^2(t)$. If the mixer switches abruptly with a 50% duty cycle and if R_L is much greater than the on-resistance of the switch and the output resistance of the LNA, compute the IP_2 of the receiver.



2

The LO signal can be represented as $\frac{1}{2} + \sum_{n=1}^{\infty} \frac{2}{n\pi} \sin\left(\frac{n\pi}{2}\right) \cos(n\omega T)$

for a two tone test we have $V_{in} = A(\cos\omega_1 t + \cos\omega_2 t)$

$\therefore$ we have $V_{IF} = \left[\alpha_1 A(\cos\omega_1 t + \cos\omega_2 t) + \alpha_2 A^2 (\cos\omega_1 t + \cos\omega_2 t)^2\right]\left[\frac{1}{2} + \frac{2}{\pi}\cos(\omega_{LO} t) + \dots\right]$

for ω_2 slightly higher than ω_1, $(\omega_2 - \omega_1)$ component at the output of the LNA appears at the output of the mixer multiplied by its DC gain and located close to DC.

$\omega_2 - \omega_1$: @LO output $\alpha_2 A^2 \cos(\omega_2 - \omega_1)$

2

@ Mixer output $\frac{1}{2}\alpha_2 A^2 \cos(\omega_2 - \omega_1)$

2

This resides in the same band as the converted RF signal $\frac{\alpha_1 A}{\pi}$

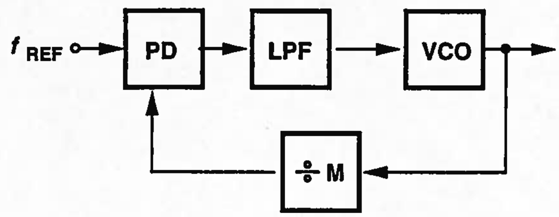
At the IP2 point $\frac{\alpha_1 A_{IP_2}}{\pi} = \frac{\alpha_2 A_{IP_2}^2}{2}$ 2

$\therefore A_{IP_2} = \frac{\alpha_1}{\alpha_2}\frac{2}{\pi}$ 2

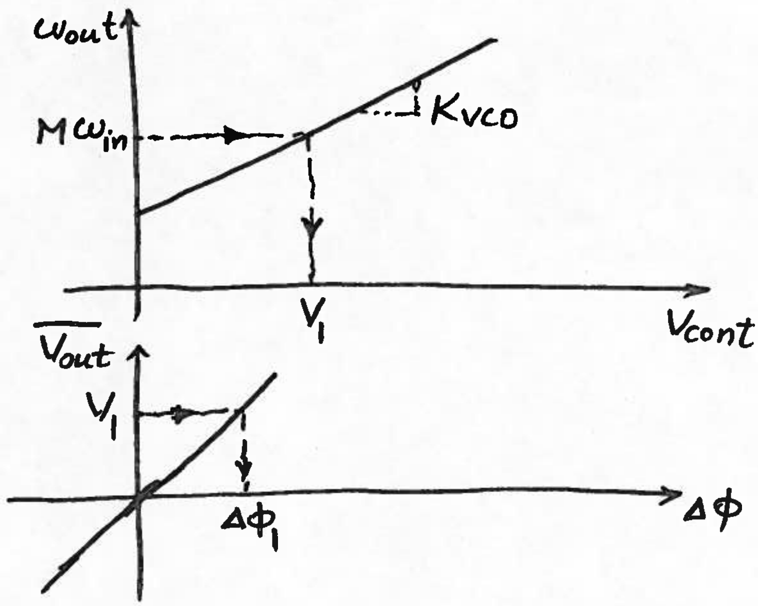
4. Consider the simple type-I PLL shown below. Assume the loop is locked.

(a) If the input frequency is known, describe in detail how you compute the control voltage and the phase error (between the two inputs of the PD).

(b) If the input frequency changes by $\Delta\omega$, compute the change in the phase error.



(a)



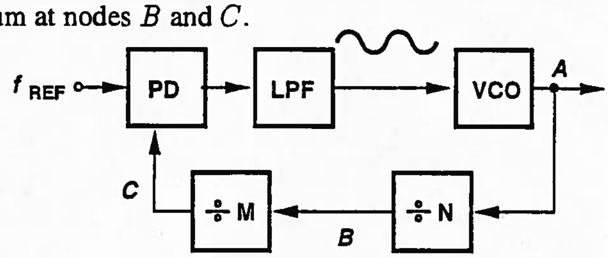
(b) If the input frequency changes by $\Delta\omega$, ω_{out} changes by $M\Delta\omega$ and the control voltage by $\frac{M\Delta\omega}{K_{VCO}}$. Thus, the input phase error changes by $\frac{M\Delta\omega}{K_{VCO}K_{PD}}$.

5. A type-I PLL is shown below, where the control voltage experiences a sinusoidal ripple with a frequency of f_{REF} and a peak amplitude of V_r. Assume the loop is locked.

(a) Explain the difference between a harmonic and a sideband.

(b) Using the narrow-band FM approximation, determine the spectrum at point A.

(c) Now determine the spectrum at nodes B and C.



(a) HARMONICS – A frequency component in a spectrum that is an integer multiple of the fundamental frequency. Appears when a sinusoid is applied to a non-linear system.

SIDEBANDS – Deterministic frequency component that appears around the carrier frequency. A generic example are tones produced by a modulation process of the original signal.

(b) (NOTE) Narrowband FM Approx.

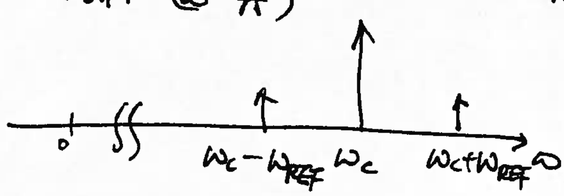
$$x_{FM}(t) = A_c \cos\left[w_c t + m\int_{-\infty}^{t} x_{BB}(t)\,dt\right] \simeq A_c \cos w_c t - A_c \sin w_c t \int x_{BB}(t)\,dt.$$

given $x_{BB}(t) = V_r \cos(\omega_{REF} t)$, $\omega_{REF} = 2\pi f_{REF}$

$$x_{FM}(t) \cong A_c \cos(\omega_c t) - \frac{A_c V_r m}{\omega_{REF}} \sin(\omega_c t)\sin(\omega_{REF} t)$$

$$= A_c \cos(\omega_c t) - \frac{A_c V_r m}{2\omega_{REF}} \cos(\omega_c - \omega_{REF})t + \frac{A_c V_r m}{2\omega_{REF}} \cos(\omega_c + \omega_{REF})t$$

(SPECTRUM @ A)



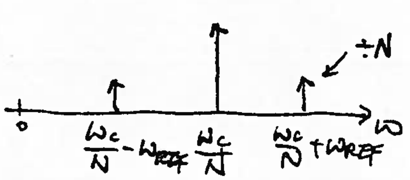
(c) When FM modulated signal is applied to a divider, (÷N)

fundamental : freq ÷ N

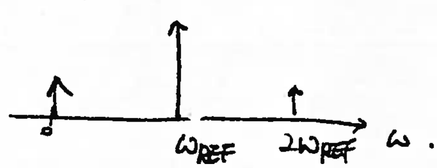
side band : Amp ÷ N

$$x_{FM} = A_c \cos\Big(\underbrace{\omega_c t + m\int x_{BB}(t)\,dt}_{\div N}\Big).$$

∴ @ B



@ C



EE215C

Final Exam

Winter 2010

Name: Solutions

Time Limit: 3 Hours

Open Book, Open Notes

1. 10

2. 10

3. 10

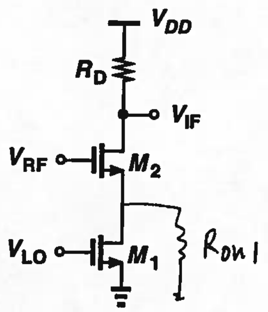
4. 10

Total: 40

1. The circuit shown below is a mixer used in traditional microwave design. Assume when M_1 is on, it has an on-resistance of R_{on1}. Also, assume abrupt edges and a 50% duty cycle for the LO and neglect channel-length modulation and body effect.

(a) Compute the voltage conversion gain of the circuit. Assume M_2 does not enter the triode region and denote its transconductance by g_{m2}.

(b) If R_{on1} is very small, determine the IP_2 of the circuit. Assume M_2 has an overdive of $V_{GS0} - V_{TH}$ in the absence of signals (when it is on).



(a) $\dfrac{-R_D}{\frac{1}{g_{m2}} + R_{on1}} \times \dfrac{2}{\pi} \times \dfrac{1}{2}$ (for single-ended)

$= \dfrac{-R_D}{\frac{1}{g_{m2}} + R_{on1}} \times \dfrac{1}{\pi}$ #

(b)

for 2 tone test $V_{in} = V_0 \cos W_1 t + V_0 \cos W_2 t$

$$\frac{1}{2}\mu_n C_{ox}\frac{W}{L}\left(V_{GS0} - V_{th} + V_0 \cos W_1 t + V_0 \cos W_2 t\right)^2$$

$$= \frac{1}{2}\mu_n C_{ox}\frac{W}{L}\Big(V_{GS}^2 + V_{th}^2 + V_0^2 \cos^2 W_1 t + V_0^2 \cos^2 W_2 t$$
$$+ V_0^2 \cos(W_1 - W_2) t + V_0^2 \cos(W_1 + W_2) t - 2V_{GS0}V_{th}$$
$$+ 2V_0 V_{GS0} \cos W_1 t + 2V_0 V_{GS0} \cos W_2 t - 2V_0 V_{th} \cos W_1 t$$
$$- 2V_0 V_{th} \cos W_2 t\Big)$$

the gain for $W_1 - W_{LO}$ & $(W_2 - W_{LO})$ at $V_{IF} = \frac{1}{\pi} \times \frac{1}{2}\mu_n C_{ox}\frac{W}{L} \cdot 2(V_{GS} - V_{th}) R_D$

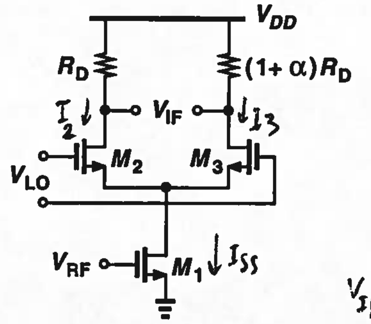
the gain for $W_1 - W_2$ at $V_{IF} = \frac{1}{2} \times \frac{1}{2}\mu_n C_{ox}\frac{W}{L} \cdot R_D$

$IIP2 = \dfrac{4(V_{GS} - V_{th})}{\pi}$ #

2. Consider the active mixer shown below, where the LO has abrupt edges and a 50% duty cycle. Also, channel-length modulation and body effect are negligible. The load resistors exhibit mismatch but the circuit is otherwise symmetric. Assume M_1 carries a bias current of I_{SS}.

(a) Determine the output offset voltage.

(b) Determine the IP_2 of the circuit in terms of the overdrive and bias current of M_1.



$I_2 = I_{SS} \times S(t)$ $\quad S(t):$

$I_3 = I_{SS} \times (1 - S(t))$ $\quad 1-S(t):$

$$V_{IF} = (1+\alpha)R_D I_3 - R_D I_2 = R_D I_{SS}(1 - 2S(t)) + \alpha R_D I_{SS}(1 - S(t))$$

$$V_{IF}\Big|_{DC} = \alpha R_D I_{SS} \times \frac{1}{2}$$

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b) Two-tone test: $V_{RF} = V_1 \cos\omega_1 t + V_1 \cos\omega_2 t + V_{GS_0}$

$$I_{SS} = \overbrace{\frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)_1}^{K} \left(V_1\cos\omega_1 t + V_1 \cos\omega_2 t + V_{GS_0} - V_{TH}\right)^2$$

$$= K\left\{ V_1^2\left[\frac{1+\cos 2\omega_1 t}{2} + \frac{1+\cos 2\omega_2 t}{2} + \cos(\omega_1+\omega_2)t + \cos(\omega_1-\omega_2)t\right] + (V_{GS_0} - V_{TH})^2 + 2(V_{GS_0} - V_{TH})V_1\left[\cos\omega_1 t + \cos\omega_2 t\right]\right\}$$

Beat Component

fundamentals

$$V_{IF}\Big|_{\text{fundamental}} = k\, 2(V_{GS_0} - V_{TH})V_1 \cdot R_D\left(\frac{2}{\pi} + \alpha\frac{1}{\pi}\right)\left[\cos(\omega_1 - \omega_{LO})t + \cos(\omega_2 - \omega_{LO})t\right]$$

$$V_{IF}\Big|_{\text{Beat}} = k V_1^2 \times \alpha R_D \times \frac{1}{2} \cdot \cos(\omega_1 - \omega_2)t$$

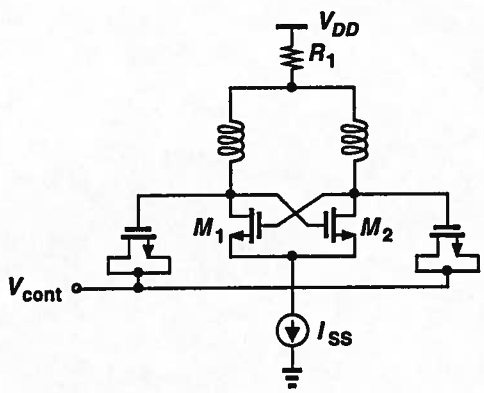
$@A_{IP_2}$: $|V_{IF}|_{\text{fundamental}} = |V_{IF}|_{\text{Beat}}$

$$\Rightarrow A_{IP_2} = \underbrace{\frac{4}{\alpha}\left(\frac{2}{\pi} + \alpha\frac{1}{\pi}\right)}_{(V_{GS_0} - V_{TH})} = \left(\frac{8}{\alpha\pi} + \frac{4}{\pi}\right)(V_{GS_0} - V_{TH})$$

3. In the VCO circuit shown below, the voltage dependence of each varactor can be expressed as $C_{var} = C_0(1 + \alpha_1 V_{var})$, where V_{var} denotes the average voltage across the varactor. Use the narrowband FM approximation in this problem. Also, neglect all other capacitances and assume the circuit oscillates at a frequency of ω_0 for the given value of V_{cont}. The dc drop across the inductors is negligible.

5 (a) Compute the "gain" from I_{SS} to the output frequency, ω_{out}. That is, assume I_{SS} changes by a small value and calculate the voltage change across the varactors and hence the change in the output frequency.

5 (b) Assume I_{SS} has a noise component that can be expressed as $I_n \cos\omega_n t$. Using the result found in (a), determine the frequency and relative magnitude of the resulting output sidebands of the oscillator.



(a)

$$\omega_{out} = \frac{1}{\sqrt{LC_{var}}} = \frac{1}{\sqrt{LC_0(1+\alpha_1 V_{var})}}$$

$$\simeq \frac{1}{\sqrt{LC_0}}\left(1 - \frac{\alpha_1 V_{var}}{2}\right) = \omega_0\left(1 - \frac{\alpha_1 V_{var}}{2}\right)$$

$$\therefore \frac{\partial \omega_{out}}{\partial I_{SS}} = \frac{\partial \omega_{out}}{\partial V_{var}} \cdot \frac{\partial V_{var}}{\partial I_{SS}} = \left(-\frac{\alpha_1 \omega_0}{2}\right)\cdot(-R_1) = \frac{\alpha_1 R_1 \omega_0}{2} \;//$$

(b) Using NB Approx.

$$V_{out}(t) = A\cos\left(\omega_0 t + \frac{\alpha_1 \omega_0 R_1}{2\omega_n}\cdot I_n \sin(\omega_n t)\right)$$

$$\simeq A\cos(\omega_0 t) - \frac{A\cdot\alpha_1\omega_0 R_1 I_n}{4\omega_n}\cos(\omega_0-\omega_n)t + \frac{A\alpha_1\omega_0 R_1 I_n}{4\omega_n}\cos(\omega_0+\omega_n)t$$

($\therefore$) freq: $\omega_0 \pm \omega_n$

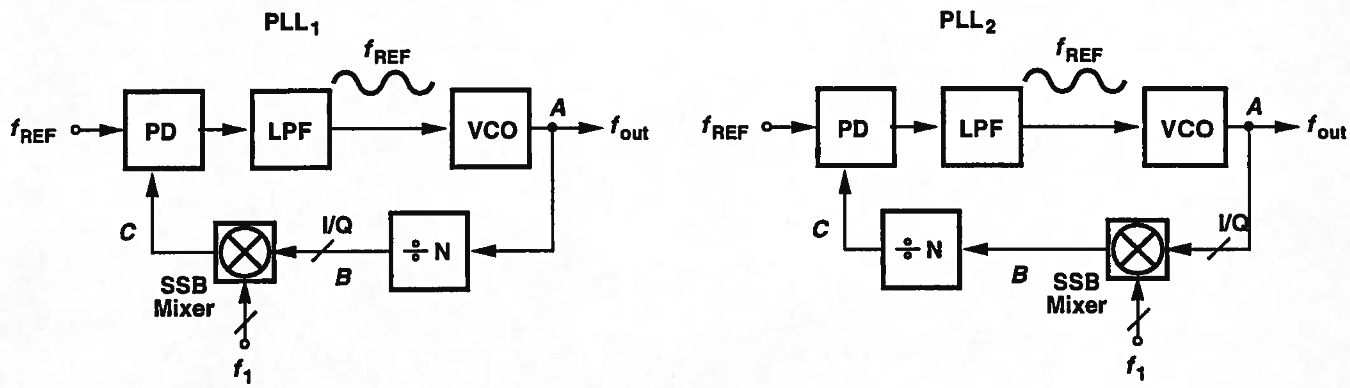
rel. mag: $\dfrac{\alpha_1 \omega_0 R_1 I_n}{4\omega_n}$

4. Two PLL configurations are shown below. Assume the SSB mixer *adds* its input frequencies. Also, assume f_1 is a constant frequency provided externally and it is less than f_{REF}. The control voltage experiences a small sinusoidal ripple with a frequency of f_{REF}. Both PLLs are locked.

3 (a) Determine the output frequencies of the two PLLs.

4 (b) Determine the spectrum at point A due to the ripple.

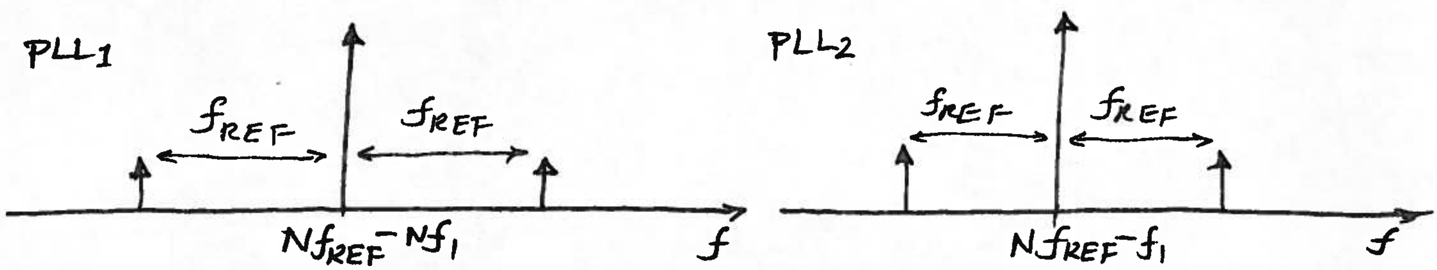
4 (c) Now determine the spectrum at nodes B and C.



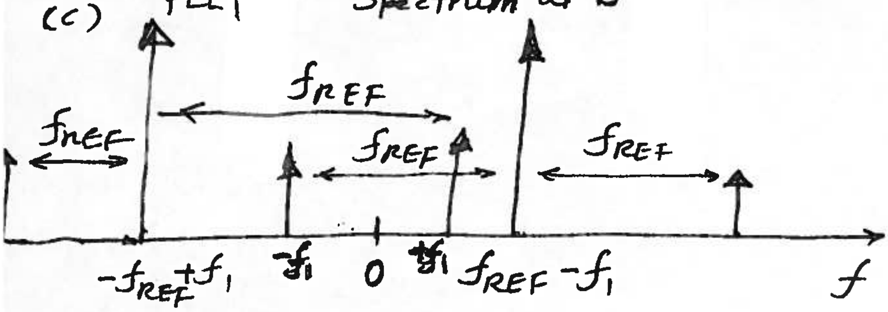
(a) PLL$_1$: $\dfrac{f_{out}}{N} + f_1 = f_{REF} \Rightarrow f_{out} = N(f_{REF} - f_1)$

PLL$_2$: $\dfrac{f_{out} + f_1}{N} = f_{REF} \Rightarrow f_{out} = N f_{REF} - f_1$

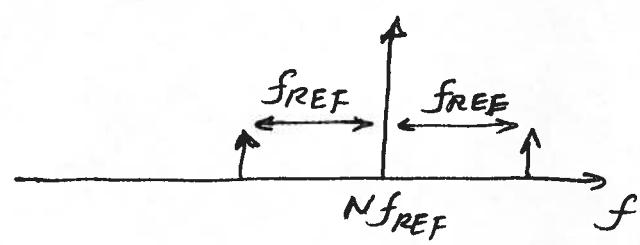
(b)



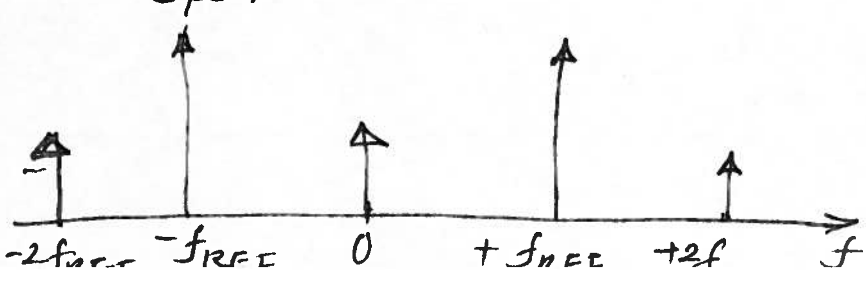
(c) PLL$_1$ Spectrum at B



PLL2 Spectrum at B



Spectrum at C



Spectrum at C