EE215C

Final Exam

Winter 2013

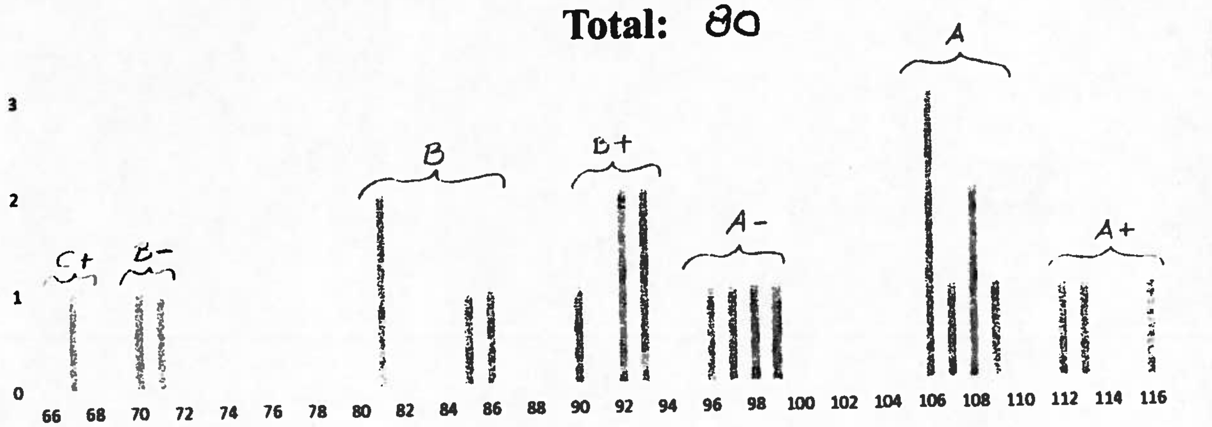
Name: Solutions

Time Limit: 3 Hours

Open Book, Open Notes

1. 20
2. 20
3. 20
4. 20

Total: 80



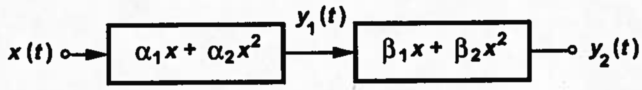
Out of 120

1. Shown below is a cascade of two second-order static systems.

(a) Determine the IIP_3 of the cascade in terms of α_j and β_j.

(b) If each stage is compressive, can the cascade be expansive? Explain in detail.

(c) Is it meaningful to define an IIP_3 for an expansive system? Explain in detail.



a) $y_2(t) = \beta_1(\alpha_1 x + \alpha_2 x^2) + \beta_2(\alpha_1 x + \alpha_2 x^2)^2$

$= \underbrace{\alpha_1\beta_1}_{k_1} x + \alpha_2\beta_1 x^2 + \alpha_1^2\beta_2 x^2 + \underbrace{2\alpha_1\alpha_2\beta_2}_{k_3} x^3 + \alpha_2^2\beta_2 x^4$

$A_{IIP} = \sqrt{\frac{4}{3}\left|\frac{k_1}{k_3}\right|} = \sqrt{\frac{2}{3}\frac{\beta_1}{\alpha_2\beta_2}}$

b) Each stage is compressive

$\rightarrow \alpha_1\alpha_2 < 0, \quad \beta_1\beta_2 < 0$

To be expansive, $k_1 \cdot k_3 > 0 \rightarrow \alpha_1\beta_1 \cdot 2\alpha_1\alpha_2\beta_2 > 0$

$\rightarrow \alpha_1 > 0$

So, if $\alpha_1 > 0$, System can be expandable

c) IIP_3 is still meaningful to define IM_3 components.

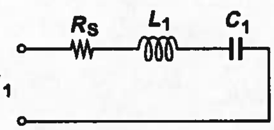
2. In this problem, we investigate an oscillator that incorporates a *series* LC tank.

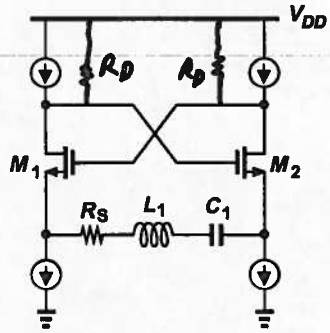
(a) Consider the series tank shown below, where R_S models the loss of the inductor. Sketch the magnitude and phase of Z_1 as a function of frequency.

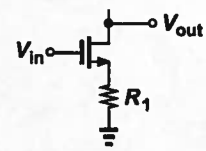
(b) Now consider the cross-coupled pair and determine its start-up condition and its frequency of oscillation. Neglect all other capacitances but not channel-length modulation. ~~(Hint: first determine the voltage gain of the degenerated stage shown on the right.)~~

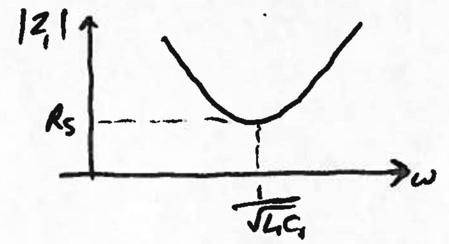
a)

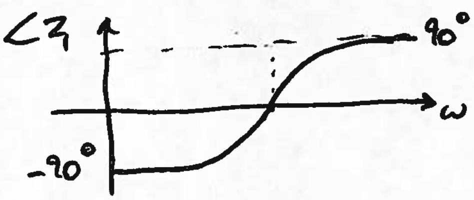
$$Z_1 = R_S - j\frac{(1-L_1C_1\omega^2)}{C_1\omega}$$





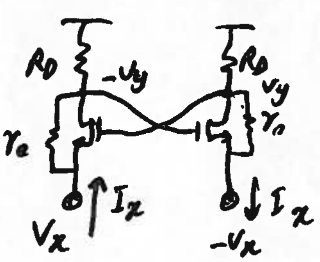






b) $f_{osc} = \frac{1}{2\pi\sqrt{L_1C_1}}$

To find the start-up condition, we calculate the impedance seen by the tank.



$$\begin{cases} -V_y = R_D I_x \\ I_x + g_m(V_y - V_x) = \frac{V_x + V_y}{r_o} \end{cases}$$

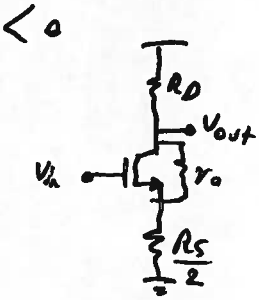
$$\Rightarrow Z_{in} = \frac{2V_x}{I_x} = \frac{2(r_o + R_D - g_m r_o R_D)}{1 + g_m r_o}$$

The negative resistance must be stronger than R_S.

$$\Rightarrow \boxed{\frac{2(r_o + R_D - g_m r_o R_D)}{1 + g_m r_o} + R_S < 0}$$

if $g_m r_o \gg 1$, $g_m R_D \gg 1 \Rightarrow \quad -2R_D + R_S < 0$

Another Method is to set the gain of the circuit on the right greater than 1.



$$\text{KVL, KCL} \Rightarrow |A_v| = \frac{g_m}{\frac{1}{r_o} + \frac{1}{R_D} + \frac{1}{r_o}\frac{R_S}{2R_D} + \frac{g_m R_S}{2R_D}}$$

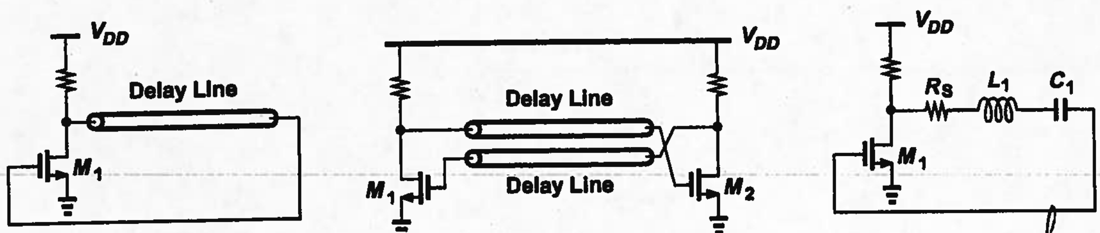
$$|A_v| > 1 \Rightarrow \frac{g_m r_o R_D}{r_o + R_D + \frac{R_S}{2} + \frac{g_m r_o R_S}{2}} > 1$$

$$\Rightarrow r_o + R_D - g_m r_o R_D + \frac{R_S}{2}(1 + g_m r_o) < 0$$

$$\Rightarrow \boxed{\frac{2(r_o + R_D - g_m r_o R_D)}{1 + g_m r_o} + R_S < 0}$$

20.

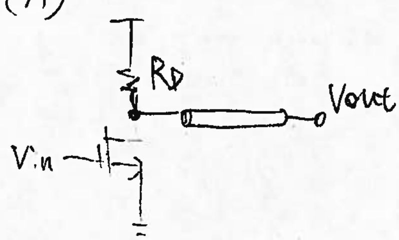
3. Explain in detail which one of the circuits shown below can oscillate. Neglect all other capacitances and channel-length modulation. For the delay lines, you can assume a zero loss, a physical length of l, and a wave velocity of v. If the circuit is capable of oscillation, determine the oscillation frequency.



The delay time of the delay line $T_d = \frac{l}{V}$

Phase introduced by T_d at some ω $= \frac{T_d}{T} \cdot 2\pi = \omega \cdot T_d$

(A)



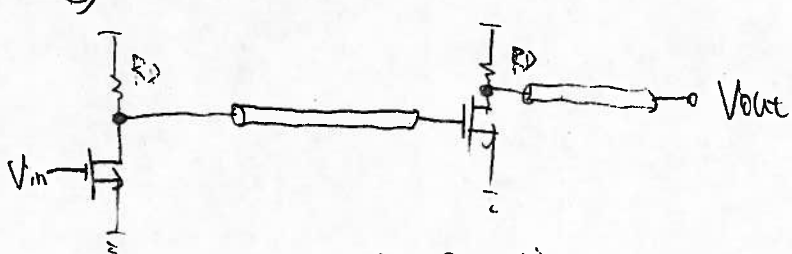
$$|H(s)| = g_m R_D$$

$$\angle H(s) = \omega T_d$$

$$\Rightarrow 2\pi f_{osc} T_d = (2N-1)\pi$$

$$\Rightarrow f_{osc} = \frac{2N-1}{2T_d} = \frac{(2N-1)V}{2l}, \quad N = 1, 2, 3 \ldots$$

(B)

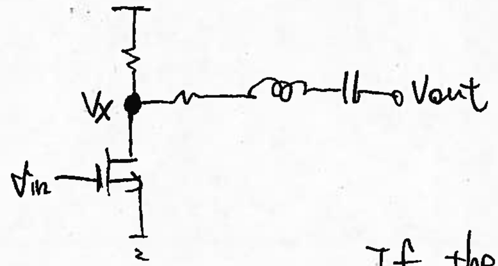


$$|H(s)| = (g_m R_D)^2$$

$$\angle H(s) = \pi + 2\omega T_d \Rightarrow \pi + 2 \cdot 2\pi f_{osc} T_d = \pi + 2 \cdot N \cdot \pi$$

$$\Rightarrow f_{osc} = \frac{N}{2T_d} = \frac{N \cdot V}{2 \cdot l}, \quad N = 1, 2, 3 \ldots$$

(c)



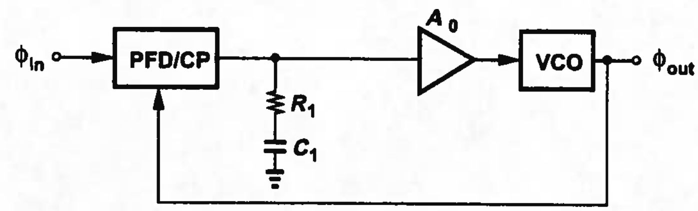
If the gate capacitance is ignored,

$V_x = V_{out}$, $\Rightarrow$ The loop can't provide $360°$ phase shift, so it can't oscillate.

4. A student decides to precede a VCO in a PLL with an amplifier as shown below. Assume the amplifier has a voltage gain of A_0.

(a) Suppose the amplifier has an input-referred offset voltage equal to V_{OS}. Does this offset cause a phase error? Explain in detail.

(b) Now suppose the amplifier is free from offset but has noise. Modeling the input-referred noise by a voltage, V_n, find the transfer function from V_n to the output phase. Sketch the magnitude of this transfer function to show its general behavior.

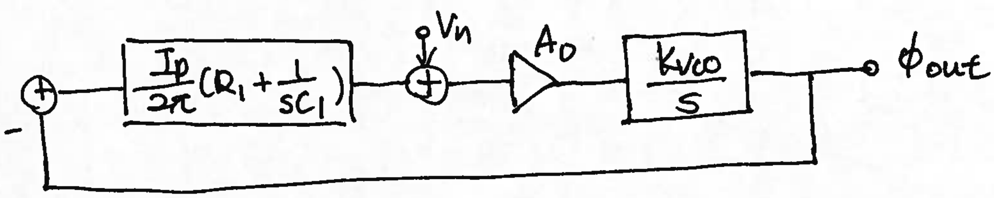


(a) NO, this effect does NOT cause a phase error.

Infinite gain of charge pump and negative feedback force a phase error to be zero with constant V_{os}.

Another point of view is that DC offset is filtered by the bandpass transfer function of (b).

(b)



$$\phi_{out} = \left(-\phi_{out}\frac{I_p}{2\pi}\left(R_1+\frac{1}{sC_1}\right) + V_n\right)\times A_0 \times \frac{K_{VCO}}{s}$$

$$\frac{\phi_{out}}{V_n} = \frac{A_0 K_{VCO}\, s}{s^2 + A_0 K_{VCO} R_1 \frac{I_p}{\pi} s + A_0 K_{VCO}\frac{I_p}{2\pi C_1}}$$