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HW 2

1. (a). Output voltage is $V_{out} = V_1 - V_1 e^{-\frac{t}{R_{on}C_H}}$ when $V_{in} = V_1$ is a constant.

Now, let $V_{out} = V_1 - 0.5\,LSB$

$$\Rightarrow V_1 e^{-\frac{t}{R_{on}C_H}} = 0.5\,LSB = 0.5 \times \frac{V_1}{2^8}$$

where $R_{on} = 400\,\Omega$, $C_H = 0.35\,pF$

$$\Rightarrow \underline{t = 0.873\,ns}$$

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(b). First we should find the initial voltage on C_H. Since the previous hold value is at the negative peak, it's obvious that $V_{out}(t=0) = -0.35V$.

According to the time domain equation: $\frac{V_{in} - V_{out}}{R_{on}} = C_H \frac{dV_{out}}{dt}$

$$\Rightarrow \frac{V_1 \sin W_{in}t - V_{out}}{R_{on}C_H} = \frac{dV_{out}}{dt}$$

Take s domain transform, $V_{out}(s) = \frac{V_1 W_{in}}{(s^2 + W_{in}^2)(1 + sR_{on}C_H)} = V_1 W_{in}\left[\frac{(As+B)}{s^2 + W_{in}^2} - \frac{C}{1 + R_{on}C_H s}\right]$

$$\Rightarrow \begin{cases} A R_{on}C_H = C \\ A + B R_{on}C_H = 0 \\ B - CW_{in}^2 = 1 \end{cases} \Rightarrow \begin{cases} A = \frac{-1}{\frac{1}{R_{on}C_H} + R_{on}C_H W_{in}^2} \\ B = \frac{1}{1 + R_{on}^2 C_H^2 W_{in}^2} \\ C = -\frac{R_{on}C_H}{\frac{1}{R_{on}C_H} + R_{on}C_H W_{in}} \end{cases}$$

Take inverse S transform, $V_{out}(t) = V_1 W_{in}\left(A\cos W_{in}t + \frac{B}{W_{in}}\sin W_{in}t - \frac{C}{R_{on}C_H} e^{-\frac{t}{R_{on}C_H}}\right)$

$$= V_1 \sin(W_{in}t - \phi) + \left(V_1 \frac{R_{on}C_H W_{in}}{1 + R_{on}^2 C_H^2 W_{in}^2} - V_1\right) e^{-\frac{t}{R_{on}C_H}} \quad (V_{out}(0) = -V_1$$

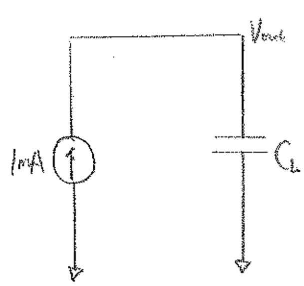
where $\arctan\phi = R_{on}C_H W_{in} = 0.0219905$

8/10 ① Since $1\,LSB = \frac{2V_1}{2^8}$, we shall make the exponential term equal to $-0.5\,LSB$

$$\Rightarrow V_1\left(\frac{R_{on}C_H W_{in}}{1 + R_{on}^2 C_H^2 W_{in}^2} - 1\right) e^{-\frac{t}{R_{on}C_H}} = -0.5 \times \frac{2V_1}{2^8}$$

$$\Rightarrow \underline{t \simeq 0.773\,ns}$$

2. (a) when S_1, S_2 turn off and S_3 turns on, $V_x < -0.2V$. So model should be:

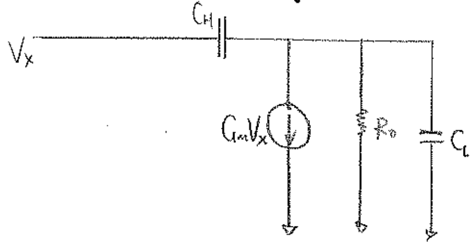


This continues until $V_x = -0.2V$ which corresponds to $V_{out} = 0.9V$

Since $I = \frac{Q}{t} = \frac{C\Delta V}{t}$

$$\Rightarrow t = \frac{C\Delta V}{I} = \frac{0.7 \times 10^{-12} \times 0.9}{10^{-3}} = 0.63\,ns$$

After $V_x \geq -0.2V$, The equivalent circuit can be shown:



According to KCL at V_{out},

$$\frac{V_{out}}{R_o} + C_L\frac{dV_{out}}{dt} + G_m(V_{out}-1.1) = 0$$

$$\Rightarrow \left(\frac{1}{R_o} + G_m\right)V_{out} + C_L\frac{dV_{out}}{dt} - 1.1G_m = 0$$

Since $G_m = 0.025\,\Omega^{-1}$, $R_o = 20k\Omega$, $C_L = 0.7pF$.

let $V_{out}(t) = A + Be^{-\frac{\frac{1}{R_o}+G_m}{C_L}t}$, $V_{out}(0) = 0.9V$

$$\Rightarrow \begin{cases} A = 1.0978 \\ B = -0.1978 \end{cases} \Rightarrow V_{out}(t) = 1.0978 - 0.1978e^{-3.58\times10^{10}t}$$

To settle to 0.5LSB for 10-bit resolution, $V_{out}(t) = 1.0978\times\left(1-0.5\times\frac{1}{2^{10}}\right)$

$\Rightarrow t \simeq 0.165\,ns$

$\Rightarrow$ Total hold settling time is $\underline{\underline{0.795\,ns}}$.

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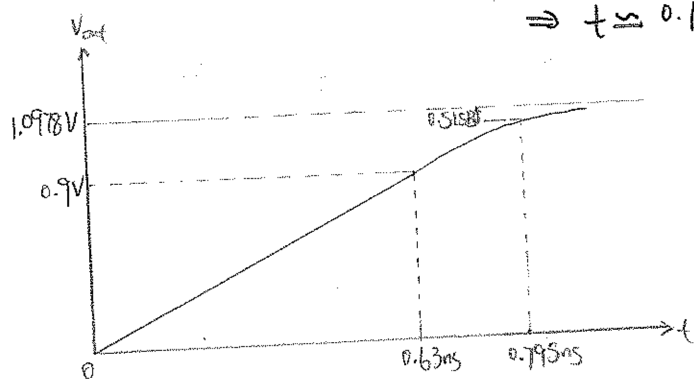
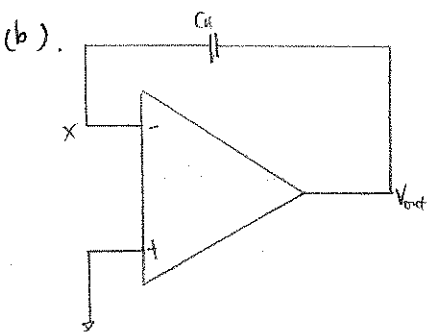


(b).



According to charge conservation at node X, we have

$$(V_{out}-V_x)C_H = C_H\cdot V_{in} \quad \& \quad V_x = -\frac{V_{out}}{A_v}$$

$$\Rightarrow \frac{V_{out}}{V_{in}} = \frac{1}{1+\frac{1}{A_v}} \simeq 1-\frac{1}{A_v} = 1-\frac{1}{G_mR_o}$$

$$\Rightarrow \underline{\underline{\text{gain error} = \frac{1}{G_mR_o} = 0.2\%}}$$

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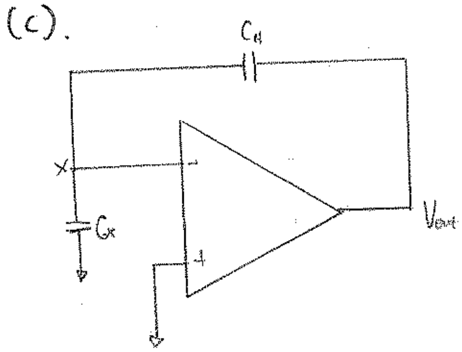
(c).



According to charge conservation at node X, we have

$$V_xC_x - (V_{out}-V_x)C_H = -V_{in}\cdot C_H$$

$$\& \quad V_x = -\frac{V_{out}}{A_v}$$

$$\Rightarrow \frac{V_{out}}{V_{in}} = \frac{C_H}{C_H+\frac{1}{A_v}(C_H+C_x)} = \frac{1}{1+\frac{1}{A_v}\cdot\frac{C_H+C_x}{C_H}} \simeq 1-\frac{1}{A_v}\cdot\frac{C_H+C_x}{C_H}$$

$$= 1-\frac{3}{2G_mR_o}$$

$$\Rightarrow \underline{\underline{\text{gain error} = \frac{3}{2G_mR_o} = 0.3\%}}.$$

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3. (a).

$$V_{GS} = \sqrt{\frac{2I_D}{\mu_n C_{ox}\frac{W}{L}}} + V_{TH} \Rightarrow \Delta V_{in} = \Delta V_{GS} = \sqrt{\frac{2I_{D1}}{\mu_n C_{ox}\frac{W}{L}}} - \sqrt{\frac{2I_{D2}}{\mu_n C_{ox}\frac{W}{L}}}$$

$$\Rightarrow \begin{cases} \Delta V_{in}^2 = \dfrac{2}{\mu_n C_{ox}\frac{W}{L}}\left(I_{SS} - 2\sqrt{I_{D1}I_{D2}}\right) \\ I_{D1} + I_{D2} = I_{SS} \end{cases}$$

$$\Rightarrow I_{D1} - I_{D2} = \frac{1}{2}\mu_n C_{ox}\frac{W}{L}\Delta V_{in}\cdot\sqrt{\frac{4I_{SS}}{\mu_n C_{ox}\frac{W}{L}} - (\Delta V_{in})^2}$$

Since $\mu_n C_{ox} = 265\times10^{-6}\ A/V^2$,

$$\therefore (0.99-0.01)\times203\times10^{-6} = \frac{1}{2}\times265\times10^{-6}\times\frac{W}{L}\times0.2\times\sqrt{\frac{4\times203\times10^{-6}}{265\times10^{-6}\times\frac{W}{L}} - 0.2^2}$$

$$\Rightarrow \frac{W}{L} = 45.9 \text{ or } 30.7$$

$$\Rightarrow W = 8.3\,\mu m \text{ or } 5.5\,\mu m$$

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According to simulation, $\underline{\underline{W = 8.3\,\mu m}}$ is the desired width to switch 99% of the tail current.

(b).

$$V_{out} = jI\left(R_L /\!/ \frac{r_o}{j}\right) = \frac{IR_L r_o}{R_L + \frac{r_o}{j}} \quad ; \text{ Ending point is } \frac{IR_L r_o}{R_L + \frac{r_o}{N}} \text{ when } j = N$$

So
$$INL = \frac{IR_L r_o}{R_L + \frac{r_o}{j}} - \frac{IR_L r_o}{R_L + \frac{r_o}{N}}\cdot\frac{j}{N} = \frac{R_L(1-\frac{j}{N})}{(R_L+\frac{r_o}{j})(R_L+\frac{r_o}{N})}\cdot IR_L r_o$$

if $r_o \gg NR_L$

we have $INL = \dfrac{IR_L^2}{r_o}j(N-j)$

$$\Rightarrow INL_{max} = \frac{IR_L^2}{4r_o}N^2 \quad \text{when } j = \frac{N}{2}$$

r_o is the cascode output impedance which equals to

$$(1+g_m r_{o1})r_{o2} + r_{o1}$$

$$g_m = \sqrt{2I_{D1}\mu_n C_{ox}\frac{W}{L}} = \sqrt{2\times0.99\times203\times10^{-6}\times265\times10^{-6}\times45.9} = 2.21\,m\Omega^{-1}$$

$$r_{o1} = \frac{1}{\lambda_1 I_D} = \frac{1}{0.37\times0.99\times203\times10^{-6}} = 13.45\,k\Omega$$

$$r_{o2} = \frac{1}{\lambda_2 I_D} = \frac{1}{0.37\times203\times10^{-6}} = 13.31\,k\Omega$$

$$\Rightarrow r_o = 422.4\,k\Omega$$

$$\Rightarrow INL_{max} = \frac{203\times10^{-6}\times50^2}{4\times422.4\times10^3}\times64^2 = \underline{\underline{1.23\,mV}}$$

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(c).

$$INL_{max} = \frac{IR_L^2}{4r_o}N^2 = 0.5LSB = 0.5\times\frac{0.65}{2^{10}} = 0.317\,mV$$

$$\Rightarrow \frac{r_o'}{r_o} = \frac{1.23}{0.317} \Rightarrow 30.72r_{o2} + r_{o1} = 1638.96\,k\Omega \Rightarrow \lambda_2' = 0.093$$

$$\Rightarrow L' = \frac{\lambda}{\lambda_2'}L = \frac{0.37}{0.093}\times0.18 = \underline{\underline{0.72\,\mu m}}$$

The minimum width is
$$W = \frac{2I_D}{\mu_n C_{ox}\frac{1}{L}\cdot V_{OD}^2} = \frac{2\times203\times10^{-6}}{265\times10^{-6}\times\frac{1}{0.72}\times0.25^2} = \underline{\underline{17.65\,\mu m}}$$

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(d) The schematic is shown below:

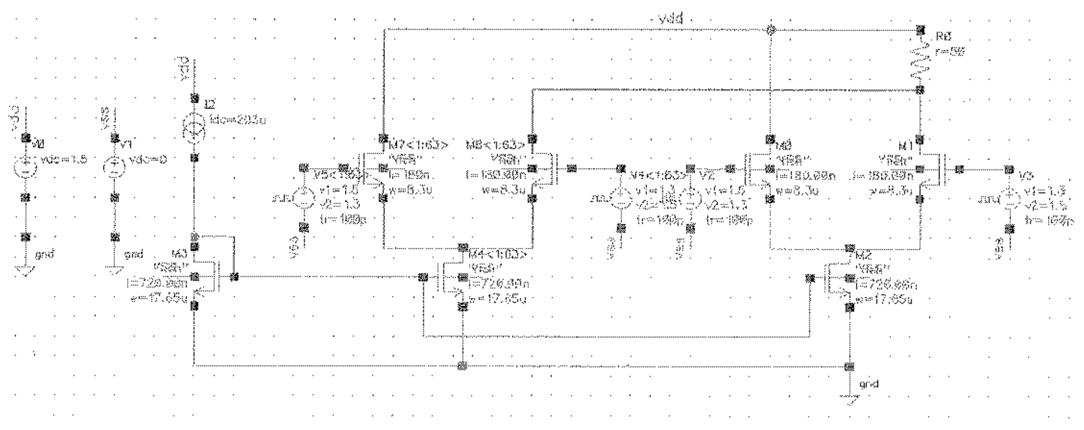

Figure 1 64 segmented DAC schematic

Transient simulation for output voltage is shown:

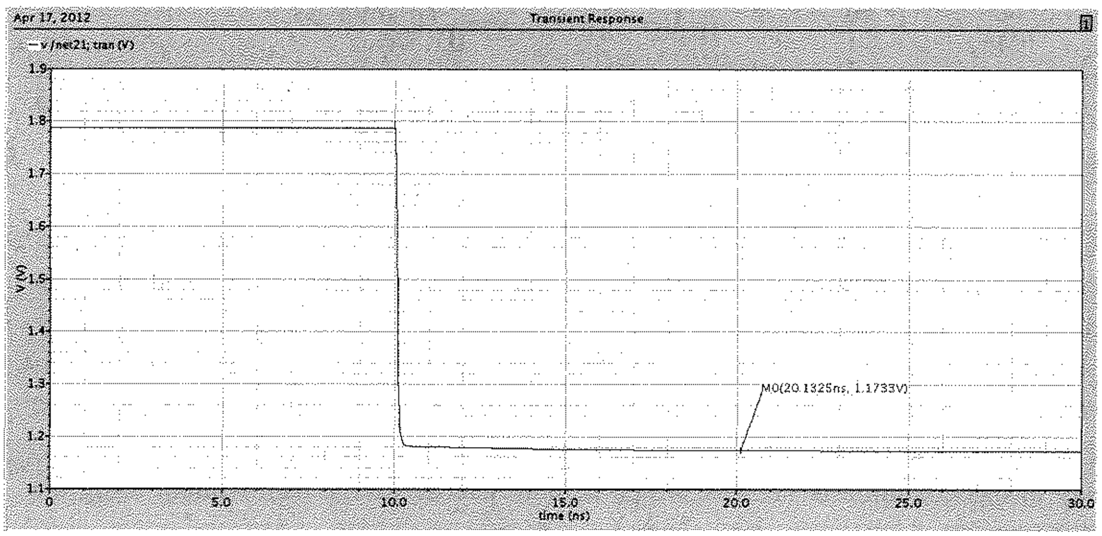

Figure 2 Output transient simulation

The final value is 1.173V, adding 0.5LSB which is 0.317mV corresponds to the voltage of 1.1733V. And settling time is:

20.13 - 10.09 = **10.04 ns** 7/15

(e) After adding a 1uF capacitor, the transient response is shown:

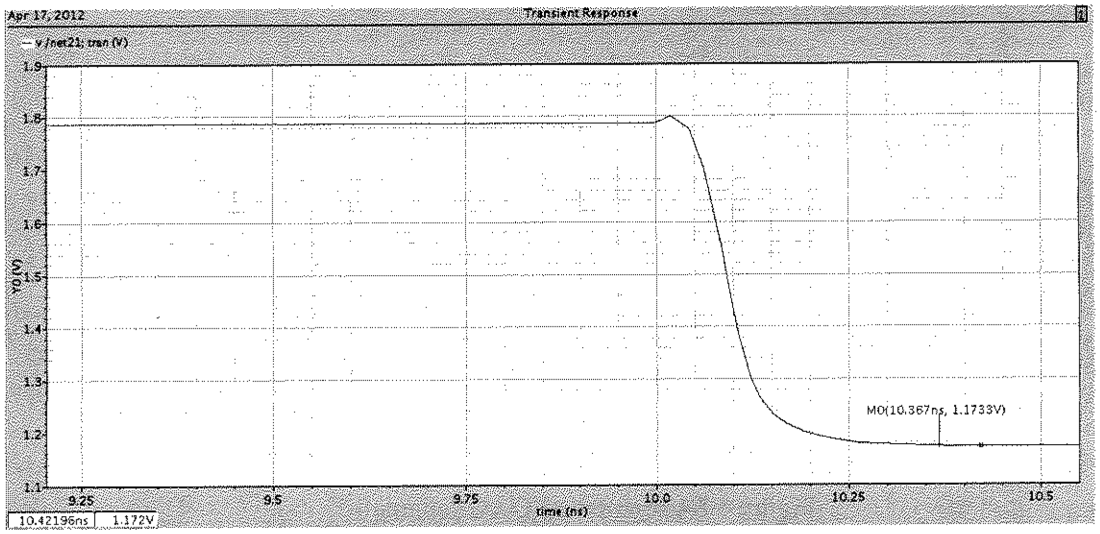

Figure 3 Output transient simulation with 1uF filtering capacitor

The settling time now drops to

$$10.367 - 10.09 = \mathbf{277ps}$$

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This capacitor improves speed dramatically because it makes the bias voltage for the tail current source relatively stable. Thus, coupling effect form the tail node to the gate bias point is reduced. Therefore, the settling speed is quite fast.

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(f) If the maximum capacitor is 4pF, I will **not use it** to increase the speed because this increases the time constant at the gate of the tail current source. Even through this cap reduces the voltage jump at the bias node, it results longer time to recover because of the increased time constant. As a result, speed may be even worse than without the capacitor. Simulation results in figure 4 have proven this analysis. It is clear that with 4pF capacitor, the instantaneous voltage drop is reduced, but the recovering time is longer. Finally, delay in this condition is 12.28ns, which is more than 2ns large than without using the capacitor.

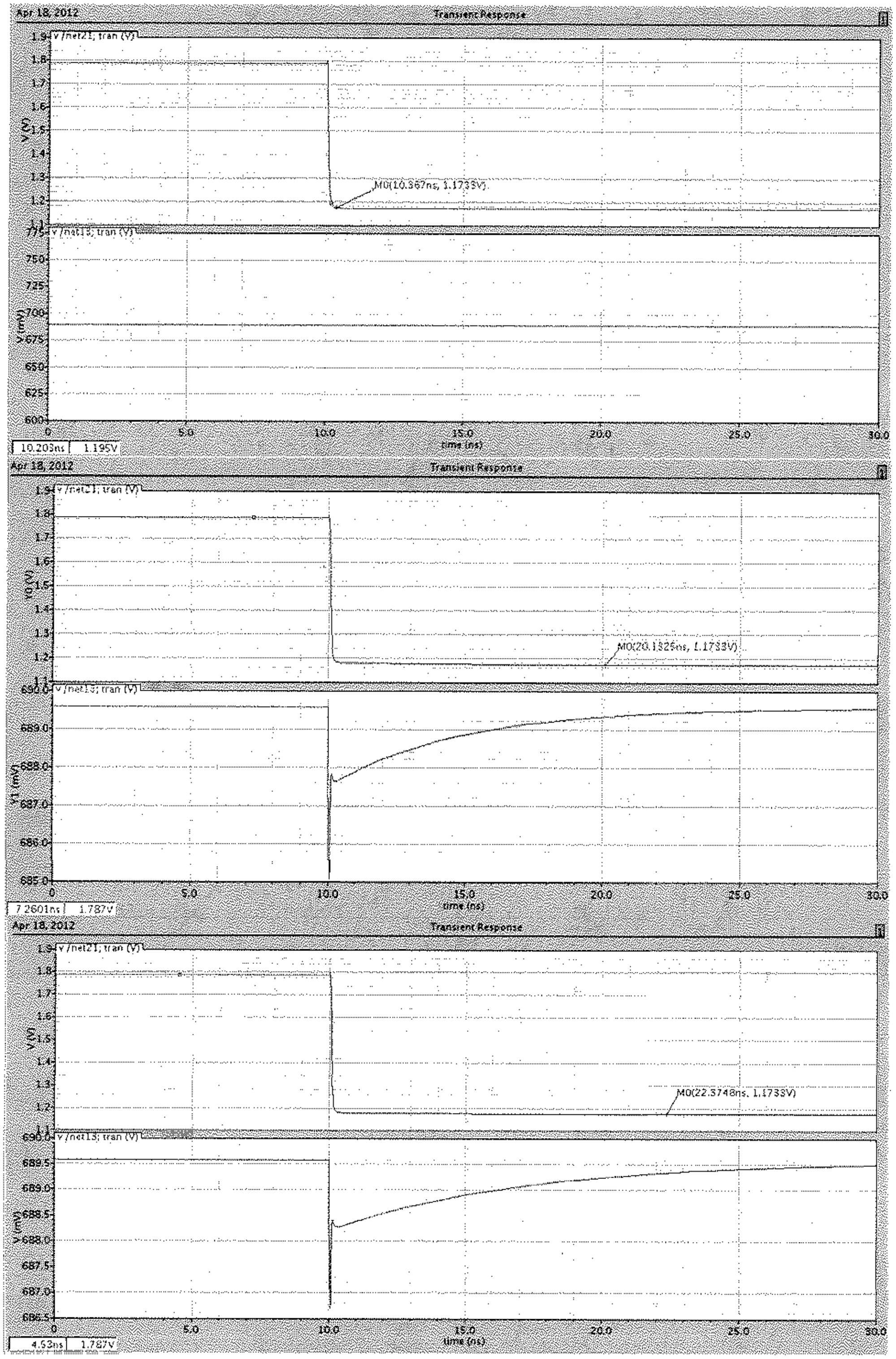

Figure 4 Transient simulation of output and gate of tail current source
(with 1uF cap, without cap, with 4pF cap)