## 12

## Introduction to Switched-Capacitor Circuits

Our study of amplifiers in previous chapters has dealt with only cases where the input signal is continuously available and applied to the circuit and the output signal is continuously observed. Called "continuous-time" circuits, such amplifiers find wide application in audio, video, and highspeed analog systems. In many situations, however, we may sense the input only at periodic instants of time, ignoring its value at other times. The circuit then processes each "sample," producing a valid output at the end of each period. Such circuits are called "discrete-time" or "sampled-data" systems.

In this chapter, we study a common class of discrete-time systems called "switched-capacitor (SC) circuits." Our objective is to provide the foundation for more advanced topics such as filters, comparators, ADCs, and DACs. Most of our study deals with switched-capacitor amplifiers but the concepts can be applied to other discrete-time circuits as well. Beginning with a general view of SC circuits, we describe sampling switches and their speed and precision issues. Next, we analyze switched-capacitor amplifiers, considering unity-gain, noninverting, and multiply-by-two topologies. Finally, we examine a switched-capacitor integrator.

### 12.1 General Considerations

In order to understand the motivation for sampled-data circuits, let us first consider the simple continuous-time amplifier shown in Fig. 12.1(a). Used extensively with bipolar op amps, this circuit presents a difficult issue if implemented in CMOS technology. Recall that, to achieve a high voltage gain, the open-loop output resistance of CMOS op amps is maximized, typically approaching hundreds of kilo-ohms. We therefore suspect that $R_{2}$ heavily drops the open-loop gain, degrading the precision of the circuit. In fact, with the aid of the simple equivalent circuit


Figure 12.1. (a) Continuous-time feedback amplifier, (b) equivalent circuit of (a).
shown in Fig. 12.1(b), we can write

$$
\begin{equation*}
-A_{v}\left(\frac{V_{\text {out }}-V_{\text {in }}}{R_{1}+R_{2}} R_{1}+V_{\text {in }}\right)-R_{\text {out }} \frac{V_{\text {out }}-V_{\text {in }}}{R_{1}+R_{2}}=V_{\text {out }}, \tag{12.1}
\end{equation*}
$$

and hence

$$
\begin{equation*}
\frac{V_{\text {out }}}{V_{\text {in }}}=-\frac{R_{2}}{R_{1}} \cdot \frac{A_{v}-\frac{R_{\text {out }}}{R_{2}}}{1+\frac{R_{\text {out }}}{R_{1}}+A_{v}+\frac{R_{2}}{R_{1}}} . \tag{12.2}
\end{equation*}
$$

Equation (12.2) implies that, compared to the case where $R_{\text {out }}=0$, the closed-loop gain suffers from inaccuracies in both the numerator and the denominator. Also, the input resistance of the amplifier, approximately equal to $R_{1}$, loads the preceding stage while introducing thermal noise.

In the circuit of Fig. 12.1(a), the closed-loop gain is set by the ratio of $R_{2}$ and $R_{1}$. In order to avoid reducing the open-loop gain of the op amp, we postulate that the resistors can be replaced by capacitors [Fig. 12.2(a)]. But, how is the bias voltage at node $X$ set? We may add a large feedback

(a)

(a)

Figure 12.2. (a) Continuous-time feedback amplifier using capacitors, (b) use of resistor to define bias point.
resistor as in Fig. 12.2(b), providing dc feedback while negligibly affecting the ac behavior of the amplifier in the frequency band of interest. Such an arrangement is indeed practical if the circuit senses only high-frequency signals. But suppose, for example, the circuit is to amplify a voltage


Figure 12.3. Step response of the amplifier of Fig. 12.2(b).
step. Illustrated in Fig. 12.3, the response contains a step change due to the initial amplification by the circuit consisting of $C_{1}, C_{2}$, and the op amp, followed by a "tail" resulting from the loss of charge on $C_{2}$ through $R_{F}$. From another point of view, the circuit may not be suited to amplify wideband signals because it exhibits a high-pass transfer function. In fact, the transfer function is given by

$$
\begin{align*}
\frac{V_{\text {out }}}{V_{\text {in }}}(s) & \approx-\frac{R_{F} \frac{1}{C_{2} s}}{R_{F}+\frac{1}{C_{2} s}} \div \frac{1}{C_{1} s}  \tag{12.3}\\
& =-\frac{R_{F} C_{1} s}{R_{F} C_{2} s+1}, \tag{12.4}
\end{align*}
$$

indicating that $V_{\text {out }} / V_{\text {in }} \approx-C_{1} / C_{2}$ only if $\omega \gg\left(R_{F} C_{2}\right)^{-1}$.
The above difficulty can be remedied by increasing $R_{F} C_{2}$, but in many applications the required values of the two components become prohibitively large. We must therefore seek other methods of establishing the bias while utilizing capacitive feedback networks.

Let us now consider the switched-capacitor circuit depicted in Fig. 12.4, where three switches control the operation: $S_{1}$ and $S_{3}$ connect the left plate of $C_{1}$ to $V_{\text {in }}$ and ground, respectively, and


Figure 12.4. Switched-capacitor amplifier.
$S_{2}$ provides unity-gain feedback. We first assume the open-loop gain of the op amp is very large and study the circuit in two phases. First, $S_{1}$ and $S_{2}$ are on and $S_{3}$ is off, yielding the equivalent


Figure 12.5. Circuit of Fig. 12.4 in (a) sampling mode, (b) amplification mode.
circuit of Fig. 12.5(a). For a high-gain op amp, $V_{B}=V_{\text {out }} \approx 0$, and hence the voltage across $C_{1}$ is approximately equal to $V_{\text {in }}$. Next, at $t=t_{0}, S_{1}$ and $S_{2}$ turn off and $S_{3}$ turns on, pulling node $A$ to ground. Since $V_{A}$ changes from $V_{\text {in0 }}$ to 0 , the output voltage must change from zero to $V_{\text {in0 }} C_{1} / C_{2}$.

The output voltage change can also be calculated by examining the transfer of charge. Note that the charge stored on $C_{1}$ just before $t_{0}$ is equal to $V_{\text {in } 0} C_{1}$. After $t=t_{0}$, the negative feedback through $C_{2}$ drives the op amp input differential voltage and hence the voltage across $C_{1}$ to zero (Fig. 12.6). The charge stored on $C_{1}$ at $t=t_{0}$ must then be transferred to $C_{2}$, producing an output


Figure 12.6. Transfer of charge from $C_{1}$ to $C_{2}$.
voltage equal to $V_{\text {in0 }} C_{1} / C_{2}$. Thus, the circuit amplifies $V_{i n 0}$ by a factor of $C_{1} / C_{2}$.
Several attributes of the circuit of Fig. 12.4 distinguish it from continuous-time implementations. First, the circuit devotes some time to "sample" the input, setting the output to zero and providing no amplification during this period. Second, after sampling, for $t>t_{0}$, the circuit ignores the input voltage $V_{i n}$, amplifying the sampled voltage. Third, the circuit configuration changes
considerably from one phase to another, as seen in Fig. 12.5(a) and (b), raising concern about its stability.

What is the advantage of the amplifier of Fig. 12.4 over that in Fig. 12.1? In addition to sampling capability, we note from the waveforms depicted in Fig. 12.5 that after $V_{\text {out }}$ settles, the current through $C_{2}$ approaches zero. That is, the feedback capacitor does not reduce the open-loop gain of the amplifier if the output voltage is given enough time to settle. In Fig. 12.1, on the other hand, $R_{2}$ continuously loads the amplifier.

The switched-capacitor amplifier of Fig. 12.4 lends itself to implementation in CMOS technology much more easily than in other technologies. This is because discrete-time operations require switches to perform sampling as well as a high input impedance to sense the stored quantities with no corruption. For example, if the op amp of Fig. 12.4 incorporates bipolar transistors at its input, the base current drawn from the inverting input in the amplification phase [Fig. 12.5(b)] creates an error in the output voltage. The existence of simple switches and a high input impedance have made CMOS technology the dominant choice for sampled-data applications.

The foregoing discussion leads to the conceptual view illustrated in Fig. 12.7 for switchedcapacitor amplifiers. In the simplest case, the operation takes place in two phases: sampling and


Figure 12.7. General view of switched-capacitor amplifier.
amplification. Thus, in addition to the analog input, $V_{i n}$, the circuit requires a clock to define each phase.

Our study of SC amplifiers proceeds according to these two phases. First, we analyze various sampling techniques. Second, we consider SC amplifier topologies.

### 12.2 Sampling Switches

### 12.2.1 MOSFETS as Switches

A simple sampling circuit consists of a switch and a capacitor [Fig. 12.8(a)]. A MOS transistor can serve as a switch [Fig. 12.8(b)] because (a) it can be on while carrying zero current, and (b) its

(a)

(b)

Figure 12.8. (a) Simple sampling circuit, (b) implementation of the switch by a MOS device.
source and drain voltages are not "pinned" to the gate voltage, i.e., if the gate voltage varies, the source or drain voltage need not follow that variation. By contrast, bipolar transistors lack both of these properties, typically necessitating complex circuits to perform sampling.

To understand how the circuit of Fig. 12.8(b) samples the input, first consider the simple cases depicted in Fig. 12.9, where the gate command, $C K$, goes high at $t=t_{0}$. In Fig.

(a)

(b)

Figure 12.9. Response of a sampling circuit to different input levels and initial conditions.
12.9(a), we assume that $V_{i n}=0$ for $t \geq t_{0}$ and the capacitor has an initial voltage equal to $V_{D D}$. Thus, at $t=t_{0}, M_{1}$ senses a gate-source voltage equal to $V_{D D}$ while its drain voltage is also equal to $V_{D D}$. The transistor therefore operates in saturation, drawing a current of $I_{D 1}=$
$\left(\mu_{n} C_{o x} / 2\right)(W / L)\left(V_{D D}-V_{T H}\right)^{2}$ from the capacitor. As $V_{\text {out }}$ falls, at some point $V_{o u t}=V_{D D}-V_{T H}$, driving $M_{1}$ into the triode region. The device nevertheless continues to discharge $C_{H}$ until $V_{o u t}$ approaches zero. We note that for $V_{\text {out }} \ll 2\left(V_{D D}-V_{T H}\right)$, the transistor can be viewed as a resistor equal to $R_{o n}=\left[\mu_{n} C_{o x}(W / L)\left(V_{D D}-V_{T H}\right)\right]^{-1}$.

Now consider the case in Fig. 12.9(b), where $V_{\text {in }}=+1 \mathrm{~V}, V_{\text {out }}\left(t=t_{0}\right)=0 \mathrm{~V}$, and $V_{D D}=3 \mathrm{~V}$. Here, the terminal of $M_{1}$ connected to $C_{H}$ acts as the source, and the transistor turns on with $V_{G S}=$ +3 V , but $V_{D S}=+1 \mathrm{~V}$. Thus, $M_{1}$ operates in the triode region, charging $C_{H}$ until $V_{\text {out }}$ approaches +1 V . For $V_{\text {out }} \approx+1 \mathrm{~V}, M_{1}$ exhibits an on-resistance of $R_{o n}=\left[\mu_{n} C_{o x}(W / L)\left(V_{D D}-V_{\text {in }}-V_{T H}\right)\right]^{-1}$.

The above observations reveal two important points. First, a MOS switch can conduct current in either direction simply by exchanging the role of its source and drain terminals. Second, as shown in Fig. 12.10, when the switch is on, $V_{\text {out }}$ follows $V_{\text {in }}$ and when the switch is off, $V_{\text {out }}$ remains


Figure 12.10. Track and hold capabilities of a sampling circuit.
constant. Thus, the circuit "tracks" the signal when $C K$ is high and "freezes" the instantaneous value of $V_{i n}$ across $C_{H}$ when $C K$ goes low.

## Example 12.1

In the circuit of Fig. 12.9(a), calculate $V_{\text {out }}$ as a function of time. Assume $\lambda=0$.
Solution. Before $V_{\text {out }}$ drops below $V_{D D}-V_{T H}, M_{1}$ is saturated and we have:

$$
\begin{align*}
V_{\text {out }}(t) & =V_{D D}-\frac{I_{D 1} t}{C_{H}}  \tag{12.5}\\
& =V_{D D}-\frac{1}{2} \mu_{n} C_{o x} \frac{W}{L}\left(V_{D D}-V_{T H}\right)^{2} \frac{t}{C_{H}} \tag{12.6}
\end{align*}
$$

After

$$
\begin{equation*}
t_{1}=\frac{2 V_{T H} C_{H}}{\mu_{n} C_{o x} \frac{W}{L}\left(V_{D D}-V_{T H}\right)^{2}} \tag{12.7}
\end{equation*}
$$

$M_{1}$ enters the triode region, yielding a time-dependent current. We therefore write:

$$
\begin{align*}
C_{H} \frac{d V_{\text {out }}}{d t} & =-I_{D 1}  \tag{12.8}\\
& =-\frac{1}{2} \mu_{n} C_{o x} \frac{W}{L}\left[2\left(V_{D D}-V_{T H}\right) V_{\text {out }}-V_{\text {out }}^{2}\right] \quad t>t_{1} \tag{12.9}
\end{align*}
$$

Rearranging (12.9), we have

$$
\begin{equation*}
\frac{d V_{\text {out }}}{\left[2\left(V_{D D}-V_{T H}\right)-V_{\text {out }}\right] V_{\text {out }}}=-\frac{1}{2} \mu_{n} \frac{C_{o x}}{C_{H}} \frac{W}{L} d t \tag{12.10}
\end{equation*}
$$

which, upon separation into partial fractions, is written as

$$
\begin{equation*}
\left[\frac{1}{V_{\text {out }}}+\frac{1}{2\left(V_{D D}-V_{T H}\right)-V_{\text {out }}}\right] \frac{d V_{\text {out }}}{V_{D D}-V_{T H}}=-\mu_{n} \frac{C_{o x}}{C_{H}} \frac{W}{L} d t . \tag{12.11}
\end{equation*}
$$

Thus,

$$
\begin{equation*}
\ln V_{o u t}-\ln \left[2\left(V_{D D}-V_{T H}\right)-V_{o u t}\right]=-\left(V_{D D}-V_{T H}\right) \mu_{n} \frac{C_{o x}}{C_{H}} \frac{W}{L}\left(t-t_{1}\right) \tag{12.12}
\end{equation*}
$$

that is,

$$
\begin{equation*}
\ln \frac{V_{\text {out }}}{2\left(V_{D D}-V_{T H}\right)-V_{\text {out }}}=-\left(V_{D D}-V_{T H}\right) \mu_{n} \frac{C_{o x}}{C_{H}} \frac{W}{L}\left(t-t_{1}\right) \tag{12.13}
\end{equation*}
$$

Taking the exponential of both sides and solving for $V_{\text {out }}$, we obtain

$$
\begin{equation*}
V_{o u t}=\frac{2\left(V_{D D}-V_{T H}\right) \exp \left[-\left(V_{D D}-V_{T H}\right) \mu_{n} \frac{C_{o x}}{C_{H}} \cdot \frac{W}{L}\left(t-t_{1}\right)\right]}{1+\exp \left[-\left(V_{D D}-V_{T H}\right) \mu_{n} \frac{C_{o x}}{C_{H}} \cdot \frac{W}{L}\left(t-t_{1}\right)\right]} \tag{12.14}
\end{equation*}
$$

In the circuit of Fig. 12.9(b), we assumed $V_{i n}=+1 \mathrm{~V}$ (Fig. 12.11). Now suppose $V_{i n}=V_{D D}$.


Figure 12.11. Maximum output level in an NMOS sampler.

How does $V_{\text {out }}$ vary with time? Since the gate and drain of $M_{1}$ are at the same potential, the transistor is saturated and we have:

$$
\begin{align*}
C_{H} \frac{d V_{\text {out }}}{d t} & =I_{D 1}  \tag{12.15}\\
& =\frac{1}{2} \mu_{n} C_{o x} \frac{W}{L}\left(V_{D D}-V_{\text {out }}-V_{T H}\right)^{2} \tag{12.16}
\end{align*}
$$

where channel-length modulation is neglected. It follows that

$$
\begin{equation*}
\frac{d V_{\text {out }}}{\left(V_{D D}-V_{\text {out }}-V_{T H}\right)^{2}}=\frac{1}{2} \mu_{n} \frac{C_{o x}}{C_{H}} \frac{W}{L} d t \tag{12.17}
\end{equation*}
$$

and hence

$$
\begin{equation*}
\left.\frac{1}{V_{D D}-V_{\text {out }}-V_{T H}}\right|_{0} ^{V o u t}=\left.\frac{1}{2} \mu_{n} \frac{C_{o x}}{C_{H}} \frac{W}{L} t\right|_{0} ^{t}, \tag{12.18}
\end{equation*}
$$

where body effect is neglected and $V_{\text {out }}(t=0)$ is assumed zero. Thus,

$$
\begin{equation*}
V_{o u t}=V_{D D}-V_{T H}-\frac{1}{\frac{1}{2} \mu_{n} \frac{C_{o x}}{C_{H}} \frac{W}{L} t+\frac{1}{V_{D D}-V_{T H}}} \tag{12.19}
\end{equation*}
$$

Equation (12.19) implies that as $t \rightarrow \infty, V_{\text {out }} \rightarrow V_{D D}-V_{T H}$. This is because as $V_{\text {out }}$ approaches $V_{D D}-V_{T H}$, the overdrive voltage of $M_{1}$ vanishes, reducing the current available for charging $C_{H}$ to negligible values. Of course, even for $V_{o u t}=V_{D D}-V_{T H}$, the transistor conducts some subthreshold current and, given enough time, eventually brings $V_{\text {out }}$ to $V_{D D}$. Nonetheless, as mentioned in Chapter 3, for typical operation speeds, it is reasonable to assume that $V_{\text {out }}$ does not exceed $V_{D D}-V_{T H}$.

The foregoing analysis demonstrates a serious limitation of MOS switches: if the input signal level is close to $V_{D D}$, then the output provided by an NMOS switch cannot track the input. From another point of view, the on-resistance of the switch increases considerably as the input and output voltages approach $V_{D D}-V_{T H}$. We may then ask: what is the maximum input level that the switch can pass to the output faithfully? In Fig. 12.11, for $V_{\text {out }} \approx V_{\text {in }}$, the transistor must operate in deep triode region and hence the upper bound of $V_{i n}$ equals $V_{D D}-V_{T H}$. As explained below, in practice $V_{i n}$ must be quite lower than this value.

## Example 12.2

In the circuit of Fig. 12.12, calculate the minimum and maximum on-resistance of $M_{1}$. Assume $\mu_{n} C_{o x}=50 \mu \mathrm{~A} / \mathrm{V}^{2}, W / L=10 / 1, V_{T H}=0.7 \mathrm{~V}, V_{D D}=3 \mathrm{~V}$, and $\gamma=0$.

Solution. We note that in the steady state, $M_{1}$ remains in the triode region because the gate voltage is higher than both $V_{\text {in }}$ and $V_{\text {out }}$ by a value greater than $V_{T H}$. If $f_{\text {in }}=10 \mathrm{MHz}$, we predict


Figure 12.12.
that $V_{\text {out }}$ tracks $V_{\text {in }}$ with a negligible phase shift due to the on-resistance of $M_{1}$ and $C_{H}$. Assuming $V_{\text {out }} \approx V_{\text {in }}$, we need not distinguish between the source and drain terminals, obtaining

$$
\begin{equation*}
R_{o n 1}=\frac{1}{\mu_{n} C_{o x} \frac{W}{L}\left(V_{D D}-V_{i n}-V_{T H}\right)} \tag{12.20}
\end{equation*}
$$

Thus, $R_{o n 1, \text { max }} \approx 1.11 \mathrm{k} \Omega$ and $R_{o n 1, \text { min }} \approx 870 \Omega$. By contrast, if the maximum input level is raised to 1.5 V , then $R_{o n 1, \max }=2.5 \mathrm{k} \Omega$.

MOS devices operating in deep triode region are sometimes called "zero-offset" switches to emphasize that they exhibit no dc shift between the input and output voltages of the simple sampling circuit of Fig. 12.8(b). ${ }^{1}$ This is evident from examples of Fig. 12.9, where the output eventually becomes equal to the input. Nonexistent in bipolar technology, the zero offset property proves crucial in precise sampling of analog signals.

We have thus far considered only NMOS switches. The reader can verify that the foregoing principles apply to PMOS switches as well. In particular, as shown in Fig. 12.13, a PMOS transistor fails to operate as a zero-offset switch if its gate is grounded and its drain terminal senses an input


Figure 12.13. Sampling circuit using PMOS switch.
voltage of $\left|V_{T H P}\right|$ or less. In other words, the on-resistance of the device rises rapidly as the input and output levels drop to $\left|V_{T H P}\right|$ above ground.

[^0]
### 12.2.2 Speed Considerations

What determines the speed of the sampling circuits of Fig. 12.8? We must first define the speed here. Illustrated in Fig. 12.14, a simple, but versatile measure of speed is the time required for


Figure 12.14. Definition of speed in a sampling circuit.
the output voltage to go from zero to the maximum input level after the switch turns on. Since $V_{\text {out }}$ would take infinite time to become equal to $V_{\text {in } 0}$, we consider the output settled when it is within a certain "error band," $\Delta V$, around the final value. For example, we say the output settles to $0.1 \%$ accuracy after $t_{S}$ seconds, meaning that in Fig. $12.14, \Delta V / V_{i n 0}=0.1 \%$. Thus, the speed specification must be accompanied by an accuracy specification as well. Note that after $t=t_{S}$, we can consider the source and drain voltages to be approximately equal.

From the circuit of Fig. 12.14, we surmise that the sampling speed is given by two factors: the on-resistance of the switch and the value of the sampling capacitor. Thus, to achieve a higher speed, a large aspect ratio and a small capacitor must be used. However, as illustrated in Fig. 12.12, the on-resistance also depends on the input level, yielding a greater time constant for more positive inputs (in the case of NMOS switches). From Eq. (12.20), we plot the on-resistance of the switch as a function of the input level [Fig. 12.15(a)], noting the sharp rise as $V_{i n}$ approaches


Figure 12.15. On-resistance of (a) NMOS and (b) PMOS devices as a function of input voltage.
$V_{D D}-V_{T H}$. For example, if we restrict the variation of $R_{o n}$ to a range of 4 to 1 , then the maximum
input level is given by

$$
\begin{equation*}
\frac{1}{\mu_{n} C_{o x} \frac{W}{L}\left(V_{D D}-V_{i n, \max }-V_{T H}\right)}=\frac{4}{\mu_{n} C_{o x} \frac{W}{L}\left(V_{D D}-V_{T H}\right)} . \tag{12.21}
\end{equation*}
$$

That is,

$$
\begin{equation*}
V_{i n, \max }=\frac{3}{4}\left(V_{D D}-V_{T H}\right) . \tag{12.22}
\end{equation*}
$$

This value falls around $V_{D D} / 2$, translating to severe swing limitations. Note that the device threshold voltage directly limits the voltage swings. ${ }^{2}$

In order to accommodate greater voltage swings in a sampling circuit, we first observe that a PMOS switch exhibits an on-resistance that decreases as the input voltage becomes more positive [Fig. 12.15(b)]. It is then plausible to employ "complementary" switches so as to allow rail-to-tail swings. Shown in Fig. 12.16(a), such a combination requires complementary clocks, producing

(a)

(b)

Figure 12.16. (a) Complementary switch, (b) on-resistance of the complementary switch.
an equivalent resistance:

$$
\begin{align*}
R_{o n, e q} & =R_{o n, N} \| R_{o n, P}  \tag{12.23}\\
& =\frac{1}{\mu_{n} C_{o x}\left(\frac{W}{L}\right)_{N}\left(V_{D D}-V_{i n}-V_{T H N}\right)} \| \frac{1}{\mu_{n} C_{o x}\left(\frac{W}{L}\right)_{P}\left(V_{i n}-\left|V_{T H P}\right|\right)}  \tag{12.24}\\
& =\frac{1}{\mu_{n} C_{o x}\left(\frac{W}{L}\right)_{N}\left(V_{D D}-V_{T H N}\right)-\left[\mu_{n} C_{o x}\left(\frac{W}{L}\right)_{N}-\mu_{p} C_{o x}\left(\frac{W}{L}\right)_{P}\right] V_{i n}-\mu_{p} C_{o x}\left(\frac{W}{L}\right)_{P} V_{T H P}} \tag{12.25}
\end{align*}
$$

Interestingly, if $\mu_{n} C_{o x}(W / L)_{N}=\mu_{p} C_{o x}(W / L)_{P}$, then $R_{o n, e q}$ is independent of the input level. ${ }^{3}$ Fig. 12.16(b) plots the behavior of $R_{o n, e q}$ in the general case, revealing much less variation than that corresponding to each switch alone.

[^1]For high-speed input signals, it is critical that the NMOS and PMOS switches in Fig. 12.16(a) turn off simultaneously so as to avoid ambiguity in the sampled value. If, for example, the NMOS device turns off $\Delta t$ seconds earlier than the PMOS device, then the output voltage tends to track the input for the remaining $\Delta t$ seconds, but with a large, input-dependent time constant (Fig. 12.17). This effect gives rise to distortion in the sampled value. For moderate precision, the simple circuit


Figure 12.17. Distortion generated if complementary switches do not turn off simultaneously.
shown in Fig. 12.18 provides complementary clocks by duplicating the delay of inverter $I_{1}$ through


Figure 12.18. Simple circuit generating complementary clocks.
the gate $G_{2}$.

### 12.2.3 Precision Considerations

Our foregoing study of MOS switches indicates that a larger $W / L$ or a smaller sampling capacitor results in a higher speed. In this section, we show that these methods of increasing the speed degrade the precision with which the signal is sampled.

Three mechanisms in MOS transistor operation introduce error at the instant the switch turns off. We study each effect individually.

Channel Charge Injection Consider the sampling circuit of Fig. 12.19 and recall that for a MOSFET to be on, a channel must exist at the oxide-silicon interface. Assuming $V_{\text {in }} \approx V_{\text {out }}$, we


Figure 12.19. Charge injection when a switch turns off.
use our derivations in Chapter 2 to express the total charge in the inversion layer as

$$
\begin{equation*}
Q_{c h}=W L C_{o x}\left(V_{D D}-V_{i n}-V_{T H}\right), \tag{12.26}
\end{equation*}
$$

where $L$ denotes the effective channel length. When the switch turns off, $Q_{c h}$ exits through the source and drain terminals, a phenomenon called "channel charge injection."

The charge injected to the left side on Fig. 12.19 is absorbed by the input source, creating no error. On the other hand, the charge injected to the right side is deposited on $C_{H}$, introducing an error in the voltage stored on the capacitor. For example, if half of $Q_{c h}$ is injected onto $C_{H}$, the resulting error equals

$$
\begin{equation*}
\Delta V=\frac{W L C_{o x}\left(V_{D D}-V_{i n}-V_{T H}\right)}{2 C_{H}} \tag{12.27}
\end{equation*}
$$

Illustrated in Fig. 12.20, the error for an NMOS switch appears as a negative "pedestal" at the


Figure 12.20. Effect of charge injection.
output. Note that the error is directly proportional to $W L C_{o x}$ and inversely proportional to $C_{H}$.
An important question that arises now is: why did we assume in arriving at (12.27) that exactly half of the channel charge in injected onto $C_{H}$ ? In reality, the fraction of charge that exits through the source and drain terminals is a relatively complex function of various parameters such as the impedance seen at each terminal to ground and the transition time of the clock [1, 2]. Investigations
of this effect have not yielded any rule of thumb that can predict the charge splitting in terms of such parameters. Furthermore, in many cases, these parameters, e.g., the clock transition time, are poorly controlled. Also, most circuit simulation programs model charge injection quite inaccurately. As a worst-case estimate, we can assume that the entire channel charge is injected onto the sampling capacitor.

How does charge injection affect the precision? Assuming all of the charge is deposited on the capacitor, we express the sampled output voltage as

$$
\begin{equation*}
V_{o u t} \approx V_{i n}-\frac{W L C_{o x}\left(V_{D D}-V_{i n}-V_{T H}\right)}{C_{H}} \tag{12.28}
\end{equation*}
$$

where the phase shift between the input and output is neglected. Thus,

$$
\begin{equation*}
V_{o u t}=V_{i n}\left(1+\frac{W L C_{o x}}{C_{H}}\right)-\frac{W L C_{o x}}{C_{H}}\left(V_{D D}-V_{T H}\right), \tag{12.29}
\end{equation*}
$$

suggesting that the output deviates from the ideal value through two effects: a non-unity gain equal to $1+W L C_{o x} / C_{H},{ }^{4}$ and a constant offset voltage $-W L C_{o x}\left(V_{D D}-V_{T H}\right) / C_{H}$ (Fig. 12.21). In other words, since we have assumed channel charge is a linear function of the input voltage, the circuit exhibits only gain error and dc offset.


Figure 12.21. Input/output characteristic of sampling circuit in the presence of charge injection.

In the foregoing discussion, we tacitly assumed that $V_{T H}$ is constant. However, for NMOS switches (in an $n$-well technology), body effect must be taken into account. ${ }^{5}$ Since $V_{T H}=$ $V_{T H 0}+\gamma\left(\sqrt{2 \phi_{B}+V_{B S}}-\sqrt{2 \phi_{B}}\right)$, and $V_{B S} \approx-V_{i n}$, we have

$$
\begin{align*}
V_{o u t} & =V_{i n}-\frac{W L C_{o x}}{C_{H}}\left(V_{D D}-V_{i n}-V_{T H 0}-\gamma \sqrt{2 \phi_{B}+V_{i n}}-\gamma \sqrt{2 \phi_{B}}\right),  \tag{12.30}\\
& =V_{i n}\left(1-\frac{W L C_{o x}}{C_{H}}\right)-\gamma \frac{W L C_{o x}}{C_{H}} \sqrt{2 \phi_{B}+V_{i n}}-\frac{W L C_{o x}}{C_{H}}\left(V_{D D}-V_{T H 0}-\gamma \sqrt{2 \phi 132.31}\right)
\end{align*}
$$

[^2]It follows that the nonlinear dependence of $V_{T H}$ upon $V_{i n}$ introduces nonlinearity in the input/output characteristic.

In summary, charge injection contributes three types of errors in MOS sampling circuits: gain error, dc offsets, and nonlinearity. In many applications, the first two can be tolerated or corrected whereas the last cannot.

It is instructive to consider the speed-precision trade-off resulting from charge injection. Representing the speed by a simple time constant $\tau$ and the precision by the error $\Delta V$ due to charge injection, we define a figure of merit as $F=(\tau \Delta V)^{-1}$. Writing

$$
\begin{align*}
\tau & =R_{o n} C_{H}  \tag{12.32}\\
& =\frac{1}{\mu_{n} C_{o x}(W / L)\left(V_{D D}-V_{i n}-V_{T H}\right)} C_{H}, \tag{12.33}
\end{align*}
$$

and

$$
\begin{equation*}
\Delta V=\frac{W L C_{o x}}{C_{H}}\left(V_{D D}-V_{i n}-V_{T H}\right) \tag{12.34}
\end{equation*}
$$

we have

$$
\begin{equation*}
F=\frac{\mu_{n}}{L^{2}} . \tag{12.35}
\end{equation*}
$$

Thus, to the first order, the trade-off is independent of the switch width and the sampling capacitor.

Clock Feedthrough In addition to channel charge injection, a MOS switch couples the clock transitions to the sampling capacitor through its gate-drain or gate-source overlap capacitance. Depicted in Fig. 12.22, the effect introduces an error in the sampled output voltage. Assuming the


Figure 12.22. Clock feedthrough in a sampling circuit.
overlap capacitance is constant, we express the error as

$$
\begin{equation*}
\Delta V=V_{C K} \frac{W C_{o v}}{W C_{o v}+C_{H}}, \tag{12.36}
\end{equation*}
$$

where $C_{o v}$ is the overlap capacitance per unit width. The error $\Delta V$ is independent of the input level, manifesting itself as a constant offset in the input/output characteristic. As with charge injection, clock feedthrough leads to a trade-off between speed and precision as well.
$k T / C$ Noise Recall from Example 7.1 that a resistor charging a capacitor gives rise to a total rms noise voltage of $\sqrt{k T / C}$. As shown in Fig. 12.23, a similar effect occurs in sampling circuits.


Figure 12.23. Thermal noise in a sampling circuit.

The on-resistance of the switch introduces thermal noise at the output and, when the switch turns off, this noise is stored on the capacitor along with the instantaneous value of the input voltage. It can be proved that the rms voltage of the sampled noise in this case is still approximately equal to $\sqrt{k T / C}[3,4]$.

The problem of $K T / C$ noise limits the performance in many high-precision applications. In order to achieve a low noise, the sampling capacitor must be sufficiently large, thus loading other circuits and degrading the speed.

### 12.2.4 Charge Injection Cancellation

The dependence of charge injection upon the input level and the trade-off expressed by (12.35) make it necessary to seek methods of cancelling the effect of charge injection so as to achieve a higher $F$. We consider a few such techniques here.

To arrive at the first technique, we postulate that the charge injected by the main transistor can be removed by means of a second transistor. As shown in Fig. 12.24, a "dummy" switch, $M_{2}$, driven by $\overline{C K}$ is added to the circuit such that after $M_{1}$ turns off and $M_{2}$ turns on, the channel


Figure 12.24. Addition of dummy device to reduce charge injection and clock feedthrough.
charge deposited by the former on $C_{H}$ is absorbed by the latter to create a channel. Note that both the source and drain of $M_{2}$ are connected to the output node.

How do we ensure that the charge injected by $M_{1}, \Delta q_{1}$, is equal to that absorbed by $M_{2}, \Delta q_{2}$ ? Suppose half of the channel charge of $M_{1}$ is injected onto $C_{H}$, i.e.,

$$
\begin{equation*}
\Delta q_{1}=\frac{W_{1} L_{1} C_{o x}}{2}\left(V_{C K}-V_{i n}-V_{T H 1}\right) \tag{12.37}
\end{equation*}
$$

Since $\Delta q_{2}=W_{2} L_{2} C_{o x}\left(V_{C K}-V_{i n}-V_{T H 2}\right)$, if we choose $W_{2}=0.5 W_{1}$ and $L_{2}=L_{1}$, then $\Delta q_{2}=\Delta q_{1}$. Unfortunately, the assumption of equal splitting of charge between source and drain is generally invalid, making this approach less attractive.

Interestingly, with the choice $W_{2}=0.5 W_{1}$ and $L_{2}=L_{1}$, the effect of clock feedthrough is suppressed. As depicted in Fig. 12.25, the total charge in $V_{\text {out }}$ is zero because


Figure 12.25. Clock feedthrough suppression by dummy switch.

$$
\begin{equation*}
-V_{C K} \frac{W_{1} C_{o v}}{W_{1} C_{o v}+C_{H}+2 W_{2} C_{o v}}+V_{C K} \frac{2 W_{2} C_{o v}}{W_{1} C_{o v}+C_{H}+2 W_{2} C_{o v}}=0 . \tag{12.38}
\end{equation*}
$$

Another approach to lowering the effect of charge injection incorporates both PMOS and NMOS devices such that the opposite charge packets injected by the two cancel each other (Fig. 12.26). For $\Delta q_{1}$ to cancel $\Delta q_{2}$, we must have $W_{1} L_{1} C_{o x}\left(V_{C K}-V_{i n}-V_{T H N}\right)=W_{2} L_{2} C_{o x}\left(V_{i n}-\left|V_{T H P}\right|\right)$.


Figure 12.26. Use of complementary switches to reduce charge injection.

Thus, the cancellation occurs for only one input level. Even for clock feedthrough, the circuit does not provide complete cancellation because the gate-drain overlap capacitance of NFETs is not equal to that of PFETs.

Our knowledge of the advantages of differential circuits suggests that the problem of charge injection may be relieved through differential operation. As shown in Fig. 12.27, we surmise that the charge injection appears as a common-mode disturbance. But, writing $\Delta q_{1}=W L C_{o x}\left(V_{C K}-\right.$


Figure 12.27. Differential sampling circuit.
$\left.V_{i n 1}-V_{T H 1}\right)$, and $\Delta q_{2}=W L C_{o x}\left(V_{C K}-V_{i n 2}-V_{T H 2}\right)$, we recognize that $\Delta q_{1}=\Delta q_{2}$ only if $V_{i n 1}=V_{i n 2}$. In other words, the overall error is not suppressed for differential signals. Nevertheless, this technique both removes the constant offset and lowers the nonlinear component. This can be understood by writing

$$
\begin{align*}
\Delta q_{1}-\Delta q_{2} & =W L C_{o x}\left[\left(V_{i n 2}-V_{i n 1}\right)+\left(V_{T H 2}-V_{T H 1}\right)\right]  \tag{12.39}\\
& =W L C_{o x}\left[V_{i n 2}-V_{i n 1}+\gamma\left(\sqrt{2 \phi_{F}+V_{i n 2}}-\sqrt{2 \phi_{F}+V_{i n 1}}\right)\right] . \tag{12.40}
\end{align*}
$$

Since for $V_{i n 1}=V_{i n 2}, \Delta q_{1}-\Delta q_{2}=0$, the characteristic exhibits no offset. Also, the nonlinearity of body effect now appears in both square-root terms of (12.40), leading to only odd-order distortion (Chapter ??).

The problem of charge injection continues to limit the speed-precision envelope in sampled-data systems. Many cancellation techniques have been introduced but each leading to other trade-offs. One such technique, called "bottom-plate sampling," is widely used in switched-capacitor circuits and is described later in this chapter.

### 12.3 Switched-Capacitor Amplifiers

As mentioned in Section 12.1 and exemplified by the circuit of Fig. 12.4, CMOS feedback amplifiers are more easily implemented with a capacitive feedback network than a resistive one. Having examined sampling techniques, we are now ready to study a number of switched-capacitor amplifiers. Our objective is to understand the underlying principles as well as the speed-precision trade-offs encountered in the design of each circuit.

Before studying SC amplifiers, it is helpful to briefly look at the physical implementation of capacitors in CMOS technology. A simple capacitor structure is shown in Fig. 12.28(a), where the "top plate" is realized by a polysilicon layer and the "bottom plate" by a heavily-doped $n^{+}$region.


Figure 12.28. (a) Monolithic capacitor structure, (b) circuit model of (a) including parasitic capacitance to the substrate.

The dielectric is the thin oxide layer used in MOS devices as well. ${ }^{6}$ An important concern in using this structure is the parasitic capacitance between each plate and the substrate. In particular, the bottom plate suffers from substantial junction capacitance to the underlying $p$ region - typically about 10 to $20 \%$ of the oxide capacitance. For this reason, we usually model the capacitor as in Fig. 12.28(b). Monolithic capacitors are described in more detail in Chapters ?? and ??.

### 12.3.1 Unity-Gain Sampler/Buffer

While a unity-gain amplifier can be realized with no resistors or capacitors in the feedback network [Fig. 12.29(a)], for discrete-time applications, it still requires a sampling circuit. We may therefore


Figure 12.29. (a) Unity-gain buffer, (b) sampling circuit followed by unity-gain buffer.
conceive the circuit shown in Fig. 12.29(b) as a sampler/buffer. However, the input-dependent charge injected by $S_{1}$ onto $C_{H}$ limits the accuracy here.

Now consider the topology depicted in Fig. 12.30(a), where three switches control the sampling and amplification modes. In the sampling mode, $S_{1}$ and $S_{2}$ are on and $S_{3}$ is off, yielding the topology

[^3]

Figure 12.30. (a) Unity-gain sampler, (b) circuit of (a) in sampling mode, (c) circuit of (a) in amplification mode.
shown in Fig. 12.30(b). Thus, $V_{\text {out }}=V_{X} \approx 0$, and the voltage across $C_{H}$ tracks $V_{\text {in }}$. At $t=t_{0}$, when $V_{\text {in }}=V_{0}, S_{1}$ and $S_{2}$ turn off and $S_{3}$ turns on, placing the capacitor around the op amp and entering the circuit into the amplification mode [Fig. 12.30(c)]. Since the op amp's high gain requires that node $X$ still be a virtual ground and since the charge on the capacitor must be conserved, $V_{\text {out }}$ rises to a value approximately equal to $V_{0}$. This voltage is therefore "frozen" and it can be processed by subsequent stages.

With proper timing, the circuit of Fig. 12.30(a) can substantially alleviate the problem of channel charge injection. As Fig. 12.31 illustrates in "slow motion," in the transition from the

(a)

(b)

(c)

Figure 12.31. Operation of the unity-gain sampler in slow motion.
sampling mode to the amplification mode, $S_{2}$ turns off slightly before $S_{1}$ does. We carefully
examine the effect of the charge injected by $S_{2}$ and $S_{1}$. When $S_{2}$ turns off, it injects a charge packet $\Delta q_{2}$ onto $C_{H}$, producing an error equal to $\Delta q_{2} / C_{H}$. However, this charge is quite independent of the input level because node $X$ is a virtual ground. For example, if $S_{2}$ is realized by an NMOS device whose gate voltage equals $V_{C K}$, then $\Delta q_{2}=W L C_{o x}\left(V_{C K}-V_{T H}-V_{X}\right)$. Although body effect makes $V_{T H}$ a function of $V_{X}, \Delta q_{2}$ is relatively constant because $V_{X}$ is quite independent of $V_{i n}$.

The constant magnitude of $\Delta q_{2}$ means that channel charge of $S_{2}$ introduces only an offset (rather than gain error or nonlinearity) in the input/output characteristic. As described below, this offset can easily be removed by differential operation. But, how about the charge injected by $S_{1}$ onto $C_{H}$ ? Let us set $V_{i n}$ to zero and suppose $S_{1}$ injects a charge packet $\Delta q_{1}$ onto node $P$ [Fig. 12.32(a)]. If the capacitance connected from $X$ to ground (including the input capacitance of the op amp) is


Figure 12.32. Effect of charge injected by $S_{1}$ with (a) zero and (b) finite op amp input capacitance, (c) transition of circuit to amplification of mode.
zero, $V_{P}$ and $V_{X}$ jump to infinity. To simplify the analysis, we assume a total capacitance equal to $C_{X}$ from $X$ to ground [Fig. 12.32(b)], and we will see shortly that its value does not affect the results. In Fig. 12.32(b), each of $C_{H}$ and $C_{X}$ carries a charge equal to $\Delta q_{1}$. Now, as shown in Fig. 12.32(c), we place $C_{H}$ around the op amp, seeking to obtain the resulting output voltage.

To calculate the output voltage, we must make an important observation: the total charge at node $X$ cannot change after $S_{2}$ turns off because no path exists for electrons to flow into or out of this node. Thus, if before $S_{1}$ turns off, the total charge on the right plate of $C_{H}$ and the top plate of $C_{X}$ is zero, it must still add up to zero after $S_{1}$ injects charge because no resistive path is connected to $X$. The same holds true after $C_{H}$ is placed around the op amp.

Now consider the circuit of Fig. 12.32(c), assuming the the total charge at node $X$ is zero. We can write $C_{X} V_{X}-\left(V_{\text {out }}-V_{X}\right) C_{H}=0$, and $V_{X}=-V_{\text {out }} / A_{v 1}$. Thus, $-\left(C_{X}+C_{H}\right) V_{\text {out }} / A_{v 1}-$
$V_{\text {out }} C_{H}=0$, i.e., $V_{\text {out }}=0$. Note that this result is independent of $\Delta q_{1}$, capacitor values, or the gain of the op amp, thereby revealing that the charge injection by $S_{1}$ introduces no error if $S_{2}$ turns off first.

In summary, in Fig. 12.30(a), after $S_{2}$ turns off, node $X$ "floats," maintaining a constant total charge regardless of the transitions at other nodes of the circuit. As a result, after the feedback configuration is formed, the output voltage is not influenced by the charge injection due to $S_{1}$. From another point of view, node $X$ is a virtual ground at the moment $S_{2}$ turns off, freezing the instantaneous input level across $C_{H}$ and yielding a charge equal to $V_{0} C_{H}$ on the left plate of $C_{H}$. After settling with feedback, node $X$ is again a virtual ground, forcing $C_{H}$ to still carry $V_{0} C_{H}$ and hence the output voltage to be approximately equal to $V_{0} C_{H}$.

The effect of the charge injected by $S_{1}$ can be studied from yet another perspective. Suppose in Fig. 12.32(c), the output voltage is finite and positive. Then, since $V_{X}=V_{\text {out }} /\left(-A_{v 1}\right), V_{X}$ must be finite and negative, requiring negative charge on the top plate of $C_{X}$. For the total charge at $X$ to be zero, the charge on the left plate of $C_{H}$ must be positive and that on its right plate negative, giving $V_{\text {out }}<0$. Thus, the only valid solution is $V_{\text {out }}=0$.

The third switch in Fig. 12.30(a), $S_{3}$, also merits attention. In order to turn on, $S_{3}$ must establish an inversion layer at its oxide interface. Does the required channel charge come from $C_{H}$ or from the op amp? We note from the foregoing analysis that after the feedback circuit has settled, the charge on $C_{H}$ equals $V_{0} C_{H}$, unaffected by $S_{3}$. The channel charge of this switch is therefore entirely supplied by the op amp, introducing no error.

Our study of Fig. 12.30(a) thus far suggests that, with proper timing, the charge injected by $S_{1}$ and $S_{3}$ is unimportant and the channel charge of $S_{2}$ results in a constant offset voltage. Fig. 12.33 depicts a simple realization of the clock edges to ensure $S_{1}$ turns off after $S_{2}$ does.


Figure 12.33. Generation of proper clock edges for unity-gain sampler.

The input-independent nature of the charge injected by the reset switch allows complete cancellation by differential operation. Illustrated in Fig. 12.34, such an approach employs a differential op amp along with two sampling capacitors so that the charge injected by $S_{2}$ and $S_{2}^{\prime}$ appears as a


Figure 12.34. Differential realization of unity-gain sampler.
common-mode disturbance at nodes $X$ and $Y$. This is in contrast to the behavior of the differential circuit shown in Fig. 12.27, where the input-dependent charge injection still leads to nonlinearity. In reality, $S_{2}$ and $S_{2}^{\prime}$ exhibit a finite charge injection mismatch, an issue resolved by adding another switch, $S_{e q}$, that turns off slightly after $S_{2}$ and $S_{2}^{\prime}$ (and before $S_{1}$ and $S_{1}^{\prime}$ ), thereby equalizing the charge at nodes $X$ and $Y$.

Precision Considerations The circuit of Fig. 12.30(a) operates as a unity-gain buffer in the amplification mode, producing an output voltage approximately equal to the voltage stored across the capacitor. How close to unity is the gain here? As a general case, we assume the op amp exhibits a finite input capacitance $C_{i n}$ and calculate the output voltage when the circuit goes from the sampling mode to the amplification mode (Fig. 12.35). Owing to the finite gain of the op amp, $V_{X} \neq 0$ in the amplification mode, giving a charge equal to $C_{i n} V_{X}$ on $C_{i n}$. The


Figure 12.35. Equivalent circuit for accuracy calculations.
conservation of charge at $X$ requires that $C_{i n} V_{X}$ come from $C_{H}$, raising the charge on $C_{H}$ to $C_{H} V_{0}+C_{i n} V_{X} .{ }^{7}$ It follows that the voltage across $C_{H}$ equals $\left(C_{H} V_{0}+C_{i n} V_{X}\right) / C_{H}$. We therefore

[^4]write $V_{\text {out }}-\left(C_{H} V_{0}+C_{\text {in }} V_{X}\right) / C_{H}=V_{X}$ and $V_{X}=-V_{\text {out }} / A_{v 1}$. Thus,
\[

$$
\begin{align*}
V_{\text {out }} & =\frac{V_{0}}{1+\frac{1}{A_{v 1}}\left(\frac{C_{i n}}{C_{H}}+1\right)}  \tag{12.41}\\
& \approx V_{0}\left[1-\frac{1}{A_{v 1}}\left(\frac{C_{i n}}{C_{H}}+1\right)\right] . \tag{12.42}
\end{align*}
$$
\]

As expected, if $C_{i n} / C_{H} \ll 1$, then $V_{\text {out }} \approx V_{0} /\left(1+A_{v 1}^{-1}\right)$. In general, however, the circuit suffers from a gain error of approximately $-\left(C_{i n} / C_{H}+1\right) / A_{v 1}$, suggesting that the input capacitance must be minimized even if speed is not critical. Recall from Chapter 9 that to increase $A_{v 1}$, we may choose a large width for the input transistors of the op amp, but at the cost of higher input capacitance. An optimum device size must therefore yield minimum gain error rather than maximum $A_{v 1}$.

## Example 12.3

In the circuit of Fig. 12.35, $C_{i n}=0.5 \mathrm{pF}$ and $C_{H}=2 \mathrm{pF}$. What is the minimum op amp gain that guarantees a gain error of $0.1 \%$ ?

Solution. Since $C_{i n} / C_{H}=0.25$, we have $A_{v 1, \text { min }}=1000 \times 1.25=1250$.

Speed Considerations Let us first examine the circuit in the sampling mode [Fig. 12.36(a)]. What is the time constant in this phase? The total resistance in series with $C_{H}$ is given by $R_{o n 1}$


Figure 12.36. (a) Unity-gain sampler in sampling mode, (b) equivalent circuit of (a).
and the resistance between $X$ and ground, $R_{X}$. Using the simple op amp model shown in Fig. 12.36(b), where $R_{0}$ denotes the open-loop output impedance of the op amp, we have

$$
\begin{equation*}
\left(I_{X}-G_{m} V_{X}\right) R_{0}+I_{X} R_{o n 2}=V_{X}, \tag{12.43}
\end{equation*}
$$

that is,

$$
\begin{equation*}
R_{X}=\frac{R_{0}+R_{o n 2}}{1+G_{m} R_{0}} . \tag{12.44}
\end{equation*}
$$

Since typically $R_{\text {on } 2} \ll R_{0}$ and $G_{m} R_{0} \gg 1$, we have $R_{X} \approx 1 / G_{m}$. For example, in a telescopic op amp employing differential to single-ended conversion, $G_{m}$ equals the transconductance of each input transistor.

The time constant in the sampling mode is thus equal to

$$
\begin{equation*}
\tau_{s a m}=\left(R_{o n 1}+\frac{1}{G_{m}}\right) C_{H} \tag{12.45}
\end{equation*}
$$

The magnitude of $\tau_{\text {sam }}$ must be sufficiently small to allow settling in the test case of Fig. 12.14 to the required precision.

Now let us consider the circuit as it enters the amplification mode. Shown in Fig. 12.37 along with both the op amp input capacitance and the load capacitance, the circuit must begin



Figure 12.37. Time response of unity-gain sampler in amplification mode.
with $V_{\text {out }} \approx 0$ and eventually produce $V_{\text {out }} \approx V_{0}$. If $C_{\text {in }}$ is relatively small, we can assume that the voltages across $C_{L}$ and $C_{H}$ do not change instantaneously, concluding that if $V_{\text {out }} \approx 0$ and $V_{C H} \approx V_{0}$, then $V_{X}=-V_{0}$ at the beginning of the amplification mode. In other words, the input difference sensed by the op amp initially jumps to a large value, possibly causing the op amp to slew. But, let us first assume the op amp can be modeled by a linear model and determine the output response.

To simplify the analysis, we represent the charge on $C_{H}$ by an explicit series voltage source, $V_{S}$, that goes from zero to $V_{0}$ at $t=t_{0}$ while $C_{H}$ carries no charge itself (Fig. 12.38). The objective is to obtain the transfer function $V_{\text {out }}(s) / V_{S}(s)$ and hence the step response. We have

$$
\begin{equation*}
V_{\text {out }}\left(\frac{1}{R_{0}}+C_{L} s\right)+G_{m} V_{X}=\left(V_{S}+V_{X}-V_{o u t}\right) C_{H} s \tag{12.46}
\end{equation*}
$$

Also, since the current through $C_{i n}$ equals $V_{X} C_{i n} s$,

$$
\begin{equation*}
V_{X} \frac{C_{i n} s}{C_{H} s}+V_{X}+V_{S}=V_{o u t} . \tag{12.47}
\end{equation*}
$$



Figure 12.38. Equivalent circuit of unity-gain circuit in amplification mode.

Calculating $V_{X}$ from (12.47) and substituting in (12.46), we arrive at the transfer function:

$$
\begin{equation*}
\frac{V_{\text {out }}}{V_{S}}(s)=R_{0} \frac{\left(G_{m}+C_{\text {in }} s\right) C_{H}}{R_{0}\left(C_{L} C_{i n}+C_{i n} C_{L}+C_{H} C_{L}\right) s+G_{m} R_{0} C_{H}+C_{H}+C_{\text {in }}} . \tag{12.48}
\end{equation*}
$$

Note that for $s=0$, (12.48) reduces to a form similar to (12.41). Since typically $G_{m} R_{0} C_{H} \gg$ $C_{H}, C_{i n}$, we can simplify (12.48) as

$$
\begin{equation*}
\frac{V_{\text {out }}}{V_{S}}(s)=\frac{\left(G_{m}+C_{i n} s\right) C_{H}}{\left(C_{L} C_{i n}+C_{i n} C_{L}+C_{H} C_{L}\right) s+G_{m} C_{H}} . \tag{12.49}
\end{equation*}
$$

Thus, the response is characterized by a time constant equal to

$$
\begin{equation*}
\tau_{a m p}=\frac{C_{L} C_{i n}+C_{i n} C_{L}+C_{H} C_{L}}{G_{m} C_{H}} \tag{12.50}
\end{equation*}
$$

which is independent of the op amp output resistance. This is because a higher $R_{0}$ leads to a greater loop gain, evetually yielding a constant closed-loop speed. If $C_{i n} \ll C_{L}, C_{H}$, then (12.50) reduces to $C_{L} / G_{m}$, an expected result because with negligible $C_{i n}$, the output resistance of the unity-gain buffer is equal to $1 / G_{m}$.

We now study the slewing behavior of the circuit, considering a telescopic op amp as an example. Upon entering the amplification mode, the circuit may experience a large step at the inverting input (Fig. 12.37). As shown in Fig. 12.39, the tail current of the op amp's input differential pair is then steered to one side, charging the capacitance seen at the output. Since $M_{2}$ is off during slewing, $C_{i n}$ is negligible and the slew rate is approximately equal to $I_{S S} / C_{L}$. The slewing continues until $V_{X}$ is sufficiently close to the gate voltage of $M_{1}$, after which point the settling progresses with the time constant given in (12.50).

Our foregoing studies reveal that the input capacitance of the op amp degrades both the speed and the precision of the unity-gain sampler/buffer. For this reason, the bottom plate of $C_{H}$ in Fig. 12.30 is usually driven by the input signal or the output of the op amp and the top plate is connected to node $X$ (Fig. 12.40), minimizing the parasitic capacitance seen from node $X$ to ground. This technique is called "bottom-plate sampling."


Figure 12.39. Unity-gain sampler during slewing.


Figure 12.40. Connection of capacitor to the unity-gain sampler.

It is instructive to compare the performance of the sampling circuits shown in Figs. 12.29(b) and 12.30(a). In Fig. 12.29(b), the sampling time constant is smaller because it depends on only the on-resistance of the switch. More importantly, in Fig. 12.29(b), the amplification after the switch turns off is almost instantaneous whereas in Fig. 12.30, it requires a finite settling time. However, the critical advantage of the unity-gain sampler is the input-independent charge injection.

### 12.3.2 Noninverting Amplifier

In this section, we revisit the amplifier of Fig. 12.4, studying its speed and precision properties. Repeated in Fig. 12.41(a), the amplifier operates as follows. In the sampling mode, $S_{1}$ and $S_{2}$ are on and $S_{3}$ is off, creating a virtual ground at $X$ and allowing the voltage across $C_{1}$ to track the input voltage [Fig. 12.41(b)]. At the end of the sampling mode, $S_{2}$ turns off first, injecting a constant charge, $\Delta q_{2}$, onto node $X$. Subsequently, $S_{1}$ turns off and $S_{3}$ turns on [Fig. 12.41(c)]. Since $V_{P}$ goes from $V_{\text {in0 }}$ to 0 , the output voltage changes from 0 to approximately $V_{\text {in } 0}\left(C_{1} / C_{2}\right)$, providing a voltage gain equal to $C_{1} / C_{2}$. We call the circuit a "noninverting amplifier" because the final output has the same polarity as $V_{i n 0}$ and the gain can be greater than unity.


Figure 12.41. (a) Noninverting amplifier, (b) circuit of (a) in sampling mode, (c) transition of circuit to amplification mode.

As with the unity-gain circuit of Fig. 12.30(a), the noninverting amplifier avoids inputdependent charge injection by proper timing, namely, turning $S_{2}$ off before $S_{1}$ (Fig. 12.42). After $S_{2}$ is off, the total charge at node $X$ remains constant, making the circuit insensitive to


Figure 12.42. Transition of noninverting amplifier to amplification mode.
charge injection of $S_{1}$ or charge "absorption" of $S_{3}$. Let us first study the effect of $S_{1}$ carefully. As illustrated in Fig. 12.43, the charge injected by $S_{1}, \Delta q_{1}$, changes the voltage at node $P$ by approximately $\Delta V_{P}=\Delta q_{1} / C_{1}$, and hence the output voltage by $-\Delta q_{1} C_{1} / C_{2}$. However, after $S_{3}$ turns on, $V_{P}$ drops to zero. Thus, the overall change in $V_{P}$ is equal to $0-V_{\text {in } 0}=-V_{\text {in0 }}$, producing an overall change in the output equal to $-V_{\text {in } 0}\left(-C_{1} / C_{2}\right)=V_{\text {in0 }} C_{1} / C_{2}$.

The key point here is that $V_{P}$ goes from a fixed voltage, $V_{0}$, to another, 0 , with an intermediate perturbation due to $S_{1}$. Since the output voltage of interest is measured after node $P$ is connected to ground, the charge injected by $S_{1}$ does not affect the final output. From another perspective, as shown in Fig. 12.44, the charge on the right plate of $C_{1}$ at the instant $S_{2}$ turns off is approximately


Figure 12.43. Effect of charge injected by $S_{1}$.


Figure 12.44. Charge redistribution in noninverting amplifier.
equal to $-V_{i n 0} C_{1}$. Also, the total charge at node $X$ must remain constant after $S_{2}$ turns off. Thus, when node $P$ is connected to ground and the circuit settles, the voltage across $C_{1}$ and hence its charge are nearly zero, and the charge $-V_{i n 0} C_{1}$ must reside on the left plate of $C_{2}$. In other words, the output voltage is approximately equal to $V_{\text {in } 0}$ regardless of the intermediate excursions at node $P$.

The foregoing discussion indicates that two other phenomena have no effect on the final output. First, from the time $S_{2}$ turns off until the time $S_{1}$ turns off, the input voltage may change significantly (Fig. 12.45) without introducing any error. In other words, the sampling instant is defined by the turn-off of $S_{2}$. Second, when $S_{3}$ turns on, it requires some channel charge but since the final value of $V_{P}$ is zero, this charge is unimportant. Neither of these effects introduces error because the total charge at node $X$ is conserved and $V_{P}$ is eventually set by a fixed (zero) potential. To emphasize that $V_{P}$ is initially and finally determined by fixed voltages, we say node $P$ is "driven" or node $P$ switches from a low-impedance node to another low-impedance node. Here the term low-impedance distinguishes node $P$, at which charge is not conserved, from "floating" nodes such as $X$, where charge is conserved.


Figure 12.45. Effect of input change after $S_{2}$ turns off.

In summary, proper timing in Fig. 12.41(a) ensures that node $X$ is perturbed by only the charge injection of $S_{2}$, making the final value of $V_{\text {out }}$ free from errors due to $S_{1}$ and $S_{3}$. The constant offset due to $S_{2}$ can be suppressed by differential operation (Fig. 12.46).


Figure 12.46. Differential realization of noninverting amplifier.

## Example 12.4

In the differential circuit of Fig. 12.46, suppose the equalizing switch is not used and $S_{2}$ and $S_{2}^{\prime}$ exhibit a threshold voltage mismatch of 10 mV . If $C_{1}=1 \mathrm{pF}, C_{2}=0.5 \mathrm{pF}, V_{T H}=0.6 \mathrm{~V}$, and for all switches $W L C_{o x}=50 \mathrm{fF}$, calculate the dc offset measured at the output assuming all of the channel charge of $S_{2}$ and $S_{2}^{\prime}$ is injected onto $X$ and $Y$, respectively.

Solution. Simplifying the circuit as in Fig. 12.47, we have $V_{\text {out }} \approx \Delta q / C_{2}$, where $\Delta q=$ $W L C_{o x} \Delta V_{T H}$. Note that $C_{1}$ does not appear in the result because $X$ is a virtual ground, i.e., the voltage across $C_{1}$ changes only negligibly. Thus, the injected charge resides primarily on the left plate of $C_{2}$, giving an output error voltage equal to $\Delta V_{o u t}=W L C_{o x} \Delta V_{T H} / C_{2}=1 \mathrm{mV}$.


Figure 12.47.

Precision Considerations As mentioned above, the circuit of Fig. 12.41(a) provides a nominal voltage gain of $C_{1} / C_{2}$. We now calculate the actual gain if the op amp exhibits a finite open-loop gain equal to $A_{v 1}$. Depicted in Fig. 12.48 along with the input capacitance of the op amp, the


Figure 12.48. Equivalent circuit of noninverting amplifier during amplification.
circuit amplifies the input voltage change such that:

$$
\begin{equation*}
\left(V_{\text {out }}-V_{X}\right) C_{2} s=V_{X} C_{i n} s+\left(V_{X}-V_{\text {in }}\right) C_{1} s \tag{12.51}
\end{equation*}
$$

Since $V_{\text {out }}=-A_{v 1} V_{X}$, we have

$$
\begin{equation*}
\frac{V_{\text {out }}}{V_{\text {in }}}=\frac{-C_{1}}{C_{2}+\frac{C_{2}+C_{1}+C_{\text {in }}}{A_{v 1}}} . \tag{12.52}
\end{equation*}
$$

For large $A_{v 1}$,

$$
\begin{equation*}
\frac{V_{\text {out }}}{V_{\text {in }}} \approx-\frac{C_{1}}{C_{2}}\left(1-\frac{C_{2}+C_{1}+C_{\text {in }}}{C_{2}} \cdot \frac{1}{A_{v 1}}\right), \tag{12.53}
\end{equation*}
$$

implying that the amplifier suffers from a gain error of $\left(C_{2}+C_{1}+C_{i n}\right) /\left(C_{2} A_{v 1}\right)$. Note that the gain error increases with the nominal gain $C_{1} / C_{2}$.

Comparing (12.42) with (12.53), we note that with $C_{H}=C_{2}$ and for a nominal gain of unity, the noninverting amplifier exhibits greater gain error than does the unity-gain sampler. This is because
the feedback factor equals $C_{2} /\left(C_{1}+C_{i n}+C_{2}\right)$ in the former and $C_{H} /\left(C_{H}+C_{i n}\right)$ in the latter. For example, if $C_{i n}$ is negligible, the unity-gain buffer's gain error is half that of the noninverting amplifier.

Speed Considerations The smaller feedback factor in Fig. 12.48 suggests that the time response of the amplifier may be slower than that of the unity-gain sampler. This is indeed true. Consider the equivalent circuit shown in Fig. 12.49(a). Since the only difference between this circuit and that

(a)

(b)

Figure 12.49. (a) Equivalent circuit of noninverting amplifier in amplification mode, (b) circuit of (a) with $V_{i n}, C_{1}$, and $C_{i n}$ replaced by a Thevenin equivalent.
in Fig. 12.38 is the capacitor $C_{1}$, which is connected from node $X$ to an ideal voltage source, we expect that (12.50) gives the time constant of this amplifier as well if $C_{i n}$ is replaced by $C_{i n}+C_{1}$. But for a more rigorous analysis, we substitute $V_{i n}, C_{1}$, and $C_{i n}$ in Fig. 12.49(a) by a Thevenin equivalent as in Fig. 12.49(b), where $\alpha=C_{1} /\left(C_{1}+C_{i n}\right)$, and $C_{\epsilon q}=C_{1}+C_{i n}$, and note that

$$
\begin{equation*}
V_{X}=\left(\alpha V_{\text {in }}-V_{o u t}\right) \frac{C_{e q}}{C_{e q}+C_{2}}+V_{\text {out }} . \tag{12.54}
\end{equation*}
$$

Thus,

$$
\begin{equation*}
\left[\left(\alpha V_{\text {in }}-V_{\text {out }}\right) \frac{C_{e q}}{C_{e q}+C_{2}}+V_{\text {out }}\right] G_{m}+V_{\text {out }}\left(\frac{1}{R_{0}}+C_{L} s=\left(\alpha V_{\text {in }}-V_{\text {out }}\right) \frac{C_{e q} C_{2}}{C_{e q}+C_{2}} s\right. \tag{12.55}
\end{equation*}
$$

and hence

$$
\begin{equation*}
\frac{V_{\text {out }}}{V_{\text {in }}}(s)=\frac{-C_{e q} \frac{C_{1}}{C_{1}+C_{i n}}\left(G_{m}-C_{2} s\right) R_{0}}{C_{2} G_{m} R_{0}+C_{e q}+C_{2}+R_{0}\left[C_{L}\left(C_{e q}+C_{2}\right)+C_{e q} C_{2}\right] s} . \tag{12.56}
\end{equation*}
$$

Note that for $s=0$, (12.56) reduces to (12.52). For a large $G_{m} R_{0}$, we can simplify (12.56) to

$$
\begin{equation*}
\frac{V_{\text {out }}}{V_{\text {in }}}(s) \approx \frac{-C_{e q} \frac{C_{1}}{C_{1}+C_{i n}}\left(G_{m}-C_{2} s\right) R_{0}}{R_{0}\left(C_{L} C_{e q}+C_{L} C_{2}+C_{e q} C_{2}\right) s+G_{m} R_{0} C_{2}} \tag{12.57}
\end{equation*}
$$

obtaining a time constant of

$$
\begin{equation*}
\tau_{a m p}=\frac{C_{L} C_{e q}+C_{L} C_{2}+C_{e q} C_{2}}{G_{m} C_{2}} \tag{12.58}
\end{equation*}
$$

which is the same as the time constant of Fig. 12.37 if $C_{i n}$ is replaced by $C_{i n}+C_{1}$. Note the direct dependence of $\tau_{a m p}$ upon the nominal gain, $C_{1} / C_{2}$.

It is instructive to examine the amplifier's time constant for the special case $C_{L}=0$. Equation (12.58) yields $\tau_{a m p}=\left(C_{1}+C_{i n}\right) / G_{m}$, a value independent of the feedback capacitor. This is because, while a larger $C_{2}$ introduces heavier loading at the output, it also provides a greater feedback factor.

### 12.3.3 Precision Multiply-by-Two Circuit

The circuit of Fig. 12.41(a) can operate with a relatively high closed-loop gain, but it suffers from speed and precision degradation due to the low feedback factor. In this section, we study a topology that provides a nominal gain of two while achieving a higher speed and lower gain error [5]. Shown in Fig. 12.50(a), the amplifier incorporates two equal capacitors, $C_{1}=C_{2}=C$. In the sampling mode, the circuit is configured as in Fig. 12.50(b), establishing a virtual ground at $X$ and allowing the voltage across $C_{1}$ and $C_{2}$ to track $V_{i n}$. In the transition to the amplification mode, $S_{3}$ turns off first, $C_{1}$ is placed around the op amp, and the left plate of $C_{2}$ is switched to ground [Fig. 12.50(c)]. Since at the moment $S_{3}$ turns off, the total charge on $C_{1}$ and $C_{2}$ equals $2 V_{\text {in0 }} C$ (if the charge injected by $S_{3}$ is neglected), and since the voltage across $C_{2}$ approaches zero in the amplification mode, the final voltage across $C_{1}$ and hence the output voltage are approximately equal to $2 V_{i n 0}$. This can also be seen from the slow motion illustration of Fig. 12.51.

The reader can show that the charge injected by $S_{1}$ and $S_{2}$ and absorbed by $S_{4}$ and $S_{5}$ is unimportant and that injected by $S_{3}$ introduces a constant offset. The offset can be suppressed by differential operation.

The speed and precision of the multiply-by-two circuit are expressed by (12.58) and (12.53), respectively, but the advantage of the circuit is the higher feedback factor for a given closed-loop gain. Note, however, that the input capacitance of the multiply-by-two circuit in the sampling mode is higher.


Figure 12.50. (a) Multiply-by-two circuit, (a) circuit of (a) in sampling mode, (b) circuit of (a) in amplification mode.


Figure 12.51. Transition of multiply-by-two-circuit to amplification mode in slow motion.

### 12.4 Switched-Capacitor Integrator

Integrators are used in many analog systems. Examples include filters and oversampled analog-todigital converters. Fig. 12.52 depicts a continuous-time integrator, whose output can be expressed


Figure 12.52. Continuous-time integrator.
as

$$
\begin{equation*}
V_{o u t}=-\frac{1}{R C_{F}} \int V_{i n} d t \tag{12.59}
\end{equation*}
$$

if the op amp gain is very large. For sampled-data systems, we must devise a discrete-time counterpart of this circuit.

Before studying SC integrators, let us first point out an interesting property. Consider a resistor connected between two nodes [Fig. 12.53(a)], carrying a current equal to $\left(V_{A}-V_{B}\right) / R$. The role


Figure 12.53. (a) Continuous-time and (b) discrete-time resistors.
of the resistor is to take a certain amount of charge from node $A$ every second and move it to node B. Can we perform the same function by a capacitor? Suppose in the circuit of Fig. 12.53(b), capacitor $C_{S}$ is alternately connected to nodes $A$ and $B$ at a clock rate $f_{C K}$. The average current flowing from $A$ to $B$ is then equal to the charge moved in on clock period:

$$
\begin{align*}
\overline{I_{A B}} & =\frac{C_{S}\left(V_{A}-V_{B}\right)}{f_{C K}^{-1}}  \tag{12.60}\\
& =C_{S} f_{C K}\left(V_{A}-V_{B}\right) \tag{12.61}
\end{align*}
$$

We can therefore view the circuit as a "resistor" equal to $\left(C_{S} f_{C K}\right)^{-1}$. Recognized by James Clark Maxwell, this property formed the foundation for many modern switched-capacitor circuits.

Let us now replace resistor $R$ in Fig. 12.52 by its discrete-time equivalent, arriving at the integrator of Fig. 12.54(a). We note that in every clock cycle, $C_{1}$ absorbs a charge equal to $C_{1} V_{\text {in }}$


Figure 12.54. (a) Discrete-time integrator, (b) response of circuit to a constant input voltage.
when $S_{1}$ is on and deposits the charge on $C_{2}$ when $S_{2}$ is on (node $X$ is a virtual ground). For example, if $V_{i n}$ is constant, the output changes by $V_{i n} C_{1} / C_{2}$ every clock cycle [Fig. 12.54(b)]. Approximating the staircase waveform by a ramp, we note that the circuit behaves as an integrator.

The final value of $V_{\text {out }}$ in Fig. 12.54(a) after every clock cycle can be written as

$$
\begin{equation*}
V_{\text {out }}\left(k T_{C K}\right)=V_{\text {out }}\left[(k-1) T_{C K}\right]-V_{\text {in }}\left[(k-1) T_{C K}\right] \cdot \frac{C_{1}}{C_{2}}, \tag{12.62}
\end{equation*}
$$

where the gain of the op amp is assumed large. Note that the small-signal settling time constant as charge is transferred from $C_{1}$ to $C_{2}$ is given by (12.50).

The integrator of Fig. 12.54(a) suffers from two important drawbacks. First, the inputdependent charge injection of $S_{1}$ introduces nonlinearity in the charge stored on $C_{1}$ and hence the output voltage. Second, the nonlinear capacitance at node $P$ resulting from the source/drain junctions of $S_{1}$ and $S_{2}$ leads to a nonlinear charge-to-voltage conversion when $C_{1}$ is switched to $X$. This can be understood with the aid of Fig. 12.55, where the charge stored on the total junction capacitance, $C_{j}$, is not equal to $V_{i n 0} C_{j}$, but rather equal to


Figure 12.55. Effect of junction capacitance nonlinearity in SC integrator.

$$
\begin{equation*}
q_{c j}=\int_{0}^{V i n 0} C_{j} d V \tag{12.63}
\end{equation*}
$$

Since $C_{j}$ is a function of voltage, $q_{c j}$ exhibits a nonlinear dependence on $V_{i n 0}$, thereby creating a nonlinear component at the output after the charge is transferred to the integration capacitor.

An integrator topology that resolves both of the foregoing issues is shown in Fig. 12.56(a). We study the circuit's operation in the sampling and integration modes. As shown in Fig. 12.56(b), in the sampling mode $S_{1}$ and $S_{3}$ are on and $S_{2}$ and $S_{4}$ are off, allowing the voltage across $C_{1}$ to track $V_{\text {in }}$ while the op amp and $C_{2}$ hold the previous value. In the transition to the integration mode, $S_{3}$ turns off first, injecting a constant charge onto $C_{1}, S_{1}$ turns off next, and subsequently $S_{2}$ and $S_{4}$ turn on [Fig. 12.56(c)]. The charge stored on $C_{1}$ is therefore transferred to $C_{2}$ through the virtual ground node.

Since $S_{3}$ turns off first, it introduces only a constant offset, which can be suppressed by differential operation. Moreover, because the left plate of $C_{1}$ is "driven" (Section 12.3.2), the charge injection or absorption of $S_{1}$ and $S_{2}$ contributes no error. Also, since node $X$ is a virtual ground, the charge injected or absorbed by $S_{4}$ is constant and independent of $V_{i n}$.


Figure 12.56. (a) Parasitic-insensitive integrator, (b) circuit of (a) in sampling mode, (c) circuit of (a) in integration mode.

How about the nonlinear junction capacitance of $S_{3}$ and $S_{4}$ ? We observe that the voltage across this capacitance goes from near zero in the sampling mode to virtual ground in the integration mode. Since the voltage across the nonlinear capacitance changes by a very small amount, the resulting nonlinearity is negligible.

### 12.5 Switched-Capacitor Common-Mode Feedback

Our study of common-mode feedback in Chapter 9 suggested that sensing the output CM level by means of resistors lowers the differential voltage gain of the circuit considerably. We also observed that sensing techniques using MOSFETs that operate as source followers or variable resistors suffer from a limited linear range. Switched-capacitor CMFB networks provide an alternative that avoids both of these difficulties (but the circuit must be refreshed periodically.)

In switched-capacitor common-mode feedback, the outputs are sensed by capacitors rather than resistors. Figure 12.57 depicts a simple example, where equal capacitors $C_{1}$ and $C_{2}$ reproduce at node $X$ the average of the changes in each output voltage. Thus, if $V_{\text {out } 1}$ and $V_{\text {out } 2}$ experience a, say, positive CM change, then $V_{X}$ and hence $I_{D 5}$ increase, pulling $V_{\text {out } 1}$ and $V_{\text {out } 2}$ down. The output CM level is then equal to $V_{G S 2}$ plus the voltage across $C_{1}$ and $C_{2}$.

How is the voltage across $C_{1}$ and $C_{2}$ defined? This is typically carried out when the amplifier is in the sampling (or reset) mode and can be accomplished as shown in Fig. 12.58. Here, during CM level definition, the amplifier differential input is zero and switch $S_{1}$ is on. Transistors $M_{6}$ and $M_{7}$ operate as a linear sense circuit because their gate voltages are nominally equal. Thus, the circuit settles such that the ouput CM level is equal to $V_{G S 6,7}+V_{G S 5}$. At the end of this mode, $S_{1}$


Figure 12.57. Simple SC common-mode feedback.


Figure 12.58. Definition of the voltage across $C_{1}$ and $C_{2}$.
turns off, leaving a voltage equal to $V_{G S 5,6}$ across $C_{1}$ and $C_{2}$. In the amplification mode, $M_{6}$ and $M_{7}$ may experience a large nonlinearity but they do not impact the performance of the main circuit because $S_{1}$ is off.

In applications where the output CM level must be defined more accurately than in the above example, the topology shown in Fig. 12.59 may be used. Here, in the reset mode, one plate of $C_{1}$


Figure 12.59. Alternative topology for definition of output CM level.
and $C_{2}$ is switched to $V_{C M}$ while the other is connected to the gate of $M_{6}$. Each capacitor therefore sustains a volatge equal to $V_{C M}-V_{G S 6}$. In the amplification mode, $S_{2}$ and $S_{3}$ are on and the other switches are off, yielding an output CM level equal to $V_{C M}-V_{G S 6}+V_{G S 5}$. Proper definition of $I_{D 3}$ and $I_{D 4}$ with respect to $I_{R E F}$ can guarantee that $V_{G S 5}=V_{G S 6}$ and hence the output CM level is equal to $V_{C M}$.

With large output swings, the speed of the CMFB loop may in fact influence the settling of the differential output [6]. For this reason, part of the tail current of the differential pairs in Figs. 12.58 and 12.59 can be provided by a constant current source so that $M_{5}$ makes only small adjustments to the circuit.

## Problems

Unless otherwise stated, in the following problems, use the device data shown in Table 2.1 and assume $V_{D D}=3 \mathrm{~V}$ where necessary. Also, assume all transistors are in saturation.
12.1 The circuit of Fig. 12.2(a) is designed with $C_{1}=2 \mathrm{pF}$ and $C_{2}=0.5 \mathrm{pF}$.
(a) Assuming $R_{F}=\infty$ but the op amp has an output resistance $R_{\text {out }}$, derive the transfer function $V_{\text {out }}(s) / V_{\text {in }}(s)$.
(b) If the op amp is ideal, determine the minimum value of $R_{F}$ that guarantees a gain error of $1 \%$ for an input frequency of 1 MHz .
12.2 Suppose in Fig. 12.5(a), the op amp is characterized by a transconductance $G_{m}$ and an output resistance $R_{\text {out }}$.
(a) Determine the transfer function $V_{o u t} / V_{\text {in }}$ in this mode.
(b) Plot the waveform at node $B$ if $V_{i n}$ is a $100-\mathrm{MHz}$ sinusoid with a peak amplitude of 1 V , $C_{1}=1 \mathrm{pF}, G_{m}=1 /(100 \Omega)$, and $R_{\text {out }}=20 \mathrm{k} \Omega$.
12.3 In Fig. 12.5(b), node $A$ is in fact connected to ground through a switch (Fig. 12.4). If the switch introduces a series resistance $R_{o n}$ and the op amp is ideal, calculate the time constant of the circuit in this mode. What is the total energy dissipated in the switch as the circuit enters the amplification mode and $V_{\text {out }}$ settles to its final value?
12.4 The circuit of Fig. 12.9(a) is designed with $(W / L)_{1}=20 / 0.5$ and $C_{H}=1 \mathrm{pF}$.
(a) Using Eqs. (12.7) and (12.14), calculate the rime required for $V_{\text {out }}$ to drop to +1 mV .
(b) Approximating $M_{1}$ by a linear resistor equal to $\left[\mu_{n} C_{o x}(W / L)_{1}\left(V_{D D}-V_{T H}\right)\right]^{-1}$, calculate the time required for $V_{\text {out }}$ to drop to +1 mV and compare the result with that obtained in part (a).
12.5 The circuit of Fig. 12.11 cannot be characterized by a single time constant because the resistance charging $C_{H}$ (equal to $1 / g_{m 1}$ if $\gamma=0$ ) varies with the output level. Assume $(W / L)_{1}=20 / 0.5$ and $C_{H}=1 \mathrm{pF}$.
(a) Using Eq. (12.19), calculate the time required for $V_{\text {out }}$ to reach 2.1 V .
(b) Sketch the transconductance of $M_{1}$ versus time.
12.6 In the circuit of Fig. 12.8(b), $(W / L)_{1}=20 / 0.5$ and $C_{H}=1 \mathrm{pF}$. Assume $\lambda=\gamma=0$ and $V_{i n}=V_{0} \sin \omega_{i n} t+V_{m}$, where $\omega_{i n}=2 \pi \times(100 \mathrm{MHz})$.
(a) Calculate $R_{o n 1}$ and the phase shift from the input to the output if $V_{0}=V_{m}=10 \mathrm{mV}$.
(b) Repeat part (a) if $V_{0}=10 \mathrm{mV}$ but $V_{m}=1 \mathrm{~V}$. The variation of the phase shift translates to distortion.
12.7 Describe an efficient SPICE simulation that yields the plot of $R_{o n, e q}$ for the circuit of Fig. 12.16.
12.8 The sampling network of Fig. 12.16 is designed with $(W / L)_{1}=20 / 0.5$ and $(W / L)_{2}=$ $60 / 0.5$. If $V_{\text {in }}=0$ and the initial value of $V_{\text {out }}$ is +3 V , estimate the time required for $V_{\text {out }}$ to drop to +1 mV .
12.9 In the circuit of Fig. 12.19, $(W / L)_{1}=20 / 0.5$ and $C_{H}=1 \mathrm{pF}$. Calculate the maximum error at the output due to charge injection. Compare this error with that resulting from clock feedthrough.
12.10 The circuit of Fig. 12.60 samples the input on $C_{1}$ when $C K$ is high and connects $C_{1}$ and $C_{2}$


Figure 12.60.
when $C K$ is low. Assume $(W / L)_{1}=(W / L)_{2}$ and $C_{1}=C_{2}$.
(a) If the initial voltages across $C_{1}$ and $C_{2}$ are zero and $V_{\text {in }}=2 \mathrm{~V}$, plot $V_{\text {out }}$ versus time for many clock cycles. Neglect charge injection and clock feedthrough.
(b) What is the maximum error in $V_{\text {out }}$ due to charge injection and clock feedthrough of $M_{1}$ and $M_{2}$ ? Assume the channel charge of $M_{2}$ splits equally between $C_{1}$ and $C_{2}$.
(c) Determine the $k T / C$ noise at the output after $M_{2}$ turns off.
12.11 For $V_{\text {in }}=V_{0} \sin \omega_{0} t+V_{0}$, where $V_{0}=0.5 \mathrm{~V}$ and $\omega_{0}=2 \pi \times(10 \mathrm{MHz})$, plot the output waveforms of the circuits shownin Fig. 12.29(b) and 12.30(a). Assume a clock frequency of 50 MHz .
12.12 In Fig. 12.45, $S_{1}$ turns off $\Delta t$ seconds after $S_{2}$ and $S_{3}$ turns on $\Delta t$ seconds after $S_{1}$ turns off. Plot the output waveform, taking into account the charge injection and clock feedthough of $S_{1}-S_{3}$.
12.13 The circuit of Fig. 12.48 is designed with $C_{1}=2 \mathrm{pF}, C_{i n}=0.2 \mathrm{pF}$ and $A_{v}=1000$. What is the maximum nominal gain, $C_{1} / C_{2}$, that the circuit can provide with a gain error of $1 \%$ ?
12.14 In Problem 12.13, what is the maximum nominal gain if $G_{m}=1 /(100 \Omega)$ and the circuit must achieve a time constant of 2 ns in the amplification mode?
12.15 The integrator of Fig. 12.54 is designed with $C_{1}=C_{2}=1 \mathrm{pF}$ and a clock frequency of 100 MHz . Neglecting charge injection and clock feedthrough, sketch the output if the input is a $10-\mathrm{MHz}$ sinusoid with a peak amplitude of 0.5 V . Approximating $C_{1}, S_{1}$, and $S_{2}$ by a resistor, estimate the output amplitude.
12.16 Consider the switched-capacitor amplifier depicted in Fig. 12.61, where the common-mode


Figure 12.61.
feedback is not shown. Assume $(W / L)_{1-4}=50 / 0.5, I_{S S}=1 \mathrm{~mA}, C_{1}=C_{2}=2 \mathrm{pF}$, $C_{3}=C_{4}=0.5 \mathrm{pF}$, and the output CM level is 1.5 V . Neglect the transistor capacitances.
(a) What is the maximum allowable output voltage swing in the amplification mode?
(b) Determine the gain error of the amplifier.
(c) What is the small-signal time constant in the amplification mode?
12.17 Repeat Problem 12.16 if the gate-source capacitance of $M_{1}$ and $M_{2}$ is not neglected.
12.18 A differential circuit incorporating a well-designed common-mode feedback network exhibits the open-loop input-output characteristic shown in Fig. 12.62(a). In some circuits, however, the characteristic appears as in Fig. 12.62(b). Explain how this effect occurs.


Figure 12.62.
12.19 In the common-mode feedback network of Fig. 12.58, assume $W / L=50 / 0.5$ for all transistors, $I_{D 5}=1 \mathrm{~mA}$, and $I_{D 6,7}=50 \mu \mathrm{~A}$. Determine the allowable range of the input common-mode level.
12.20 Repeat Problem 12.19 if $(W / L)_{5,6}=10 / 0.5$.
12.21 Suppose in the common-mode feedback network of Fig. 12.58, $S_{1}$ injects a charge of $\Delta q$ onto the gate of $M_{5}$. How much do the gate voltage of $M_{5}$ and the output common-mode level change due to this error?
12.22 In the circuit of Fig. 12.63, each op amp is represented by a Norton equivalent and characterized by $G_{m}$ and $R_{\text {out }}$. The output currents of two op amps are summed at node $Y$ [7]. (The circuit is shown in the amplification mode.) Note that the main amplifier and the auxiliary amplifier are identical and the error amplifier senses the voltage variation at node $X$ and injects a proportional current into node $Y$. Assume $G_{m} R_{\text {out }} \gg 1$.
(a) Calculate the gain error of the circuit.
(b) Repeat part (a) if the auxiliary and error amplifiers are eliminated and compare the results.


Figure 12.63.

## References

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[^0]:    ${ }^{1}$ We assume the circuit following the sampler draws no input dc current.

[^1]:    ${ }^{2}$ By contrast, the output swing of cascode stages is typically limited by overdrive voltages rather than the threshold voltage.
    ${ }^{3}$ In reality, $V_{T H N}$ and $V_{T H P}$ vary with $V_{i n}$ through body effect but we ignore this variation here.

[^2]:    ${ }^{4}$ The voltage gain is greater than unity because the pedestal becomes smaller as the input level rises.
    ${ }^{5}$ Even for PMOS switches, the $n$-well is connected to the most positive supply voltage because the source and drain terminals of the switch may interchange during sampling.

[^3]:    ${ }^{6}$ The oxide thickness in this type of amplifier is typically thicker than that of MOS gate area because silicon dioxide grows faster on a heavily-doped material.

[^4]:    ${ }^{7}$ The charge on $C_{H}$ increases because moving positive charge from the left plate of $C_{H}$ to the top plate of $C_{i n}$ leads to a more positive voltage across $C_{H}$.

