Sample HW#2 sol.

1 : 20
2 : 30

1. (Circuit a)

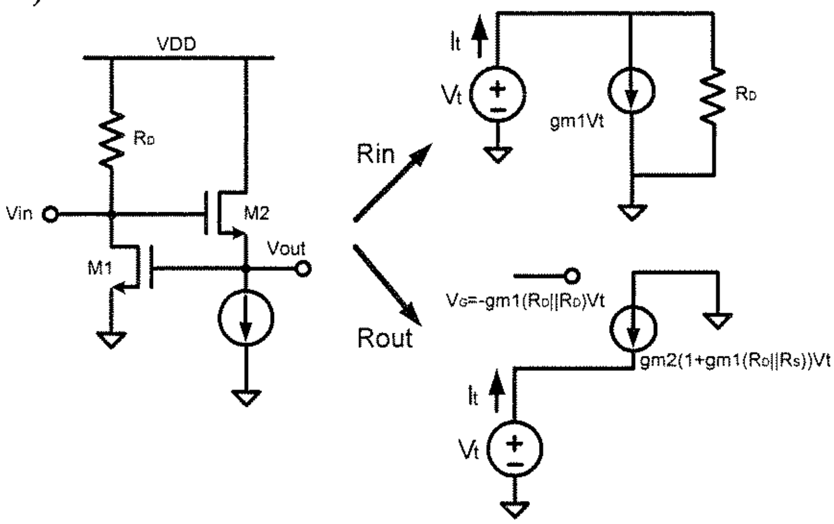

Figure 1. Rin and Rout for Circuit a

As shown in figure 1, we can derive input resistance and output resistance of circuit a. In circuit a, the gain from the gate of M2 to the source of M2 is unity since it is a perfect source follower. In other words, M1 can be considered as a diode connected transistor when we find the input resistance.

$$R_{in} = \frac{1}{g_{m1}} \parallel R_D = \frac{R_D}{1+g_{m1}R_D}$$

When we calculate the output resistance, we need to consider the circuit experience a negative feedback with loop gain of $g_{m1}(R_S \parallel R_D)$. The output resistance, therefore, is divided by $1+g_{m1}(R_S \parallel R_D)$.

$$R_{out} = \frac{1}{g_{m2}} \frac{1}{1+g_{m1}(R_S \parallel R_D)}$$

The total gain $A = \frac{R_{in}}{R_S+R_{in}} = \frac{R_D}{R_S+R_D+g_{m1}R_SR_D}$.

Therefore,

$$\overline{V_{n,out}^2} = 4kT\left[R_S\left(\frac{R_{in}}{R_S+R_{in}}\right)^2 + \frac{1}{R_D}(R_{in} \parallel R_S)^2 + \gamma g_{m1}(R_{in} \parallel R_S)^2 + \gamma g_{m2}\left(\frac{1}{g_{m2}}\frac{1}{1+g_{m1}(R_S \parallel R_D)}\right)^2\right]$$

$$\overline{V_{n,out}^2} = 4kT\left[\begin{array}{l} R_S\left(\frac{R_D}{R_S+R_D+g_{m1}R_SR_D}\right)^2 + \frac{1}{R_D}\left(\frac{R_SR_D}{R_S+R_D+g_{m1}R_SR_D}\right)^2 \\ +\gamma g_{m1}\left(\frac{R_SR_D}{R_S+R_D+g_{m1}R_SR_D}\right)^2 + \gamma g_{m2}\left(\frac{1}{g_{m2}}\frac{R_S+R_D}{R_S+R_D+g_{m1}R_SR_D}\right)^2 \end{array}\right]$$

$$NF = \frac{\overline{V_{n,out}^2}}{A^2}\frac{1}{4kTR_S} = 1 + \frac{R_S}{R_D} + \gamma g_{m1}R_S + \gamma\frac{1}{g_{m2}R_S}\left(1 + \frac{R_S}{R_D}\right)^2$$

(a) If $R_D \to \infty$, $NF = 1 + \gamma g_{m1}R_S + \gamma\frac{1}{g_{m2}R_S}$

This means that the noise generated from R_D goes to zero since the value of R_D is too big. Input resistance goes up to $\frac{1}{g_{m1}}$ and total gain increases. That is, the noise from M2 should decrease. While the noise from M2 stays the same, the signal amplitude increases. As we can see in the above equation, the noise contribution from M2 decreases as R_D goes to infinity.

(b) If $g_{m2} \to 0$, $NF \to \infty$

As $g_{m2} \to 0$, the noise contribution of M2 becomes infinity. This means that the output resistance goes up and the signal is not amplified enough to beat the noise.

1. (Circuit b)

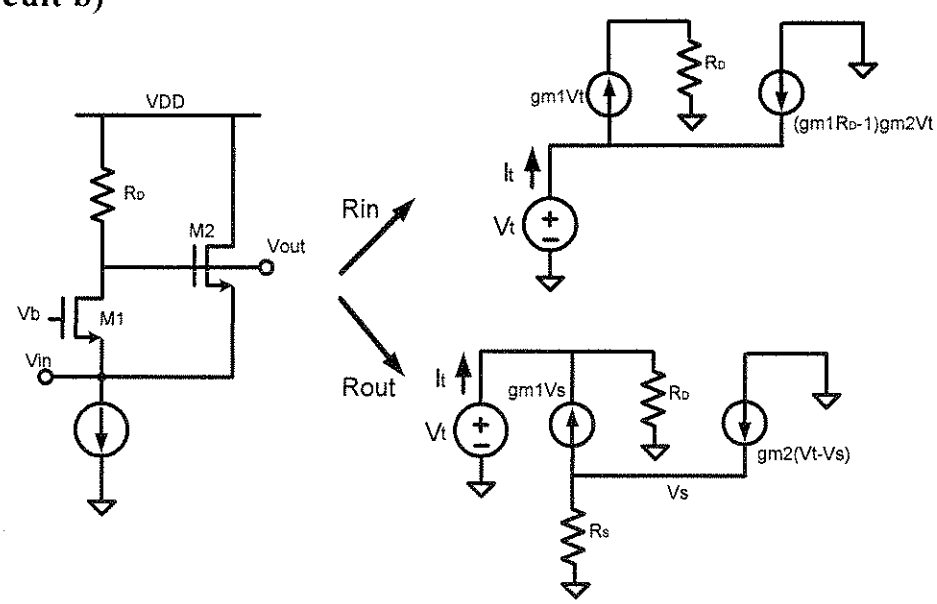

Figure 2. Rin and Rout for Circuit 2

From figure 2, we can derive input resistance and output resistance of circuit b.

$$R_{in} = \frac{1}{g_{m1} + g_{m2} - g_{m1}g_{m2}R_D}$$

$$R_{out} = \frac{(g_{m1} + g_{m2})R_S R_D + R_D}{1 + g_{m1}R_s + g_{m2}R_s - g_{m1}g_{m2}R_S R_D}$$

The total gain $A = \frac{R_{in}}{R_S + R_{in}} g_{m1} R_D = \frac{g_{m1} R_D}{1 + g_{m1} R_s + g_{m2} R_s - g_{m1} g_{m2} R_S R_D}$.

Therefore,

$$\overline{V_{n,out}^2} = 4kT\left[R_S \left(\frac{R_{in}}{R_S + R_{in}} g_{m1} R_D \right)^2 + \frac{1}{R_D} (R_{out})^2 + \gamma g_{m1} \left(\frac{-R_D - g_{m2} R_S R_D}{1 + g_{m1} R_S + g_{m2} R_S - g_{m1} g_{m2} R_S R_D} \right)^2 + \gamma g_{m2} \left(\frac{R_S R_{in}}{R_S + R_{in}} g_{m1} R_D \right)^2 \right]$$

$$\overline{V_{n,out}^2} = 4kT \left[\begin{array}{l} R_S \left(\frac{g_{m1} R_D}{1 + g_{m1} R_s + g_{m2} R_s - g_{m1} g_{m2} R_S R_D} \right)^2 + \frac{1}{R_D} \left(\frac{(g_{m1} + g_{m2}) R_S R_D + R_D}{1 + g_{m1} R_s + g_{m2} R_s - g_{m1} g_{m2} R_S R_D} \right)^2 \\ + \gamma g_{m1} \left(\frac{g_{m2} R_S R_D + R_D}{1 + g_{m1} R_s + g_{m2} R_s - g_{m1} g_{m2} R_S R_D} \right)^2 + \gamma g_{m2} \left(\frac{g_{m1} R_S R_D}{1 + g_{m1} R_s + g_{m2} R_s - g_{m1} g_{m2} R_S R_D} \right)^2 \end{array} \right]$$

$$NF = \frac{\overline{V_{n,out}^2}}{A^2} \frac{1}{4kTR_S} = 1 + \frac{1}{R_S R_D} \left(\frac{1 + (g_{m1} + g_{m2}) R_S)}{g_{m1}} \right)^2 + \frac{\gamma g_{m1}}{R_S} \left(\frac{1 + g_{m2} R_S}{g_{m1}} \right)^2 + \frac{\gamma g_{m2}}{R_S} (R_S)^2$$

$$= 1 + \frac{1}{g_{m1}^2 R_S R_D} (1 + (g_{m1} + g_{m2}) R_S))^2 + \frac{\gamma}{g_{m1} R_S} (1 + g_{m2} R_S)^2 + \gamma g_{m2} R_S$$

(a) If $R_D \to \infty$, $NF = 1 + \frac{\gamma}{g_{m1} R_S} (1 + g_{m2} R_S)^2 + \gamma g_{m2} R_S$

This means that the noise generated from R_D goes to zero since the value of R_D is too big. In other words, the noise generated at R_D cannot flow to generate noise voltage because the resistance at the output node is infinity.

(b) If $g_{m2} \to 0$, $NF = 1 + \frac{1}{g_{m1}^2 R_S R_D} (1 + g_{m1} R_S)^2 + \frac{\gamma}{g_{m1} R_S}$

As $g_{m2} \to 0$, the noise contribution of M2 becomes zero. The positive feedback generated by M2 no longer exists and the noise contribution from M1 and R_D also decreases as positive feedback becomes weaker.

2.(a)

W2 can be found by simulations. To fit the resonance frequency of the drain of M1, W2 should be 8um.

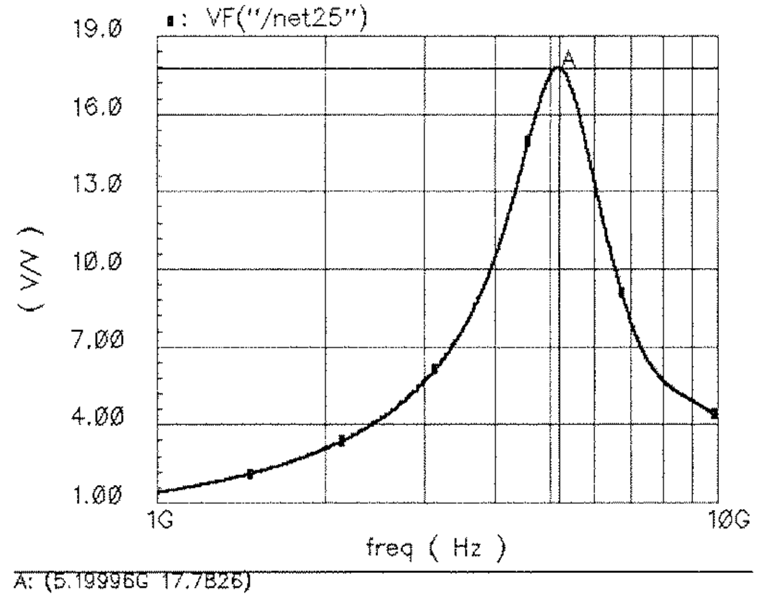

Figure 3. Gain of the First Stage @ Resonance Frequency (5.2GHz)

2.(b)

As shown in figure 4, transconductances and output resistances are obtained from Cadence DC operating point.

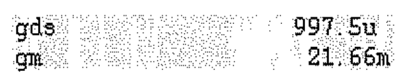

Figure 4. DC operating point of M1

With this information, the input resistance and the output resistance can be calculated.

$$R_{in} = \frac{r_{ds} + R_p}{1 + g_{m1} r_{ds}} = \frac{1002.5 + 1306.9}{1 + 0.02166 \times 1002.5} = 101.67\Omega$$

$$R_{out} = R_p \parallel (r_{ds} + R_S + g_{m1} r_{ds} R_S) = 1306 \parallel (1002.5 + 50 + 1085.7) = 810.78\Omega$$

The total gain is

$$A = \frac{R_{in}}{R_S + R_{in}} \times A_v = \frac{101.67}{151.67} \times 17.7826 = 11.92$$

The output noise voltage can be calculated as

$$\overline{V_{n,out}^2} = 4kT\left[R_S A^2 + \gamma g_{m1} \left(\frac{R_p r_{ds}}{R_S + R_p + (1 + g_{m1} R_S) r_{ds}} \right)^2 + \frac{1}{R_p} R_{out}^2 \right].$$

Thus NF is described as below.

$$NF = \frac{\overline{V_{n,out}^2}}{A^2}\frac{1}{4kTR_S} = 1 + \frac{\gamma g_{m1}}{R_S A^2}\left(\frac{R_p r_{ds}}{R_S + R_p + (1+g_{m1}R_S)r_{ds}}\right)^2 + \frac{1}{R_p R_S A^2}R_{out}^2$$

$NF \approx 1 + \gamma 0.32678 + 0.09 = 1.41678 = 1.513dB$, if we assume $\gamma = 1$.

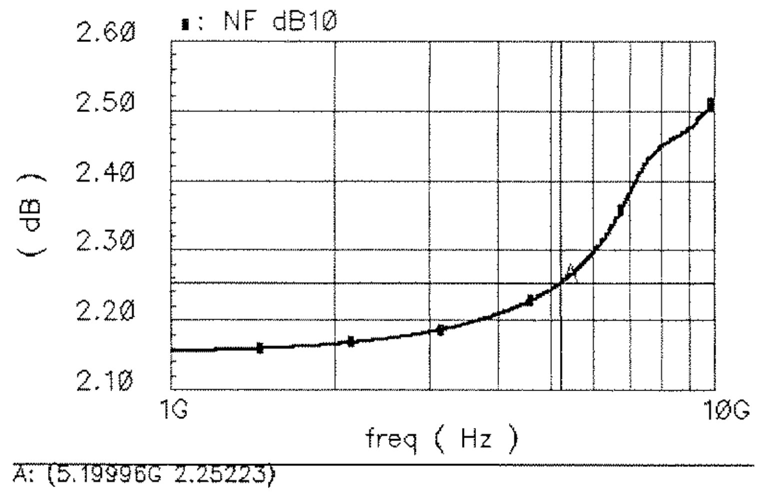

Figure 5. First stage NF simulation result

NF is obtained using S parameter analysis in Spectre as shown in figure 5. The simulation result is 2.25dB, which is 0.7dB higher than hand calculation result. This discrepancy might be due to the wrong assumption, $\gamma = 1$. In short channel devices, γ can be even larger than 2.

/PORT0	rn	1.15682e-16	59.54
/M0	id	6.39955e-17	32.94
/R1	rn	1.41732e-17	7.29
/M7	id	3.69697e-19	0.19
/R2	rn	8.63595e-20	0.04

Figure 6. Noise Contribution Table for the First Stage

Figure 6 shows the noise contribution of each element. M0 represents M1 of the first stage. M2 represents M2 of the second stage. R1 and R2 are Rp of L1 and L2 respectively. This table shows that noise from M1 is underestimated in hand calculation. Therefore, $\gamma = 2$ is more accurate value than using $\gamma = 1$ in this case.

2.(c)

Figure 7 depicts the total NF of the two stage amplifier. The simulation shows that NF=2.435dB at 5.2GHz. This value is very similar to NF of the first stage. This means that noise contribution of the second stage is very small.

The signal is amplified by the first stage whose gain is over 17. Since the signal is already amplified when it arrives at the input of the second stage, the additional noise generated from the second stage is very small compared to the signal.

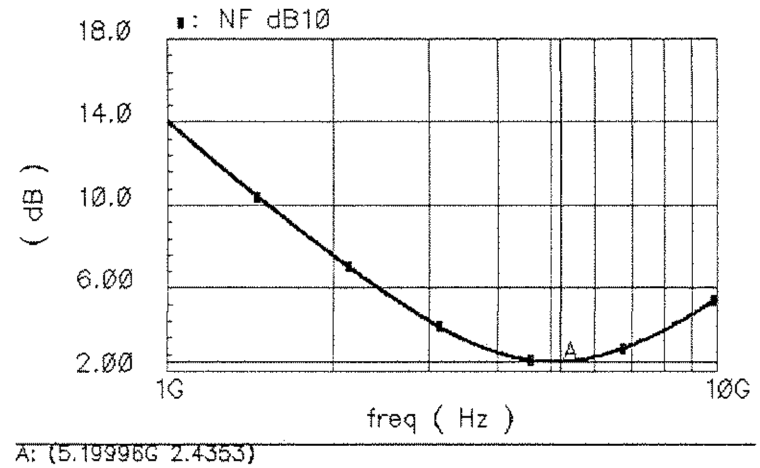

Figure 7. Total NF simulation result