EE215C

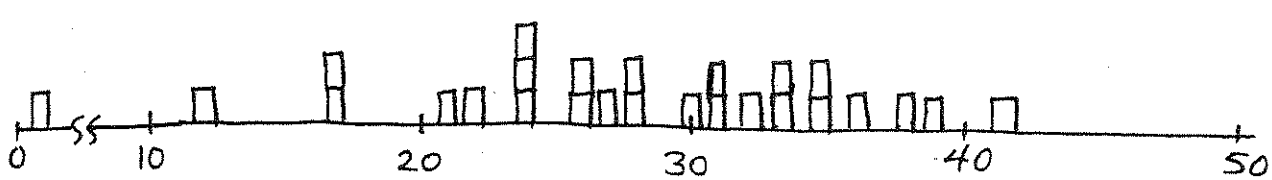
Midterm Exam
Winter 2009

Name: Solutions

Time Limit: 2 Hours
Open Book, Open Notes

1. 10
2. 15
3. 15
4. 10

Total: 50



Avg. = 27/50

1. Consider the cascade of two nonlinear stages. The first stage is described by $y = \alpha_1 x + \alpha_3 x^3$ and the second by $y = \beta_1 x + \beta_3 x^3$.

5 (a) Determine the overall input 1-dB compression point of the cascade in terms of the 1-dB compression points of each stage.

5 (b) Determine the output 1-dB compression point of the cascade in terms of α_j and β_j.

(a)

$$A_{-1dB,1} = \sqrt{0.145 \left| \frac{\alpha_1}{\alpha_3} \right|} \quad \text{(Stage 1)}$$

$$A_{-1dB,2} = \sqrt{0.145 \left| \frac{\beta_1}{\beta_3} \right|} \quad \text{(Stage 2)} \qquad \boxed{1}$$

After cascading: $y = \beta_1 (\alpha_1 x + \alpha_3 x^3) + \beta_3 (\alpha_1 x + \alpha_3 x^3)^3$

$$\Rightarrow y = (\beta_1 \alpha_1) x + (\beta_1 \alpha_3 + \beta_3 \alpha_1^3) x^3 + \cdots$$

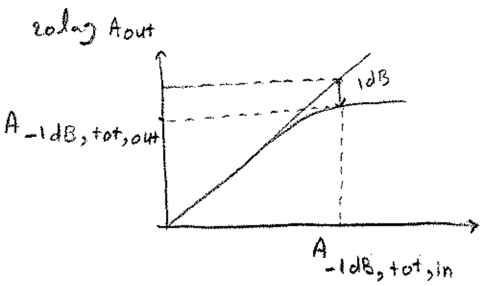
$$\Rightarrow A_{-1dB,tot} = \sqrt{0.145 \left| \frac{\alpha_1 \beta_1}{\beta_1 \alpha_3 + \beta_3 \alpha_1^3} \right|} \Rightarrow \frac{1}{A^2_{-1dB,tot}} = \frac{1}{0.145} \cdot \left[\frac{\alpha_3}{\alpha_1} + \alpha_1^2 \frac{\beta_3}{\beta_1} \right] \quad \boxed{2}$$

$$\Rightarrow \frac{1}{A^2_{-1dB,tot}} = \frac{1}{A^2_{-1dB,1}} + \frac{\alpha_1^2}{A^2_{-1dB,2}} \quad \boxed{2}$$

(b)

$$A_{-1dB,tot,in} = \sqrt{0.145 \left| \frac{\alpha_1 \beta_1}{\beta_1 \alpha_3 + \beta_3 \alpha_1^3} \right|} \quad \text{at the input} \quad \boxed{3}$$

ideal gain $= \alpha_1 \beta_1$



$$20 \log (\alpha_1 \beta_1 A_{-1dB,tot,in}) - 1\,dB = 20 \log (A_{-1dB,tot,out})$$

$$\Rightarrow 20 \log \left(\frac{A_{-1dB,tot,out}}{\alpha_1 \beta_1 A_{-1dB,tot,in}} \right) = -1\,dB \quad \boxed{2}$$

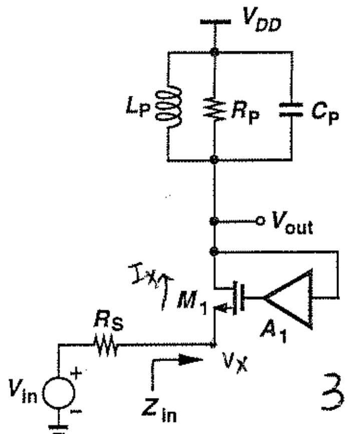
$$\Rightarrow A_{-1dB,tot,out} = 10^{-1/20} \cdot \alpha_1 \beta_1 \sqrt{0.145 \left| \frac{\alpha_1 \beta_1}{\beta_1 \alpha_3 + \beta_3 \alpha_1^3} \right|} \quad \text{at the output}$$

2. Consider the circuits shown below, where an ideal voltage amplifier with a gain of A_1 returns a fraction of the output to the gate of the input transistor. Neglect channel-length modulation, body effect, and other capacitances.

5 (a) Compute the input impedance. 3 Can the real part become negative at any frequency? 2 Explain in detail.

5 (b) Compute the noise figure with respect to a source resistance of R_S at the resonance frequency of the tank.

5 (c) Simplify the noise figure expression if the input is matched at the resonance frequency of the tank.



a) $g_m(A_1 V_{out} - V_x) = -V_{out}\left(\frac{1}{sL_p} + \frac{1}{R_p} + sC_p\right)$

$$V_{out} = \frac{g_m V_x}{g_m A_1 + \frac{1}{sL_p} + \frac{1}{R_p} + sC_p}$$

$$I_x = V_{out}\left(\frac{1}{sL_p} + \frac{1}{R_p} + sC_p\right)$$

$$= \frac{g_m V_x}{g_m A_1 + \frac{1}{sL_p} + \frac{1}{R_p} + sC_p}\left(\frac{1}{sL_p} + \frac{1}{R_p} + sC_p\right)$$

3 $$Z_{in} = \frac{V_x}{I_x} = \frac{g_m A_1 + \frac{1}{sL_p} + \frac{1}{R_p} + sC_p}{g_m\left(\frac{1}{sL_p} + \frac{1}{R_p} + sC_p\right)} \quad \#$$

$$\text{real-}Z_{in} = \frac{1}{g_m}\,\frac{\left(g_m A_1 + \frac{1}{R_p}\right)\frac{1}{R_p} + \left(\omega C_p - \frac{1}{\omega L_p}\right)^2}{\left(\frac{1}{R_p}\right)^2 + \left(\omega C_p - \frac{1}{\omega L_p}\right)^2}$$

(if $A_1 > 0$)

2 $\because \left(\omega C_p - \frac{1}{\omega L_p}\right)^2 > 0$ at all frequencies, $\Rightarrow$ real-$Z_{in} > 0$

real-Z_{in} never becomes negative. #

5 b) Output noise from R_S: $4kTR_S A_{total}^2$

Output noise from R_p: $\frac{4kT}{R_p} R_{out}^2$

Output noise from M_1: $\frac{4kT\gamma}{g_m}\left(\frac{g_m}{1+g_mR_S}\right)^2 R_{out}^2$

$$\frac{R_{out}^2}{A_{total}^2} = \frac{1}{G_m^2} = \left(\frac{1+g_mR_S}{g_m}\right)^2$$

$$F = \frac{\text{output noise}/A_{total}^2}{4kTR_S} = 1 + \frac{1}{R_pR_S}\frac{R_{out}^2}{A_{total}^2} + \frac{\gamma}{g_mR_S}\left(\frac{g_m}{1+g_mR_S}\right)^2\frac{R_{out}^2}{A_{total}^2}$$

$$= 1 + \frac{1}{R_pR_S}\left(\frac{1+g_mR_S}{g_m}\right)^2 + \frac{\gamma}{g_mR_S} = 1 + \frac{1}{R_pg_m^2R_S} + \frac{R_S}{R_p} + \frac{\gamma}{g_mR_S} \quad \#$$

$NF = 10\log(F)$ #

5 c) If $Z_{in} = R_S$ at resonate $f \Rightarrow Z_{in} = \frac{1}{g_m}(1 + g_mA_1R_p) = R_S$ (2)

$$\Rightarrow F = 1 + \frac{1}{R_pg_m(1+g_mA_1R_p)} + \frac{1+g_mA_1R_p}{g_mR_p} + \frac{\gamma}{1+g_mA_1R_p} \quad \#$$

($\because$ NF is dominated by the term with γ in this case

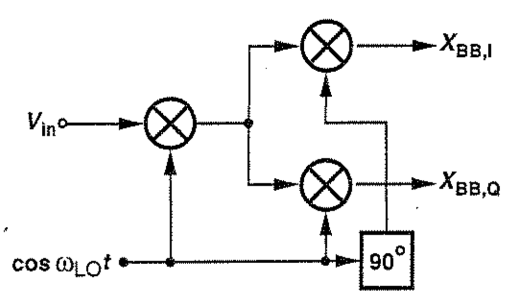
$\therefore A_1$ helps reduce the noise.)

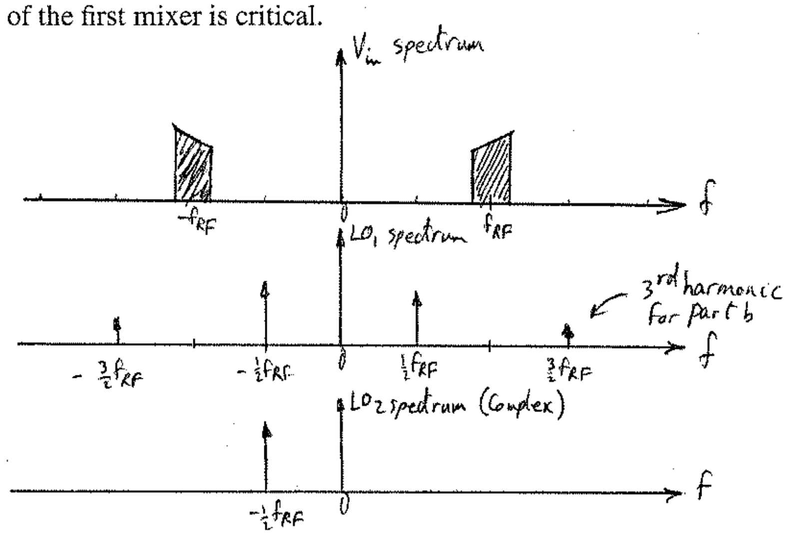
3. The receiver architecture shown below operates with an LO frequency equal to half of the input carrier frequency. Assume the input has an asymmetrically-modulated spectrum.

5 (a) Plot the IF and baseband spectra assuming ideal mixers.

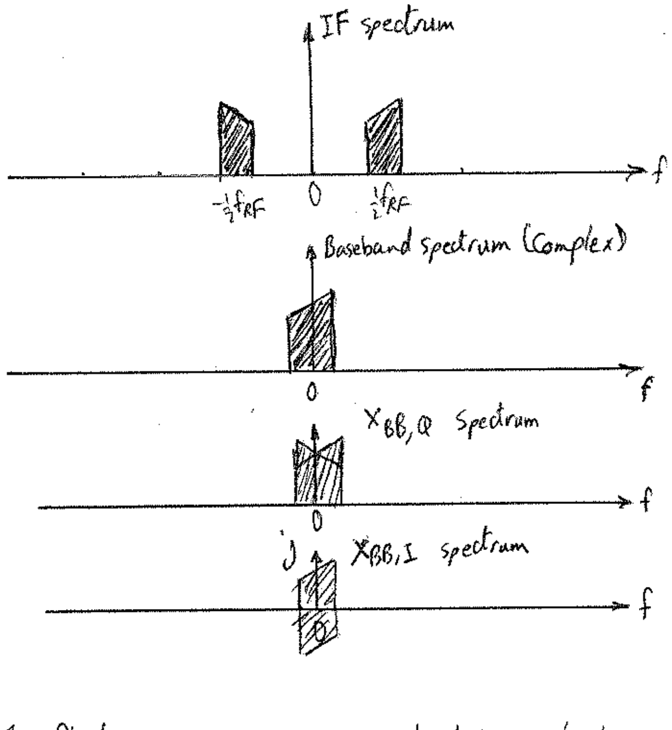
5 (b) Now suppose the first mixer experiences hard switching and introduces the third harmonic of the LO, i.e., it mixes the RF input with an LO of the form $\cos \omega_{LO}t + \alpha \cos(3\omega_{LO}t)$. Plot the IF spectrum and explain whether or not this architecture operates well with asymmetrically-modulated inputs.

5 (c) Explain why the flicker noise at the input of the first mixer is critical.

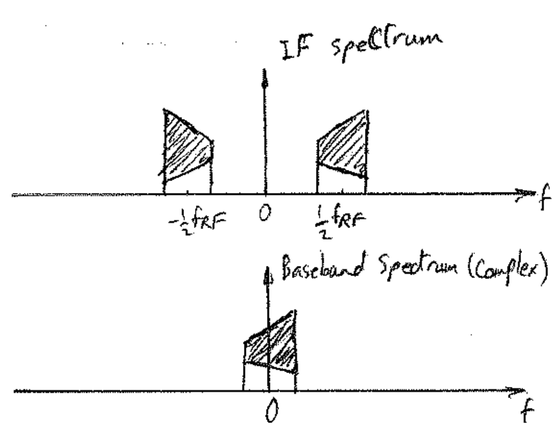




a)



b)



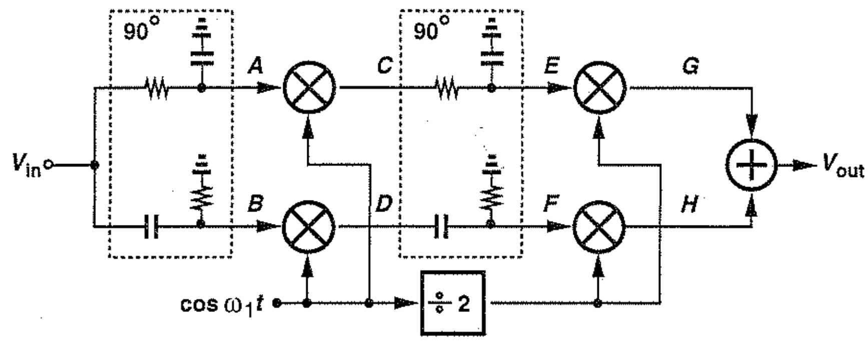
So this architectures will not work well with asymmetrically-modulated inputs in the presence of first LO third harmonic. This is because the third harmonic folds the signal on itself corrupting the information.

c) The flicker noise is present at DC which is of equal distance from the first LO as the signal. The first LO will thus upconvert it to IF and this corrupts the signal. Also, this is in the very beginning of the RF receiver where noise is so critical and gets amplified.

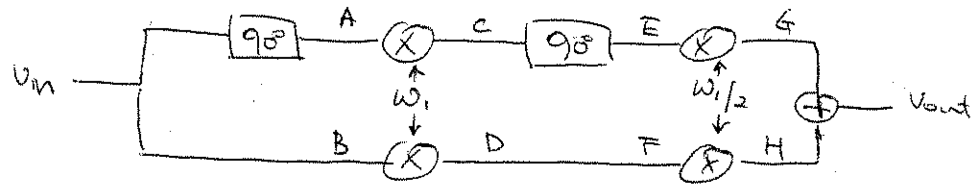
4. An engineer constructs the receiver shown below and chooses ω_1 such that the second IF is zero. (Only the I branch is shown for the sake of simplicity.) The RC-CR networks are inserted to perform a 90° phase shift at the frequency of interest.

5 (a) Does the receiver reject the image? Explain with the aid of spectra at various points in the chain.

5 (b) Does the receiver reject the image with respect to the third harmonic of the LO, i.e., the mirror image of ω_{in} with respect to $3\omega_1$? Explain with the aid of spectra at various points in the chain.



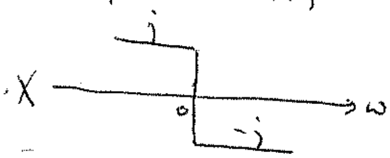
4. Simplified CKT Diagram



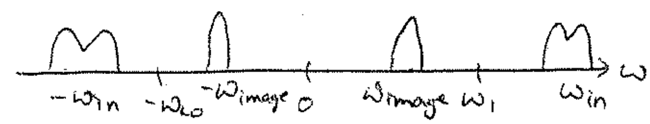
i) for zero IF,

$$\omega_{in} = \omega_1 + \frac{\omega_1}{2} = \frac{3}{2}\omega_1$$

ii) 90° phase shift,

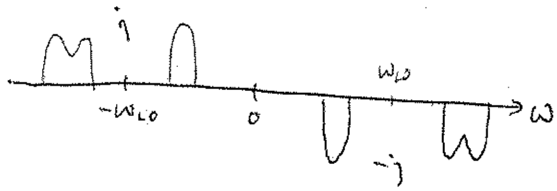


iii) X_{in}



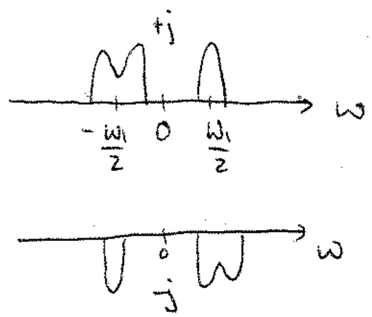
a)

X_A

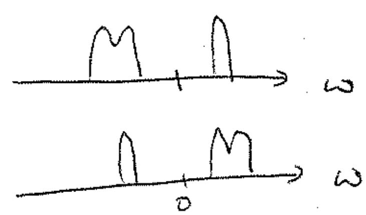


$X_B = X_{in}$

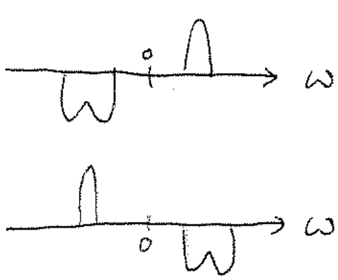
X_C



X_D

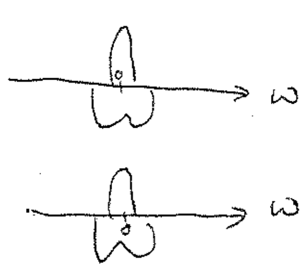


X_E

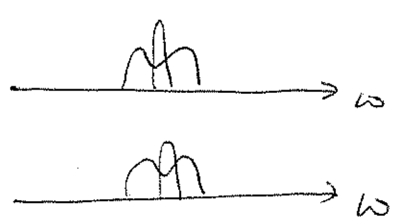


$X_F = X_D$

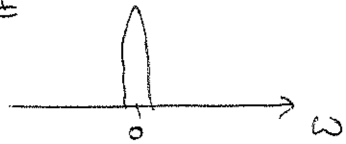
X_G



X_H

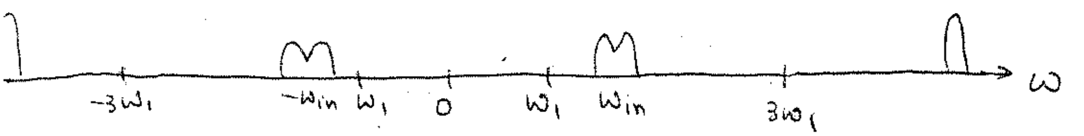


$\therefore$ X_{out}

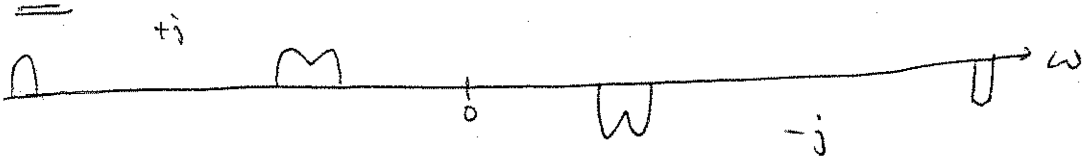


The signal will be rejected,
However if $\omega_{in} = \frac{1}{2}\omega_1$,
the image will be rejected, but only to some extent, due to the magnitude mismatch between the upper / lower branch.
(RC-CR : 90° phase shift for all freq., same Amplitude response only at 1/RC)

(b) X_{in}

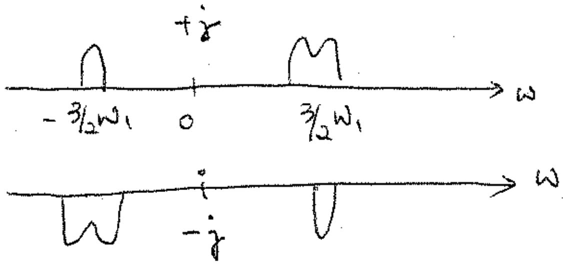


X_A

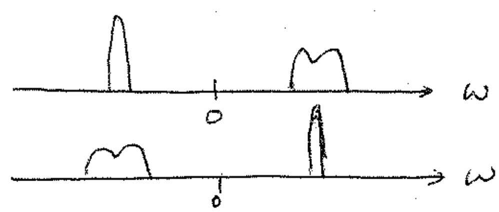


$X_B = X_{in}$,

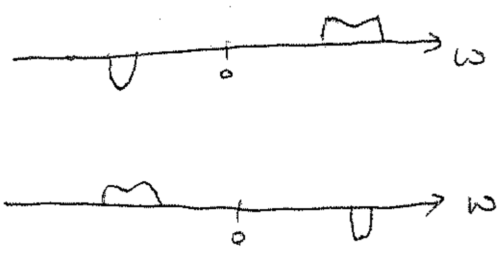
X_C



X_D

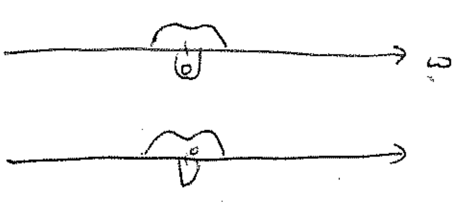


X_E

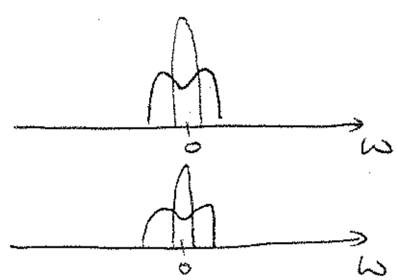


$X_F = X_D$

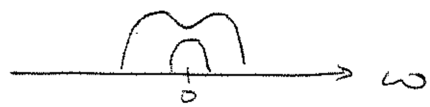
X_G



X_H



$\therefore$ $\underline{X_{out}}$



The image would be attenuated (not fully rejected because of the frequency dependent magnitude mismatch between upper/lower branch.