EE215C

Midterm Exam
Winter 2009

Name: Solutions

Time Limit: 2 Hours
Open Book, Open Notes

1. 10
2. 15
3. 15
4. 10
Total: 50

Avg. = $\frac{27}{50}$
1. Consider the cascade of two nonlinear stages. The first stage is described by \( y = \alpha_1 x + \alpha_3 x^3 \) and the second by \( y = \beta_1 x + \beta_3 x^3 \).

(a) Determine the overall input 1-dB compression point of the cascade in terms of the 1-dB compression points of each stage.

(b) Determine the output 1-dB compression point of the cascade in terms of \( \alpha_j \) and \( \beta_j \).

\[
A_{-1\text{dB},1} = \sqrt{0.145 \left| \frac{\alpha_1}{\beta_1} \right|} \quad (\text{Stage 1})
\]

\[
A_{-1\text{dB},2} = \sqrt{0.145 \left| \frac{\beta_1}{\beta_3} \right|} \quad (\text{Stage 2})
\]

After cascading,
\[ y = \beta_1 (\alpha_1 x + \alpha_3 x^3) + \beta_3 (\alpha_1 x + \alpha_3 x^3)^3 \]

\[ y = (\beta_1 \alpha_1) x + (\beta_1 \alpha_3 + \beta_3 \alpha_3^3) x^3 + \cdots \]

\[
\Rightarrow A_{-1\text{dB},\text{tot},\text{in}} = \sqrt{0.145 \left| \frac{\alpha_1 \beta_1}{\beta_1 \alpha_3 + \beta_3 \alpha_3^3} \right|} \quad \Rightarrow \frac{1}{2} = \frac{1}{A_{-1\text{dB},\text{tot},\text{in}}} \end{align}
\]

\[
A_{-1\text{dB},\text{tot},\text{out}} = \sqrt{0.145 \left| \frac{\alpha_1 \beta_1}{\beta_1 \alpha_3 + \beta_3 \alpha_3^3} \right|} \quad \text{at the input}
\]

\[
20 \log_{10} \left( \frac{A_{\text{in}}}{A_{\text{out}}} \right) = -1 \text{ dB}
\]

\[
20 \log_{10} \left( \frac{A_{-1\text{dB},\text{tot},\text{out}}}{A_{-1\text{dB},\text{tot},\text{in}}} \right) = -1 \text{ dB}
\]

\[
A_{-1\text{dB},\text{tot},\text{out}} = 10 \cdot \alpha_1 \beta_1 \sqrt{0.145 \left| \frac{\alpha_1 \beta_1}{\beta_1 \alpha_3 + \beta_3 \alpha_3^3} \right|} \quad \text{at the output}
\]
2. Consider the circuits shown below, where an ideal voltage amplifier with a gain of $A_1$ returns a fraction of the output to the gate of the input transistor. Neglect channel-length modulation, body effect, and other capacitances.

(a) Compute the input impedance. Can the real part become negative at any frequency? Explain in detail.

(b) Compute the noise figure with respect to a source resistance of $R_s$ at the resonance frequency of the tank.

(c) Simplify the noise figure expression if the input is matched at the resonance frequency of the tank.

\[
V_{	ext{out}} = \frac{V_{x} \left( \frac{1}{sL} + \frac{1}{r_p} + sC_p \right)}{g_m A_1 + \frac{1}{sL} + \frac{1}{r_p} + sC_p}
\]

\[
I_x = V_{x} \left( \frac{1}{sL} + \frac{1}{r_p} + sC_p \right)
\]

\[
Z_{in} = \frac{V_{x}}{I_x} = \frac{g_m V_{x}}{g_m A_1 + \frac{1}{sL} + \frac{1}{r_p} + sC_p}
\]

\[
\text{real-} Z_{in} = \frac{1}{g_m} \left( \frac{g_m A_1 + \frac{1}{r_p}}{\left( \frac{1}{r_p} \right)^2 + (w_C - \frac{1}{w_d})^2} \right)
\]

\[
\Rightarrow (w_C - \frac{1}{w_d})^2 > 0 \quad \text{at all frequencies,} \quad \Rightarrow \text{real-} Z_{in} > 0
\]

\[NF = 10 \log (T) \]

5b) Output noise from $R_s$: $\frac{4kT R_s}{g_m}$

Output noise from $r_p$: $\frac{4kT}{r_p}$

Output noise from $M_1$: $\frac{4kT}{g_m} \left( \frac{g_m}{1+g_m R_p} \right)^2 R_p$

\[
F = \frac{\text{output noise}}{\text{noise total}} = 1 + \frac{1}{g_m R_s} \frac{R_{out}}{\text{noise total}} + \frac{1}{g_m} \left( \frac{g_m}{1+g_m R_p} \right)^2 \frac{R_p}{\text{noise total}}
\]

\[NF = 10 \log (F) \&
\]

5c) If $Z_{in} = R_s$ at resonance $f \Rightarrow Z_{in} = \frac{1}{g_m} (1 + \frac{g_m}{r_p}) = R_s$ (2)

\[
F = 1 + \frac{1}{g_m R_s} \left( \frac{1+g_m}{r_p} \right) + \frac{1}{g_m} \frac{R_p}{r_p} + \frac{1}{g_m} \frac{R_p}{r_p}
\]

$\Rightarrow NF$ is dominated by the term with $\frac{1}{g_m}$ in this case.$\&$

$p_1$ helps reduce the noise.
3. The receiver architecture shown below operates with an LO frequency equal to half of the input carrier frequency. Assume the input has an asymmetrical-modulated spectrum.

(a) Plot the IF and baseband spectra assuming ideal mixers.

(b) Now suppose the first mixer experiences hard switching and introduces the third harmonic of the LO, i.e., it mixes the RF input with an LO of the form \( \cos \omega_{LO} t + a \cos(3\omega_{LO} t) \). Plot the IF spectrum and explain whether or not this architecture operates well with asymmetrical-modulated inputs.

(c) Explain why the flicker noise at the input of the first mixer is critical.

---

So this architecture will not work well with asymmetrical-modulated inputs in the presence of first LO third harmonic. This is because the third harmonic folds the signal on itself corrupting the information.

c) The flicker noise is present at DC which is at equal distance from the first LO as the signal. The first LO will thus upconvert it to IF and this corrupts the signal. Also, this is in the very beginning of the RF receiver where noise is so critical and gets amplified.
4. An engineer constructs the receiver shown below and chooses \( \omega_1 \) such that the second IF is zero. (Only the I branch is shown for the sake of simplicity.) The RC-CR networks are inserted to perform a 90° phase shift at the frequency of interest.

(a) Does the receiver reject the image? Explain with the aid of spectra at various points in the chain.

(b) Does the receiver reject the image with respect to the third harmonic of the LO, i.e., the mirror image of \( \omega_{in} \) with respect to 3\( \omega_1 \)? Explain with the aid of spectra at various points in the chain.
4. Simplified Circuit Diagram

1) For zero IF,
   \[ \omega_{in} = \omega_1 + \frac{\omega_1}{2} = \frac{3}{2} \omega_1 \]

2) 90° phase shift,
   \[ X \rightarrow 90° \rightarrow \omega \]

a) \[ X_A = \omega \]

b) \[ X_B = X_{in} \]

c) \[ X_C \]

d) \[ X_D \]

e) \[ X_E = X_D \]

f) \[ X_F \]

g) \[ X_H \]
The signal will be rejected, however if $\omega_n = \frac{1}{2} \omega_1$, the image will be rejected, but only to some extent, due to the mismatch between the upper/lower branch.

RC-CR: 90° phase shift for all freq., same amplitude response only at $1/RC$
The image would be attenuated (not fully rejected) because of the frequency-dependent magnitude mismatch between upper/lower branch.