The fundamentals of MHD turbulence in the limit Rm << 1

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Contents

The influence of a uniform DC field

- 3D ➲ strong anisotropy: $\tau_J = \dfrac{\rho}{\sigma B^2}$

The influence of the Hartmann walls

- Quasi-2D after: τ_{2D}
- Hartmann damping time: τ_H

Our experiments to support these ideas

Concluding remarks

Any initially quasi-isotropic eddy elongates in the B-direction

$$\frac{l_{//}}{l_{\perp}} \approx \left(\frac{t}{\tau_J}\right)^{1/2}$$

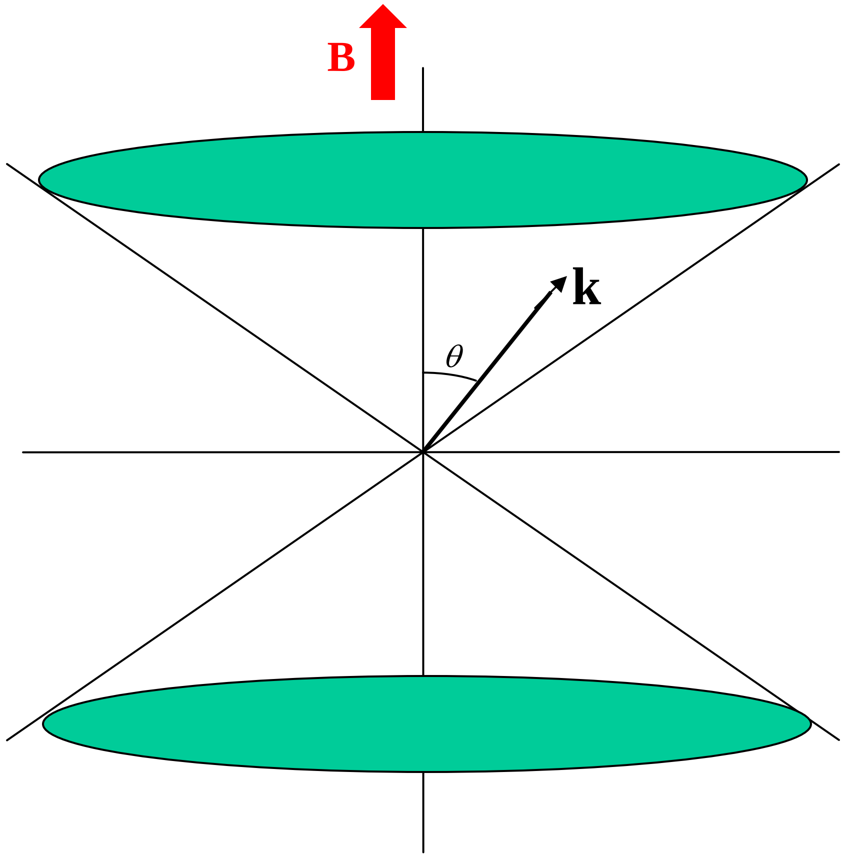
Three explanations:

In the Fourier space (AMSF, J. de Méca., 1979)
The electromagnetic diffusion (S&M, JFM, 1982)
Invariance of the angular momentum (Davidson, JFM, 1997)

Notice: the linear theory (Moffatt, JFM, 1967) predicts :

$$\langle u_{//}^2 \rangle = 2 \langle u_{\perp}^2 \rangle$$

First explanation (within the Fourier space)



$$\frac{1}{\rho} j \times B \rightarrow -\frac{\sigma B^2}{\rho} \cos^2\theta \, \hat{u}(\mathbf{k}, t)$$

$$\frac{\partial \bar{E}}{\partial t} = -\frac{\sigma B^2}{\rho} \cos^2\bar{\theta}(\mathrm{t}) \, \bar{E}$$

$$\tau_J(t) = \frac{\rho}{\sigma B^2 \cos^2\bar{\theta}(t)} \approx t$$

$$\frac{l_{//}}{l_{\perp}} \approx \left[\cos\bar{\theta}(t)\right]^{-1} = \left(\frac{\sigma B^2 t}{\rho}\right)^{1/2}$$

(Alemany, Moreau, Sulem, Frisch, J. de Méca., 1979)

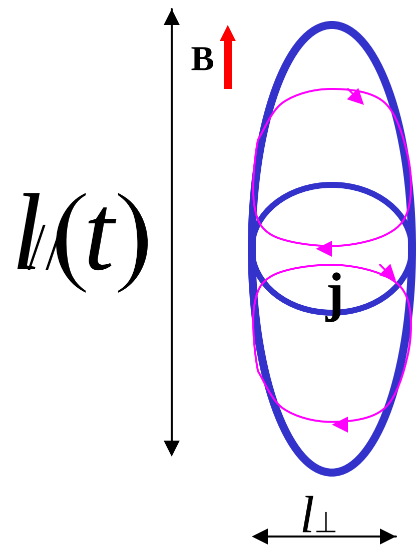
Diffusion in the B-direction

$$(B.\nabla)\mathbf{u} + \eta\Delta_\perp \mathbf{b} = 0 \rightarrow \mathbf{b} = -\mu\sigma B \Delta_\perp^{-1}\frac{\partial \mathbf{u}}{\partial z}$$

$$\frac{1}{\rho}\mathbf{j}\times\mathbf{B} = \frac{1}{\mu\rho}(\nabla\times\mathbf{b})\times\mathbf{B} \rightarrow -\frac{\sigma B^2}{\rho}\Delta_\perp^{-1}\frac{\partial^2 \mathbf{u}}{\partial z^2}$$

Then: $$\frac{\partial \mathbf{u}}{\partial t} = \frac{\sigma B^2}{\rho} l_\perp^2 \frac{\partial^2 \mathbf{u}}{\partial z^2} = D\frac{\partial^2 \mathbf{u}}{\partial z^2}$$

And: $$l_{//} \approx \sqrt{Dt} \approx l_\perp \sqrt{\frac{\sigma B^2 t}{\rho}} \approx l_\perp \left(\frac{t}{\tau_J}\right)^{1/2}$$



(Sommeria & Moreau, JFM, 1982)

Invariance of angular momentum

$$\bar{E} = \int u^2 dV \;\rightarrow\; \frac{\partial \bar{E}}{\partial t} = -\left(\frac{l_\perp}{l_{//}}\right)^2 \frac{\bar{E}}{\tau}$$

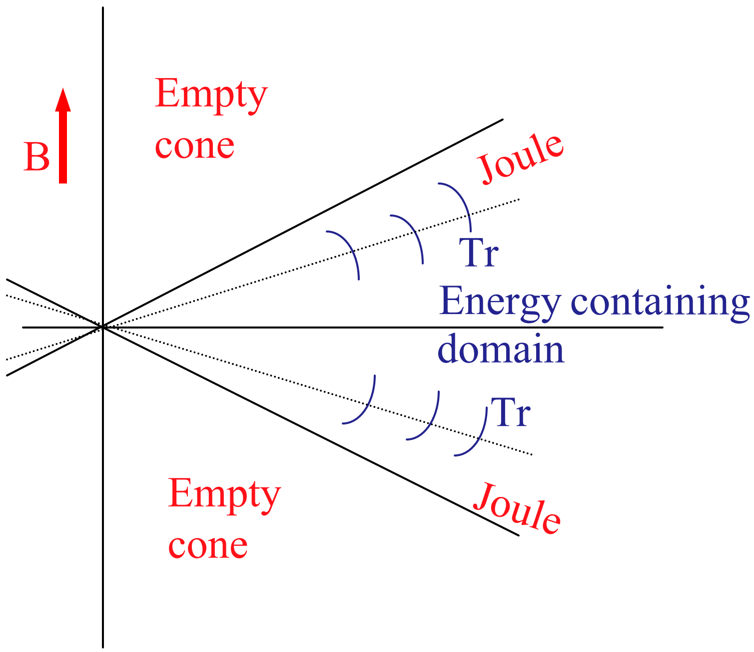
$$\mathbf{H} = \int \mathbf{r} \times \mathbf{u}\, dV \;\rightarrow\; \boxed{\frac{\partial \mathbf{H}_{//}}{\partial t} = 0} \quad \frac{\partial \mathbf{H}_\perp}{\partial t} = -\frac{\mathbf{H}_\perp}{4\tau}$$

The invariance of $H_{//}$ becomes compatible with the decrease of E and $H_\perp$ as soon as

$$\boxed{\left(\frac{l_\perp}{l_{//}}\right)^2 \approx \left(\frac{t}{\tau}\right)^{-1}}$$

(Davidson, JFM, 1995 and 1997)

Return in the Fourier space



$$\frac{\partial \bar{E}}{\partial t} \approx -\frac{\bar{E}}{t} \rightarrow \bar{E} \approx t^{-n}$$

Assume a quasi-steady equilibrium between Joule dissipation and inertia:

$$\tau_{tr} \approx \tau_J(t)$$

1. Globally:

$$\frac{l_\perp}{\bar{E}^{1/2}} \approx t \rightarrow \bar{E} \approx t^{-2}$$

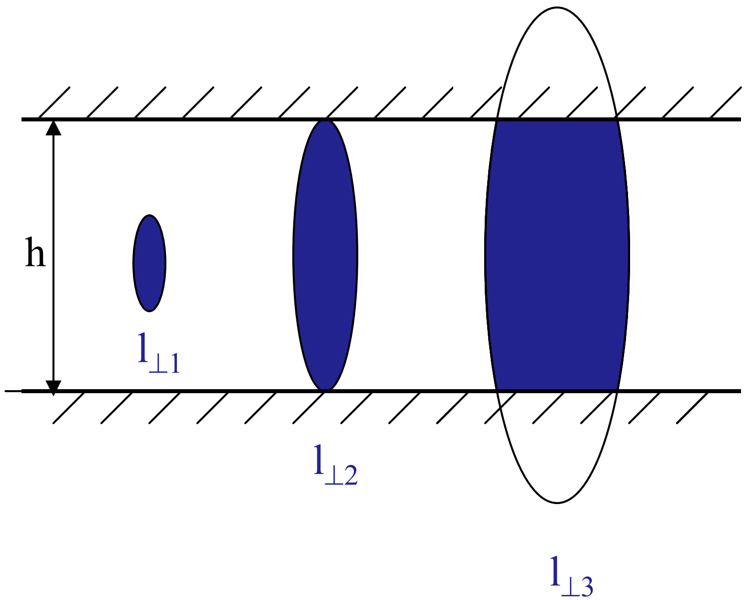
2. Locally:

$$\frac{1/k}{\sqrt{kE(k)}} \approx t \rightarrow \boxed{E(k,t) \approx t^{-2}k^{-3}}$$

No cascade at all!

(well confirmed by experiments: AMSF, 1979 and EGLW, 1998)

First influence of Ha-walls



Since: $$l_{//} \approx l_{\perp} \left(\frac{\sigma B^2 t}{\rho} \right)^{1/2}$$

for each $l_{\perp}$ there exists a time $$\tau_{2D} \approx \tau_J \frac{h^2}{l_{\perp}^2}$$

such that $$l_{//} \approx h$$

Example: h=2cm,
$\rho=10^4$ kg m^{-3}, $\sigma=10^6\ \Omega^{-1}\ m^{-1}$

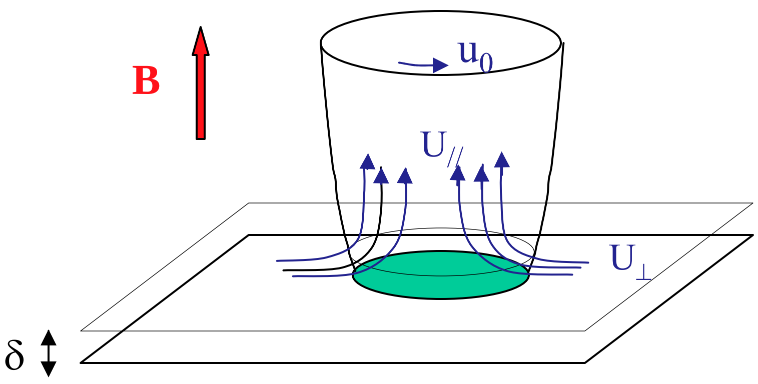
B(Tesla)	**0.1**	**1**	**10**
τ_J **(s)**	**1**	$\mathbf{10^{-2}}$	$\mathbf{10^{-4}}$
τ_{2D} **(s, 1cm)**	**4**	$\mathbf{4\ 10^{-2}}$	$\mathbf{4\ 10^{-4}}$
τ_{2D} **(s, 4cm)**	**0.25**	$\mathbf{0.25\ 10^{-2}}$	$\mathbf{0.25\ 10^{-4}}$

2. Suppression of $u_{//}$

A sort of Ekman pumping takes place
within the Hartmann layer at the scale of each vortex

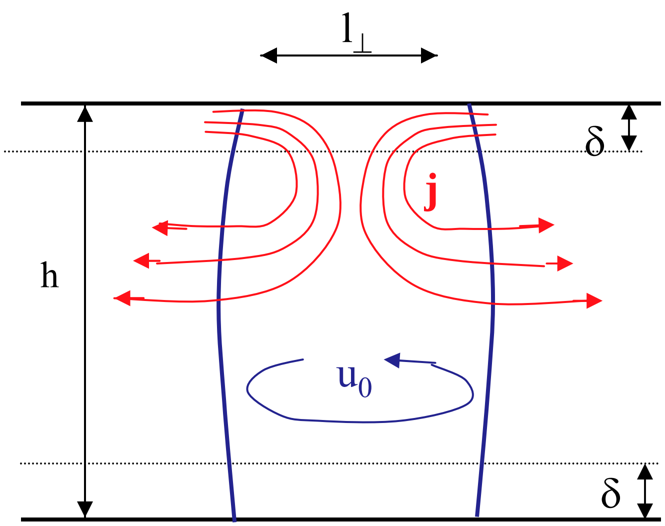
$$\delta p_0 \approx \rho u_0^2 \approx \rho \nu \frac{u_\perp}{\delta^2} l_\perp \rightarrow u_\perp \approx u_0 \frac{u_0 l_\perp}{\nu} \frac{\delta^2}{l_\perp^2}$$

$$u_{//} \approx u_\perp \frac{\delta}{l_\perp} \approx u_0 \frac{u_0 l_\perp}{\nu} \frac{\delta^3}{l_\perp^3} \rightarrow \boxed{\frac{u_{//}}{u_0} \approx \frac{\mathrm{Re}}{Ha^3} \frac{h^3}{l_\perp^3}}$$



This Ekman pumping is also responsible for a « barrel shaping » of the eddies (Bühler, JFM, 1996; Ziganov & Thess, JFM, 1998; Potherat, Sommeria & Moreau, JFM, 2000)

3. The Hartmann damping



(Sommeria & Moreau, JFM, 1982)

Theorem of kinetic energy applied to a quasi-2D eddy:

$$\frac{\partial}{\partial t}\int \rho u^2\, dV \approx -\int \frac{j^2}{\sigma}\, dV$$

$$\rightarrow \frac{h}{\tau_H}\rho u_0^2 \approx \int_0^\infty \sigma B^2 u_0^2 e^{-2z/\delta}\, dz$$

$$\rightarrow \boxed{\tau_H = n\frac{h}{B}\sqrt{\frac{\rho}{\sigma\nu}} = nHa\tau_J}$$

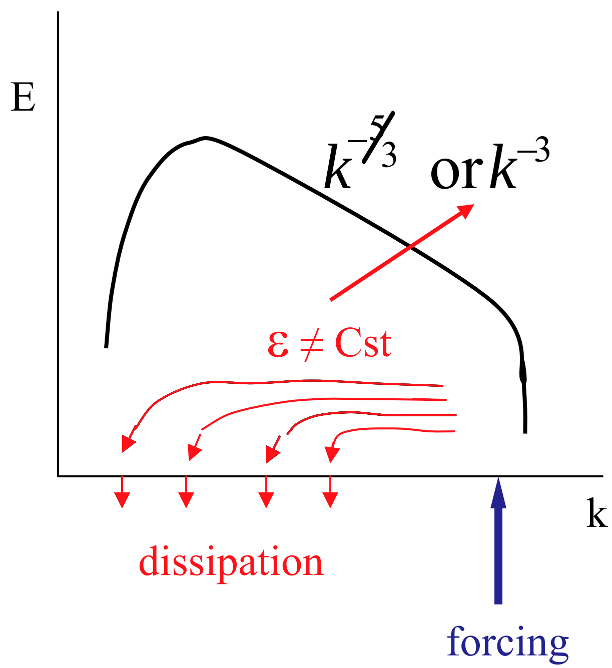
Main consequence: the SM-82 equation

$$\boxed{\frac{\partial \mathbf{u}_\perp}{\partial t} + (\mathbf{u}_\perp\cdot\nabla)\mathbf{u}_\perp = -\frac{1}{\rho}\nabla_\perp p - \frac{\mathbf{u}_\perp}{\tau_H}}$$

Recently refined by Potherat: PSM, JFM, 2000

4. Inverse energy cascade

As soon as $\tau_{2D} << \tau_{tu}$, the turbulence is 2D in the planes perpendicular to B, which are highly correlated



Then the energy cascade is inverse: $\tau_{tu} \approx \frac{l_\perp}{u_\perp}$

Hartmann damping (relevant or not): $\tau_H \approx 2\frac{h}{B}\sqrt{\frac{\rho}{\sigma\nu}}$

Local equilibrium within the inertial/damping range:

$$\tau_{tr}(k) \approx \frac{1/k}{\sqrt{kE(k)}} \approx \frac{1}{\sqrt{k^3E(k)}} \approx \tau_H$$

Then: $E \approx k^{-3}$

(Kolesnikov & Tsinober, IANauk, 1974; Lielausis, AER, 1975; Sommeria, JFM, 1986)

The key time scales in a liquid metal exp. with a moderate or a high magnetic field

$h = l_\perp = 1\ cm$

$u_\perp = 1\ cm s^{-1}$

$\rho \approx 10^4\ kg m^{-3}$

$\sigma \approx 10^6\ \Omega^{-1} m^{-1}$

$\nu \approx 10^{-7}\ m^2 s^{-1}$

$$\frac{\tau_{tu}}{\tau_H} = \frac{Ha}{Re} \frac{l_\perp^2}{h^2}$$

	B = 0.1 T	B = 5 T
$Ha = Bh\sqrt{\frac{\sigma}{\rho\nu}}$	30	1500
$\tau_J = \frac{\rho}{\sigma B^2}$	1 s	0.4 10^{-3} s
$\tau_{2D} = \frac{\rho}{\sigma B^2} \frac{h^2}{l_\perp^2}$	1 s	0.4 10^{-3} s
$\tau_{tu} = \frac{l_\perp}{u_\perp}$	1 s	1 s
$\tau_H = 2\frac{h}{B}\sqrt{\frac{\rho}{\sigma\nu}}$	60 s	1 s
$\tau_\nu = \frac{l^2}{\nu}$	10^3 s	10^3 s

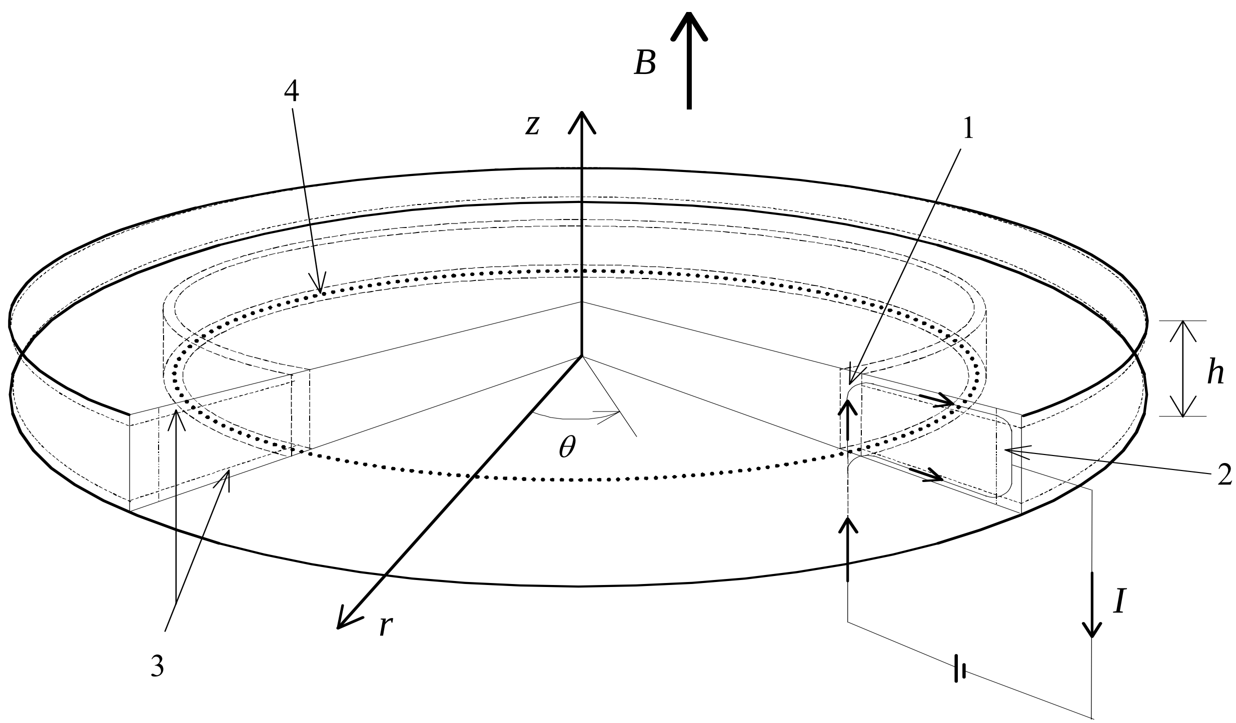
Our experiments to support these ideas

In complement to the experiments performed in **Riga** (Lielausis, AER, 1975), in **Purdue** (BL, PoF, 1967. DL, JFM, 1971), in **Beer-Sheva** (BG, JFM, 1979; SZB ExpFl, 1986), in **Karlsruh**e (MGMB, JFM, 2000), and in **Dresden** (EGLW, AIAA,1998), 3 original experiments were performed in **Grenoble**, specifically to observe and measure the **basic properties of MHD turbulence**:

Alemany (1970's): a 2 m vertical cylindrical tank in a coil ($B \leq 0.25$ T),
no Hartmann walls: anisotropy ($U_{//} > U_{\perp}$)
Sommeria (1980's): a 2 cm trunk of cylinder in a coil ($B = 0.1 - 0.2$ T),
characterization of the 2D dynamics ($U_{//} << U_{\perp}$),
MATUR (1990's): a MHD quasi-2D turbulent shear flow
a- Alboussière et al. (ETFS, 1999): moderate magnetic field ($B = 0.17$ T)
b- Messadek (JFM, 2002): high magnetic field ($B = 0.5$ to 6 T)

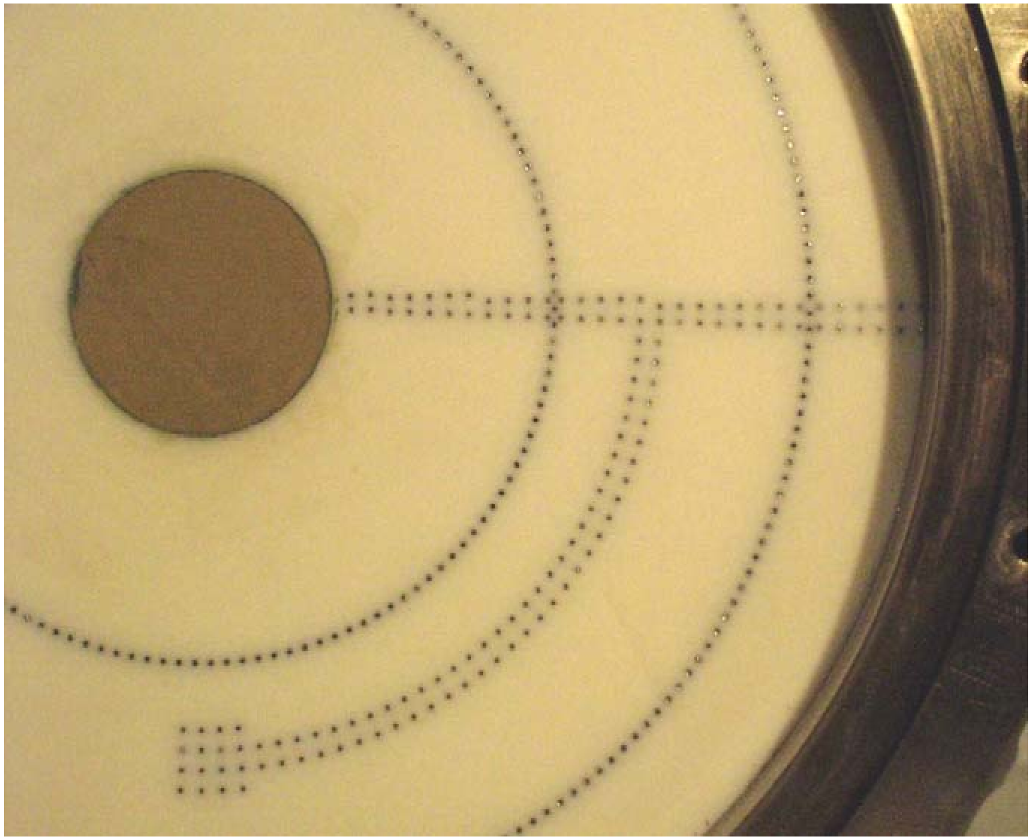
The MATUR cell

(driving mechanism & diagnostic)

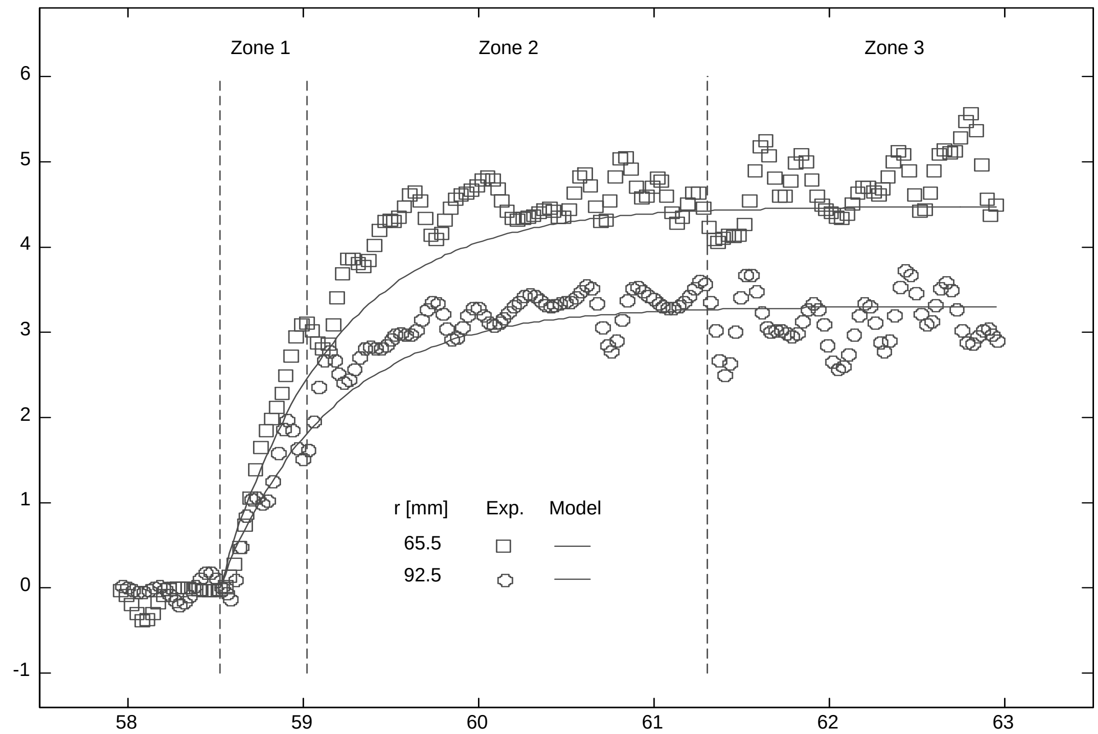


The MATUR cell

(view of the bottom wall)



MATUR: Spin up, instability and generation of turbulence



U_θ(cm/s) versus time (s) for B=4 T and I=15 A

Laminar model:

$$U(r,t)=\frac{I}{4\pi r\sqrt{\rho\nu\sigma}}\left(1-e^{-\frac{\nu\,Ha}{h^2}t}\right)$$

Mean velocity profiles measured in MATUR

QuickTime™ et un décompresseur
TIFF (LZW) sont requis pour visualiser
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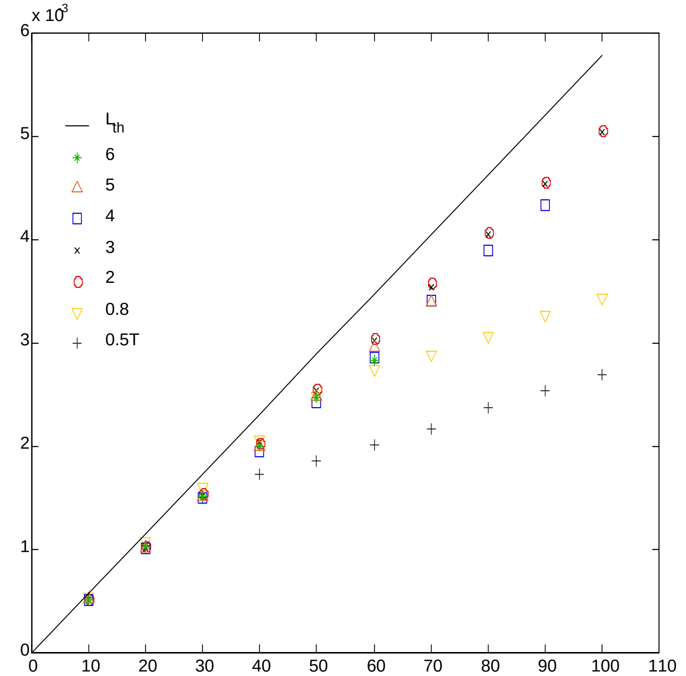
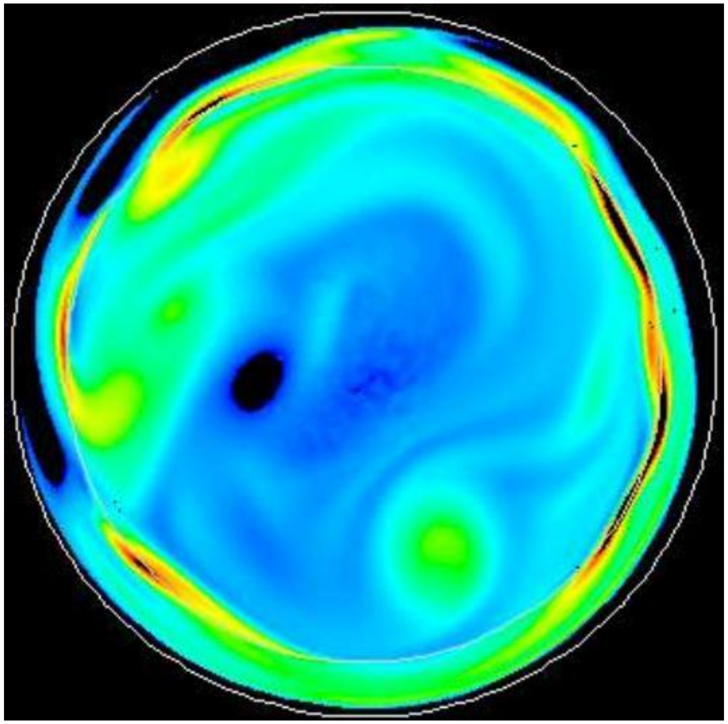


Angular momentum L (m^4s^{-1}) versus I (Amp)
for different magnetic fields (0.5 to 6 Tesla)

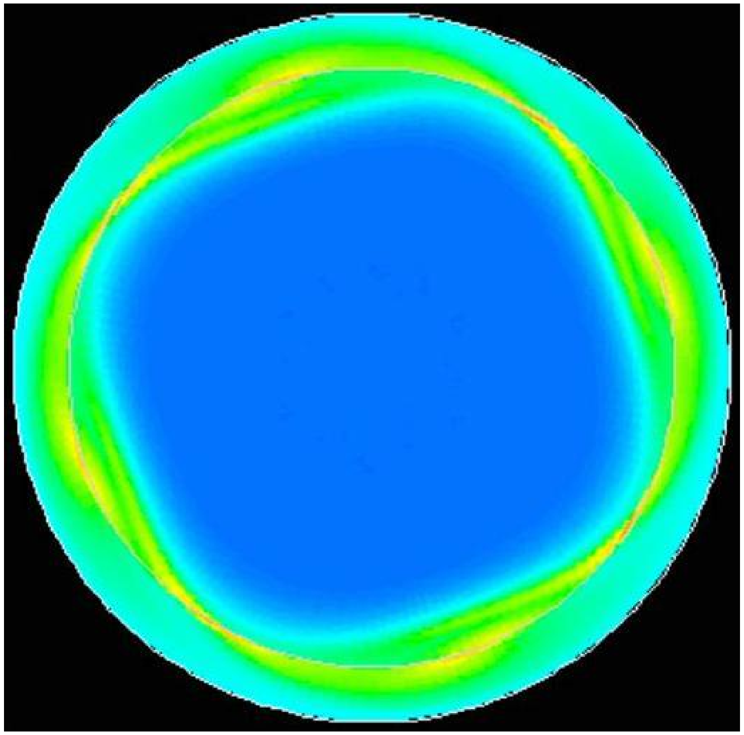
$$L_{th} = \int_0^R r^2 U_\theta(r)\,dr = \frac{IR^2}{4\pi\sqrt{\rho\nu\sigma}}$$

A refined version of SM-82 for moderate magnetic fields: PSM, JFM, 2000

$$\mathbf{v} = \frac{1}{h}\int_{-h/2}^{+h/2} \mathbf{u}\,dz \quad \rightarrow \quad \frac{\partial \mathbf{v}}{\partial t} + (\mathbf{v}\cdot\nabla)\mathbf{v} = -\frac{1}{\rho}\nabla p - \frac{\mathbf{v}}{\tau_H} - (\text{AP-NL term})$$



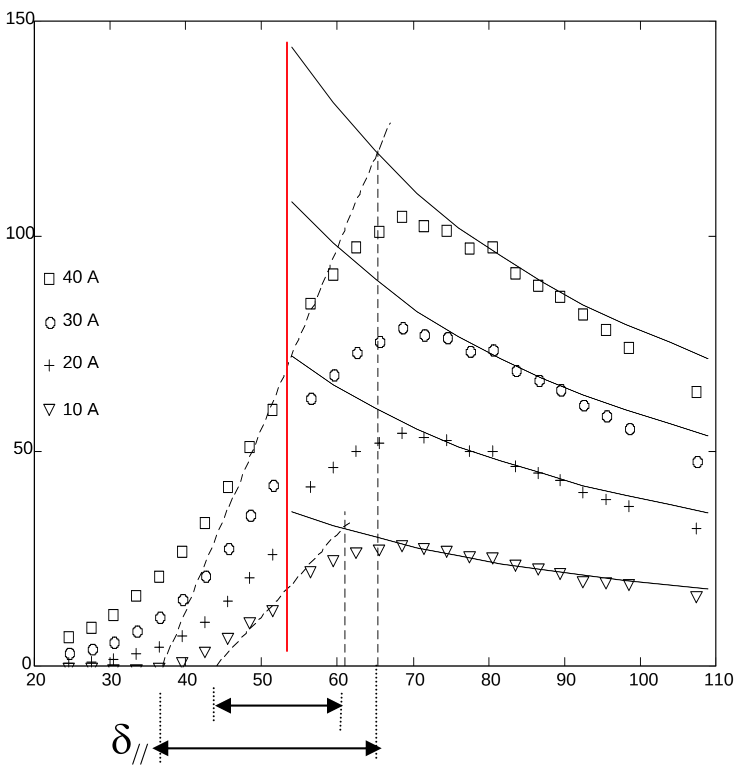
SM-82, I=30A, B=0.17T, t=70s $\approx 3\tau_H$
Instantaneous vorticity



PSM-2000, I=30A, B=0.17T, t=70s $\approx 3\tau_H$
Instantaneous vorticity

The thickness of the free shear layer does not vary as $Ha^{-1/2}$

Laminar: $U = \dfrac{I}{4\pi r\sqrt{\sigma\rho\nu}}$



Typical mean velocity profiles
for B = 3 T (Ha=900); r_{inj} = 54 mm

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Best fit with the measurements: $\dfrac{\delta_{//}}{h} = \left(\dfrac{Re}{Ha}\right)^{1/2.3}$

Radial distribution of the RMS of the fluctuations u_θ (left) and u_r (right) for B= 3T (top) and 5 T (bottom)

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Time spectra (left) and spatial spectra (right) of u_θ at r = 68.5 mm

The peaks exhibit the large scale structures. The k^{-3} log-law exhibits the damped inverse cascade

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Velocity (left) and vorticity (right) fields, reconstructed with a Taylor assumption, exhibiting the number of large coherent structures

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The number of large scale structures varies as

$$N_S \approx 80\left(\frac{Ha}{\mathrm{Re}}\right)^{2.5}$$

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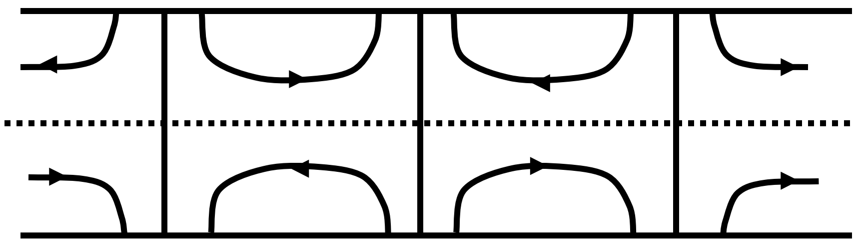
Concluding remarks

1. Significant progresses on the <u>understanding</u> of MHD turbulence

2. Next challenges: **- non-uniform magnetic fields**
- non-negligible Rm



3. No numerical model available to compute actual flows