Efficient Computation of Convolutional Decoder Reliability Without a Cyclic Redundancy Check

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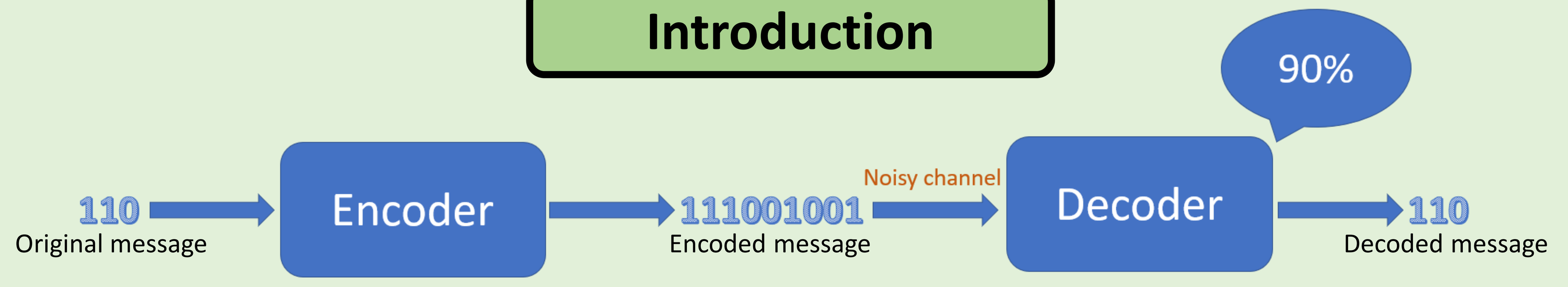
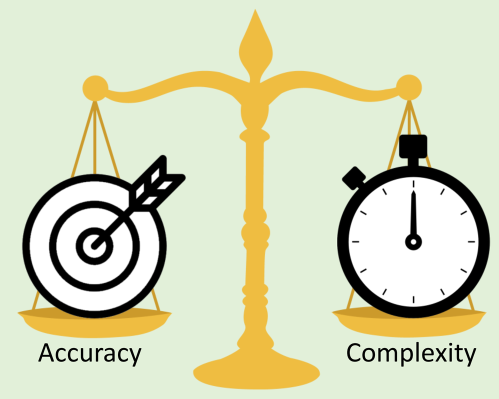
Introduction



- Some error detecting codes, such as cyclic redundancy checks (CRCs), have significant overhead for short message lengths.
- Three metrics to compute decoder reliability (likelihood of correct decoding) without overhead:
  - Reliability output Viterbi algorithm (ROVA) calculates the probability of correct decoding.
  - Accumulated information density (AID) sums the information density of each bit in the codeword.
  - Codeword information density (CID) finds the information density of the entire codeword.
- Goal: determine the best metric in terms of accuracy and complexity and develop a model for that metric to be able to control the error rate.

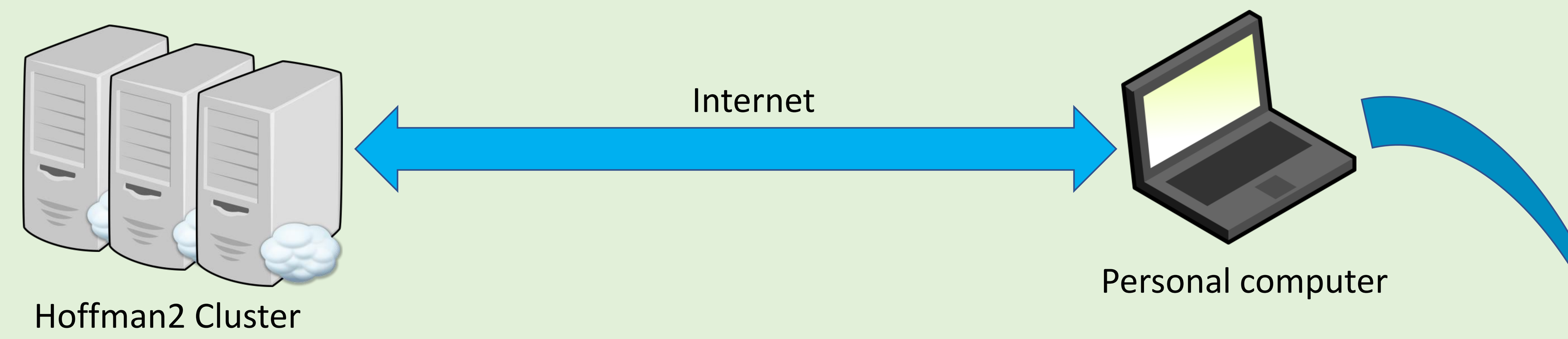
Metric	Complexity	Accuracy
ROVA	High	High
AID	Low	???
CID	Medium	???

Table 1. Preliminary information on the complexity and accuracy of ROVA, AID, and CID.



Tradeoff between accuracy and complexity.

Materials and Methods



- Data gathering:
  - Run simulations to generate histograms of ROVA, AID, and CID values.
  - Keep track of correctly and incorrectly decoded codewords.
  - Measure time taken to complete simulations using ROVA, AID, and CID separately.
- Data analysis:
  - Separation between correct and incorrect histograms tells us effectiveness of metric.
  - Determine best metric from analysis.
- Modeling:
  - Design a model to fit the distribution of the chosen metric and find:
    - Probability of undetected error P(E).
    - Probability of correctness P(C).
    - Probability of rejection P(NACK).

MATLAB
C++

Results: Data Gathering

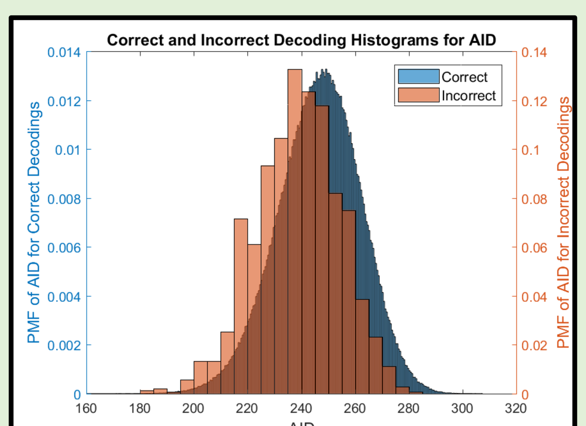


Fig. 1. Histogram of AID values of correct and incorrect decodings. The lack of separation between blue and orange curves indicates that AID is a poor metric for detecting errors.

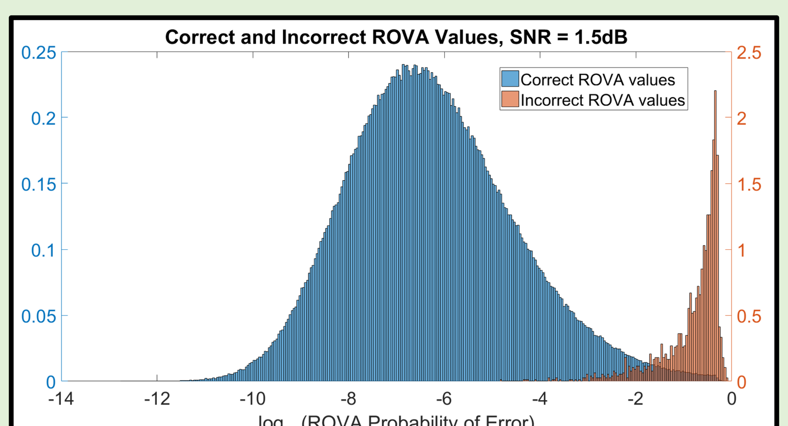


Fig. 2. Histogram of the log of ROVA error probabilities of correct and incorrect decodings. The clear separation between blue and orange curves indicates that ROVA is a good metric for detecting errors. The ROVA probability of error is the complement of ROVA.

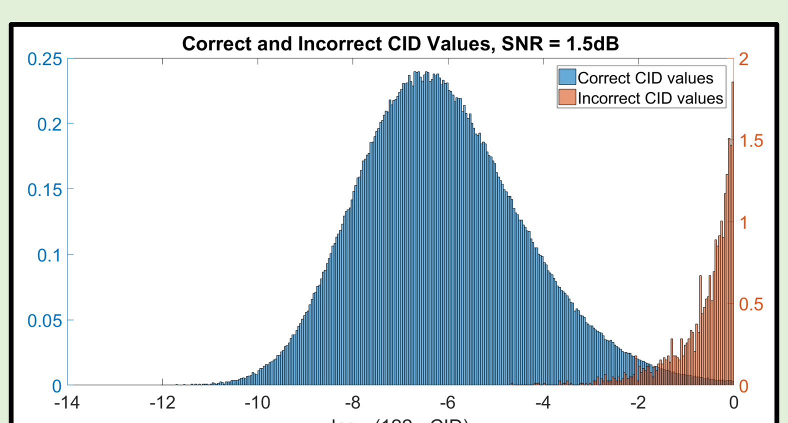


Fig. 3. Histogram of the log of CID values of correct and incorrect decodings. The clear separation between blue and orange curves indicates that CID is a good metric for detecting errors.

Although the shape of the distributions of ROVA and CID are similar, they are not the same. However, a one-to-one transformation exists between ROVA and CID, given by

$$\text{CID} = \log_2(\text{ROVA}) + \text{Message length}$$

Because a one-to-one transformation exists, ROVA and CID have an identical capability to detect errors despite having different distributions.

Results: Data Analysis

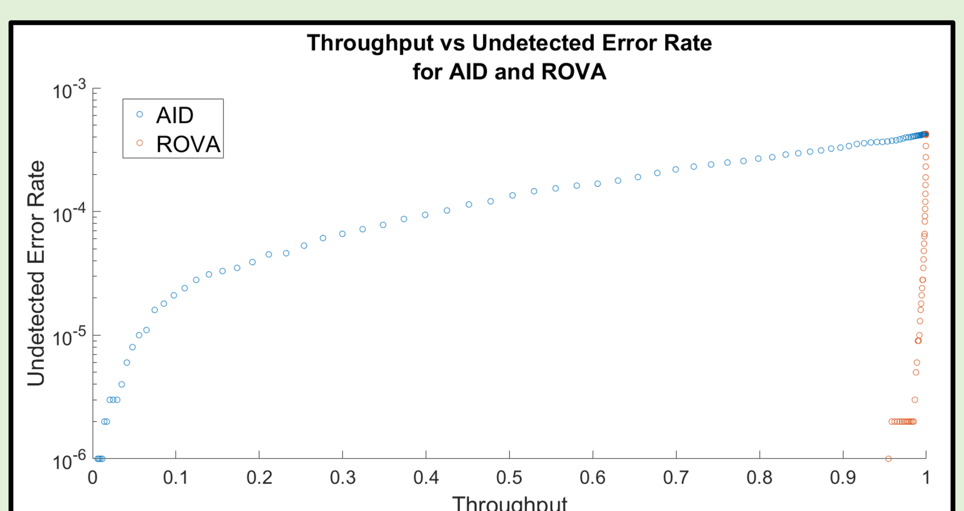


Fig. 4. Graph of throughput vs. undetected error rate for AID and ROVA. A good metric should have high throughput for a low undetected error rate, which ROVA offers and AID does not.

Throughput: ratio of accepted messages to total received messages.
Undetected error rate: ratio of errors not detected by the metric to total received messages.

	AID	CID	ROVA
Number of operations	3.0×10^6	2.7×10^6	1.8×10^7
Runtime (ms) (10000 decodings)	1.7×10^5	1.3×10^5	2.7×10^5

Table 2. Complexity and time comparisons of ROVA, AID, and CID. Note that the number of operations does not include the operations required for Viterbi decoding. CID has a much better runtime than ROVA and requires fewer operations.

Results: Modeling

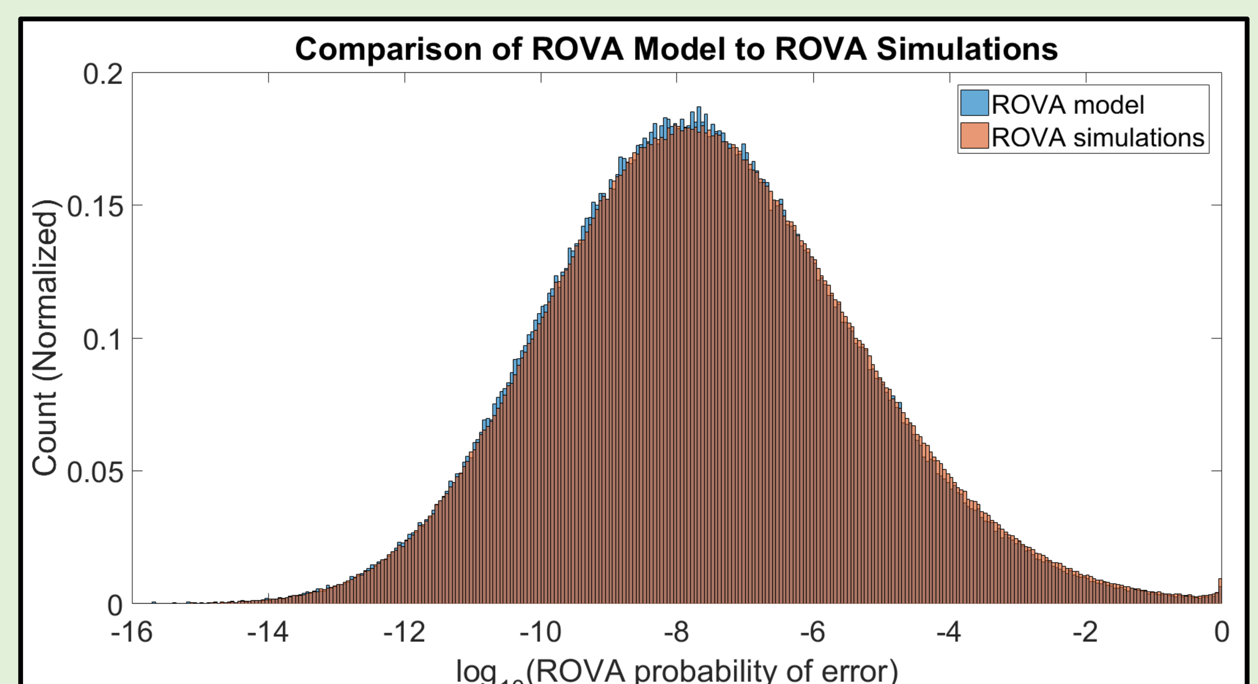


Fig. 5. Histogram of the log of ROVA values with model overlaid. The model matches the simulation extremely well. The model was generated with the following equation:

$$\text{ROVA} = \frac{P(\hat{x}^n) f_{Y|X}(y^n|\hat{x}^n)}{\sum_{x^n \in \mathcal{C}} P(x^n) f_{Y|X}(y^n|x^n)} = \frac{1}{1+E}$$

$P(\hat{x}^n)$ = probability that the decoded codeword $\hat{x}^n$ is the original codeword

$f_{Y|X}(y^n|\hat{x}^n)$ = conditional probability that y^n was received assuming $\hat{x}^n$ is the original codeword

where C is the set of all valid codewords and E is a sum of log-normal random variables.

Conclusion

- Although AID can be computed the fastest, it is too inaccurate to be used as a metric for decoder reliability.
- ROVA is extremely accurate, but it has the highest complexity, so it is impractical as a metric.
- The distributions for ROVA and CID are related by a one-to-one transformation, so CID has the same accuracy as ROVA.
- In addition to having equivalent accuracy to ROVA, CID is faster than ROVA, making it the best metric of the three to assess decoder reliability.

Future Work

- Problem: finding the distribution of CID relies on specific information about the encoder that can currently only be obtained empirically.
  - Goal: find a method to obtain this encoder information analytically.
- Problem: generating enough CID values to find P(C), P(E), P(NACK) for various noise levels takes a long time.
  - Goal: use the model to quickly obtain these values.

References

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