Information-Reduced Carrier Synchronization of BPSK and QPSK Using Soft Decision Feedback

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***Abstract*— This paper addresses the carrier-phase estimation problem under low SNR conditions as are typical of turbo- and LDPC-coded applications. In [1], [2] closed-loop carrier synchronization schemes for error-correction coded BPSK and QPSK modulation were proposed that were based on feeding back hard data decisions at the input of the loop, the purpose being to remove the modulation prior to attempting to track the carrier phase as opposed to the more conventional decision-feedback schemes that incorporate such feedback inside the loop. In this paper, we consider an alternative approach wherein the soft information from the iterative decoder of turbo or LDPC codes is instead used as the feedback.**

I. Introduction

In recent years there has been an ever-increasing interest in highly power efficient error-correction codes such as turbo codes and low density parity check (LDPC) codes. These codes approach the Shannon channel capacity of the system and operate at very low symbol signal-to-noise ratios (SNRs) thus necessitating the need for carrier synchronization schemes that likewise operate efficiently at these SNRs.

A significant research effort is underway in the area of joint decoding and carrier phase estimation. As clearly explained by Noels *et al.* [3] two somewhat distinct groups of joint decoding and synchronization algorithms have evolved. The first of these approach the problem by modifing iterative detection/decoding algorithms and/or graphs to include parameter estimation. A partial list of work on this approach includes [4]–[9].

Of particular interest has been the work of Colavolpe *et al.* [9] where phase-tracking processing nodes were introduced in the iterative decoding graph. Dauwels *et al.* also investigated [7] specially adapted message-passing update rules. Howard *et al.* [8] proposed a pilotless modulation technique for turbo-coded differential 8-PSK modulation which uses 35 iterations to compensate a $\pi/8$ phase offset at $E_b/N_o = 4.5$ dB. We also note the work of Nuriyev [6] who has adapted density evolution to evaluate the performance of joint carrier-phase estimation in a pilot-assisted environment.

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The second group of algorithms pass messages between an independent phase estimation block and an essentially unmodified iterative decoder. The resulting architectures are often said to employ *turbo synchronization* [3]. Algorithms of this type can can be found in [10]–[14].

The technique in this paper falls into this second category and has the potentially attractive feature that little modification is required with either the iterative decoder or the carrier recovery block (which consists primarily of a phase-locked loop (PLL)). Specifically, the work leverages the fact that LDPC symbol estimates can 'wipe-off' modulated symbols in a decision directed carrier recovery loop to enhance the carrier information such that a classic residual carrier PLL is able to provide increasingly accurate phase estimates over LDPC iterations. The method incurs a latency penalty (by way of increased iterations) as carrier phase is acquired. However, complexity in terms of system description and area (in the case of a real-time implementation) remains similar to that of state of the art residual carrier recovery techniques currently used for BPSK and QPSK modulation in NASA's deep-space network.

The authors in [10] propose somewhat similar work but have described a phase estimate based on the instantaneous average of an entire block of received symbols. We also note the work of Lottici *et al.* [12] who developed a blind recovery technique for QAM receivers. The work in this paper is also based on blind, or pilotless, operation and we motivate this in part by recalling a result from Anastasopoulos [4] who showed pilotless techniques to be more efficient at lower SNRs where pilot insertion loss is considerable.

The rest of this paper is organized as follows. The next section provides a detailed description of the proposed method. In Section III, we consider BPSK modulation and derive the tracking performance of the PLL in terms of its mean-square phase error when operating in the linear (high loop SNR) region as is typical. In Section IV we illustrate a digital implementation that achieves the same performance as the piecewise constant analog model considered in Section III. Results for QPSK modulation are presented in Section V. Section VI presents numerical results derived from a simulation of the BPSK scheme with a particular LDPC code. Finally, Section VII documents our conclusions.

II. System Description

The notion of iterative information-reduced carrier synchronization for coded binary phase-shift-keyed (BPSK) modulation was first introduced in [1]. The term “information-reduced” alludes to the use of an estimate of the instantaneous data symbol (and thus of the instantaneous phase modulation) to reduce the amount of randomness (and thus the amount of information) in the signal being processed in the carrier synchronizer.

Specifically, in information-reduced carrier-synchronization (IRCS), the reduction of the amount of information is accomplished by attempting to convert the received modulated carrier to an unmodulated carrier (pure tone) before applying it to a phase-tracking loop, in the hope of improving performance. Traditional IRCS systems for synchronization with carrier signals modulated by BPSK include Costas loops, data-aided loops, and demodulation/remodulation loops. The traditional systems are designed to implement various approximations of a closed-loop structure that effects maximum a posteriori (MAP) estimation of phase. The degradation of tracking performance of such a loop in the case of BPSK is represented by a quantity called the “squaring loss”, which is a measure of the degradation of the receiver signal-to-noise (SNR) ratio and is associated with the mean-squared phase error of the loop. In the case of a conventional in-phase/quadrature (I-Q) carrier-tracking loop, the mean-square phase error is a result of signal and noise cross products that are generated in the effort to remove the data modulation from the loop error signal. At low symbol SNR, the squaring loss of an I-Q loop can be severe enough to prevent tracking. Several publications based on this notion have appeared in the literature that include everything from the basic idea and accompanying analysis/performance evaluations [1] to successful application and implementation for specific block and convolutional codes [15], [16].

If the data sequence and its timing were completely known, then a BPSK signal could be converted to a pure tone simply by multiplying the BPSK signal by the data waveform. One could then track the unmodulated carrier with improved performance by use of a phase-locked loop, which does not exhibit squaring loss. Short of complete knowledge of the data waveform and in the presence of noise, the best approximation of a pure tone could be obtained by feeding back an estimate of the data waveform corresponding to tentative decisions on the data symbols. Such feedback is called “decision feedback”.

Decision feedback is used within the traditional loops, but is not used to modify the loop structures. In the proposed IRCS system (see Fig. **??**), decision feedback is introduced at the input terminal of the loop; simultaneously, the structure of the loop would be modified (in the sense that its parameters would be modified) on the basis of the associated change in data-transition statistics in the input. In this scheme, the input signal would be converted to a close approximation of a pure tone, with a resultant improvement in carrier-tracking performance over conventional I-Q loops.

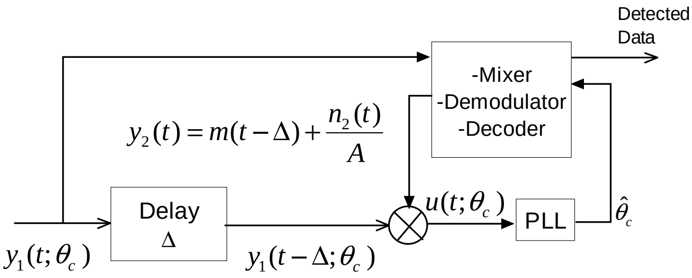


Fig. 1. Analog receiver with Information-Reduced Carrier Synchronizer Using Soft Decision Feedback for BPSK

Although initially available data-waveform estimates are generally of low quality, they can be used to initiate the IRCS process by reducing the number of data transitions at the input. Once phase lock is achieved, the improved phase estimates can be fed back to the data detector, yielding improved symbol estimates for feedback, and thereby achieving even better phase tracking. This iterative process eventually leads to virtual elimination of squaring loss, so that the performance of the system approaches that of a phase-locked loop operating on an unmodulated carrier signal.

III. Tracking Performance for BPSK Modulation

The sample analog receiver shown in Fig. 1 will aid us in the derivation of the proposed carrier synchronization method. An alternative practical implementation of this receiver is detailed in Section IV. Consider an input BPSK modulation of the form

$$y_1(t;\theta_c) = \sqrt{2P}m(t)sin\left(w_c t + \theta_c\right) + n_1(t) \quad (1)$$

where w_c and θ_c are the carrier frequency and phase. Carrier power P is affected by a bandpass AWGN process $n_1(t)$ that can be expressed as

$$n_1(t) = \sqrt{2}\left[N_{c1}(t)cos(w_c t + \theta_c) - N_{s1}(t)sin(w_c t + \theta_c)\right]) \quad (2)$$

where $N_{c1}(t), N_{s1}(t)$ have single-sided noise power spectral density (PSD) equal to N_o and

$$m(t) = \sum_{k=-\infty}^{\infty} d_k p\left(t - kT_s\right)$$

is a baseband modulation with independent, identically distributed (i.i.d.)± 1 data symbols d_k and unit rectangular pulse shape $p(t)$ of duration T_s or a root-raised cosine pulse with zero crossings a multiples of T_s. For simplicity from this point forward we will assume a rectangular pulse shape. Under this assumption matched filters will be equivalent to integrators. The next step is to delay $y_1(t;\theta_c)$ by the decoder delay Δ and multiply it by a normalized (by the signal amplitude A) [1] version of the soft decision feedback signal corresponding to the extrinsic information derived from decoding the LDPC code. This can be modeled as a Gaussian signal [17], [18] , i.e.,

$$y_2(t) = m(t - \Delta) + n_2(t)/A$$

where $n_2(t)$ can be modeled as a piecewise constant base-band noise process, namely,

$$n_2(t) = \sum_{k=-\infty}^{\infty} n_{2k} p\left(t - kT_s - \Delta\right)$$

where n_{2k} are i.i.d. zero mean Gaussian random variables (RVs) with variance σ^2. Over a single iteration (block of input symbols), A and σ^2 are assumed to be fixed. The result of this multiplication, whose intent is to remove the modulation, is

$$\begin{aligned} u(t;\theta_c) &\triangleq y_1(t-\Delta;\theta_c) y_2(t) \\ &= \sqrt{2P} sin(w_c t + \theta_c) \\ &\quad + (\sqrt{(2P)}/A) m(t-\Delta) n_2(t) sin(w_c t + \theta_c) \\ &\quad + \sqrt{2} m(t-\Delta)[N_{c1}(t) cos(w_c t + \theta_c) \\ &\quad - N_{s1}(t) sin(w_c t + \theta_c)] \\ &\quad + (\sqrt{2}/A) n_2(t)[N_{c1}(t) cos(w_c t + \theta_c) \\ &\quad - N_{s1}(t) sin(w_c t + \theta_c)] \end{aligned} \tag{3}$$

which is the sum of a pure sinusoidal tone at the carrier frequency plus a mixture of (signal × noise) and (noise × noise) terms. The signal in (3) is then input to a PLL whose voltage-controlled oscillator (VCO) output can be expressed as

$$r_{vco}(t) = \sqrt{2} cos(w_c t + \hat{\theta}_c) \tag{4}$$

then we multiply (3) and (4) together and low-pass filter (to remove frequency components at $2w_c$) to obtain

$$\begin{aligned} z(t) &= \sqrt{P} sin(\phi_c) + (\sqrt{P}/A) m(t-\Delta) n_2(t) sin(\phi_c) \\ &\quad + m(t-\Delta) \times [N_{c1}(t) cos(\phi_c) - N_{s1}(t) sin(\phi_c)] \\ &\quad + (1/A) n_2(t) \times [N_{c1}(t) cos(\phi_c) - N_{s1}(t) sin(\phi_c)] \\ &\triangleq \sqrt{P} sin(\phi_c) + v(t,\phi_c) \end{aligned}$$

where $\phi_c = \theta_c - \hat{\theta}_c$ denotes the phase error in the loop. Note that the loop tracks the phase error ϕ_c as opposed to twice the phase $2\phi_c$ error as in the more conventional Costas loop.

Next we pass $z(t)$ through a matched filter (I&D in the case of square pulses) to produce (in the k^{th} interval $(k+1)T_s \le t \le (k+2)T_s$) the piecewise constant error signal [2]

$$\begin{aligned} e_k &= \textstyle\int_{kT_s}^{(k+1)T_s} z(t) dt \\ &= T_s \sqrt{P} sin(\phi_c) + (\sqrt{P}/A) T_s d_k n_{2k} sin(\phi_c) \\ &\quad + d_k [N_{c1k} cos(\phi_c) - N_{s1k} sin(\phi_c)] \\ &\quad + (1/A) n_{2k} [N_{c1k} cos(\phi_c) - N_{s1k} sin(\phi_c)] \\ &= T_s \sqrt{P} sin(\phi_c) + v(k,\phi_c) \end{aligned} \tag{5}$$

where

$$\begin{aligned} N_{c1k} &= \textstyle\int_{kT_s}^{(k+1)T_s} N_{c1}(t) dt \\ N_{s1k} &= \textstyle\int_{kT_s}^{(k+1)T_s} N_{s1}(t) dt \end{aligned} \tag{6}$$

are zero mean Gaussian noise RVs with variance ${\sigma_n}^2 = N_o T_s/2$. Clearly from the above, the slope of the $S-$curve, K_g, is given by

$$K_g = T_s \sqrt{P} \tag{7}$$

We now compute the autocorrelation function of $v(k,\phi_c)$ (treated as a piecewise continuous process $v(t,\phi_c)$) from which we shall obtain the equivalent noise PSD affecting the loop. For operation in the neighborhood of $\phi_c = 0$, it is reasonable to consider only the autocorrelation function of $v(k,0)$. Assuming $n_1(t)$ and $n_2(t)$ are independent and the noise samples are independent from symbol interval to symbol interval, then the autocorrelation is triangular

$$R_v(\tau) = E\left\{v(t,0) v(t+\tau,0)\right\} = \left\{ \begin{array}{c} \sigma_v^2 \left(1 - \frac{|\tau|}{T_s}\right) |\tau| \le T_s \\ 0 \text{ otherwise} \end{array} \right.$$

with

$$\begin{aligned} \sigma_v^2 &= E\left\{v^2(k,0)\right\} = E\left\{N_{c1k}^2 \left(d_k + \tfrac{n_{2k}}{A}\right)^2\right\} \\ &= \sigma_n^2 (1 + \sigma^2/A^2) = \tfrac{N_o T_s}{2} \left(1 + \sigma^2/A^2\right) \end{aligned}$$

Thus, the equivalent single-sided noise PSD, N_e, is given by

$$N_e = 2 \int_{-\infty}^{-\infty} R_v(\tau,0) d\tau = N_o T_s^2 (1 + \sigma^2/A^2) \tag{8}$$

Finally, the mean-square phase error in the loop is given by

$$\sigma_{\phi_c}^2 = \frac{N_e B_L}{K_g^2} = \frac{N_o B_L}{P}\left(1 + \frac{\sigma^2}{A^2}\right) \triangleq \frac{1}{\rho S_L} \tag{9}$$

where B_L is the noise bandwidth, $\rho = P/(N_o B_L)$ is the loop SNR in a conventional PLL and

$$S_L \triangleq \left(1 + \frac{\sigma^2}{A^2}\right)^{-1} \tag{10}$$

is the degradation of the loop SNR analogous to the "squaring loss" in a conventional Costas loop. The quantity A^2/σ^2 represents the decoder *soft SNR estimate*. For iterative decoding of LDPC and turbo codes [17]–[19], the mean and the variance are related by the symmetry condition $\sigma^2 = 2A$.

[1] The purpose of the normalization is to make the signal component of the feedback independent of the extrinsic symbol information amplitude, A. This in turn results in the input to the carrier tracking loop being independent of this same amplitude thereby not affecting the choice of the loop bandwidth. In practice, one would need to estimate A per iteration to perform this normalization. For the purpose of our theoretical discussion here, we shall assume that this estimation is perfect.

[2] Without loss in generality, we herein ignore the decoder delay Δ.

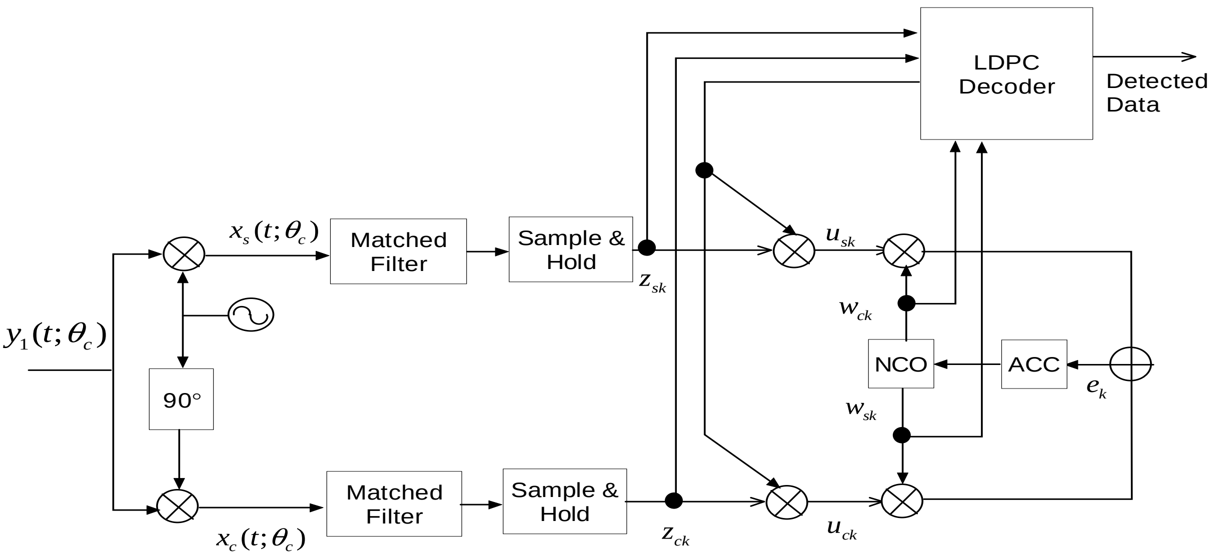


Fig. 2. Digital implementation of BPSK Receiver with Information-Reduced Carrier Synchronizer Using Soft Decision Feedback

Using this relation (10) becomes

$$S_L^{IR} \triangleq \left(1+\frac{2}{A}\right)^{-1} \tag{11}$$

As the iteration proceeds, the estimated data SNR increases and likewise the squaring loss decreases (i.e., S_L^{IR} approaches unity). By comparison, for a Costas loop, the expression for the squaring loss is given by

$$\begin{aligned} S_L^C &\triangleq (1+1/(2R_d))^{-1} \\ R_d &= P.T_s/N_o \end{aligned}$$

and thus remains fixed, independent of the iteration process, for a given symbol SNR. To numerically evaluate the performance in (11), one needs to quantify the functional dependence of the decoder soft-estimate of the data SNR and the input symbol SNR.

IV. A Practical Digital Implementation

The circuit in Fig. 2, shows a practical implementation of the carrier recovery loop presented in the previous section. Once again we assume a rectangular pulse shape although a bandwidth efficient Nyquist pulse-shape can also be used. The first difference with the block diagram in Fig. 1 is that demodulation (convert to baseband) of the input signal (1) is done with the carrier phase error still present, using the I and Q reference signals (arbitrarily assuming them to have zero phase relative to the received signal). We thus multiply the input by $\sqrt{2}sin(w_c t)$ and $\sqrt{2}cos(w_c t)$, and obtain

$$\begin{aligned} x_c^{HighF}(t;\theta_c) &= \sqrt{P}m(t).\left(sin\left(2w_c t+\theta_c\right)+sin(\theta_c)\right) \\ &\quad +\sqrt{2}n_1(t)cos(w_c t) \\ &= \sqrt{P}m(t).\left(sin\left(2w_c t+\theta_c\right)+sin(\theta_c)\right) \\ &\quad +N_{c1}(t)cos(2w_c t+\theta_c)+N_{c1}(t)cos(\theta_c) \\ &\quad -N_{s1}(t)sin(2w_c t+\theta_c)-N_{s1}(t)sin(\theta_c) \end{aligned}$$

$$\begin{aligned} x_s^{HighF}(t;\theta_c) &= \sqrt{P}m(t).\left(-cos\left(2w_c t+\theta_c\right)+cos(\theta_c)\right) \\ &\quad +\sqrt{2}n_1(t)sin(w_c t) \\ &= \sqrt{P}m(t).\left(-cos\left(2w_c t+\theta_c\right)+cos(\theta_c)\right) \\ &\quad -N_{c1}(t)sin(2w_c t+\theta_c)+N_{c1}(t)sin(\theta_c) \\ &\quad +N_{s1}(t)cos(2w_c t+\theta_c)-N_{s1}(t)cos(\theta_c) \end{aligned}$$

Applying a low-pass filter to remove frequencies at $2w_c$ yields:

$$x_c(t;\theta_c)=\sqrt{P}m(t)sin(\theta_c)+N_{c1}(t)cos(\theta_c)-N_{s1}(t)sin(\theta_c)$$
$$x_s(t;\theta_c)=\sqrt{P}m(t)cos(\theta_c)-N_{c1}(t)sin(\theta_c)-N_{s1}(t)cos(\theta_c)$$

The demodulated signals are then passed through (I&Ds) (or matched root raised cosine filters) to give

$$z_{ck}=\sqrt{P}T_s d_k sin(\theta_c)+N_{c1k}cos(\theta_c)-N_{s1k}sin(\theta_c)$$
$$z_{sk}=\sqrt{P}T_s d_k cos(\theta_c)-N_{c1k}sin(\theta_c)-N_{s1k}cos(\theta_c)$$

in the interval $(k+1)T_s \leq t \leq (k+2)T_s$. Next multiply z_{ck} and z_{sk} by the normalized soft decision feedback sample (decoder extrinsic information)

$$y_{2k}=d_k+n_{2k}/A \tag{12}$$

where as before over a given iteration $\{n_{2k}\}$ are modeled as i.i.d. zero mean Gaussian RVs with variance σ^2. The result of this multiplication removes the modulation and produces

$$\begin{aligned} u_{ck} &= z_{ck}y_{2k}=\sqrt{P}T_s sin(\theta_c) \\ &\quad +[(d_k+n_{2k}/A)(N_{c1k}cos(\theta_c)-N_{s1k}sin(\theta_c))+ \\ &\quad n_{2k}/A\sqrt{P}T_s d_k.sin(\theta_c)] \\ &= \sqrt{P}T_s sin(\theta_c)+v_{ck}, \end{aligned}$$

$$
\begin{aligned}
u_{sk} &= \sqrt{P}T_s.cos(\theta_c) \\
&\quad +[(d_k + n_{2k}/A)(-N_{c1k}sin(\theta_c) - N_{s1k}cos(\theta_c)) + \\
&\quad n_{2k}/A\sqrt{P}T_s d_k cos(\theta_c)] \\
&= \sqrt{P}T_s cos(\theta_c) + v_{sk},
\end{aligned}
$$

which is then input to a digital PLL whose number-controlled oscillator (NCO) produces an estimate of the carrier phase denoted by $\hat{\theta}_c$. Multiplying u_{ck} and u_{sk} by $w_{ck} = cos(\hat{\theta}_c)$ and $w_{sk} = sin(\hat{\theta}_c)$, respectively, and then differencing the results of these products provides the error signal

$$
\begin{aligned}
e_k &= u_{ck}w_{ck} - u_{sk}w_{sk} \\
&= \tfrac{1}{2}\sqrt{P}T_s\left(sin(\phi_c) + sin(\theta_c + \hat{\theta}_c)\right) \\
&\quad -\tfrac{1}{2}\sqrt{P}T_s\left(-sin(\phi_c) + sin(\theta_c + \hat{\theta}_c)\right) \\
&\quad v_{ck}cos(\hat{\theta}_c) - v_{sk}sin(\hat{\theta}_c) \\
&= \sqrt{P}T_s.sin(\phi_c) + v_{ck}cos(\hat{\theta}_c) - v_{sk}sin(\hat{\theta}_c)
\end{aligned} \tag{13}
$$

where as before $\phi_c = \theta_c - \hat{\theta}_c$ denotes the phase error in the loop. Comparing (13) with (5) we see that they are identical and thus the performance of the digital PLL would also be described by (9) together with (10).

V. Tracking Performance for QPSK Modulation

As we did in Section III, we now present a sample receiver for QPSK signals shown in Fig. 3. Consider an input QPSK modulation of the form

$$
\begin{aligned}
y_1(t;\theta_c) = &\sqrt{P}m_I(t)\sin(\omega_c t + \theta_c) \\
&+\sqrt{P}m_Q(t)\cos(\omega_c t + \theta_c) + n_1(t)
\end{aligned}
$$

where, analogous to (2), the noise is modeled by

$$
n_1(t) = \sqrt{2}\left[N_{c1}(t)\cos(\omega_c t + \theta_c) - N_{s1}(t)\sin(\omega_c t + \theta_c)\right]
$$

and the I and Q modulations are given by

$$
\begin{aligned}
m_I(t) &= \sum_{k=-\infty}^{\infty} d_{Ik}p(t - kT_s) \\
m_Q(t) &= \sum_{k=-\infty}^{\infty} d_{Qk}p(t - kT_s)
\end{aligned}
$$

As before, multiply $y_1(t;\theta_c)$ (again ignoring the decoder delay) by a normalized version of the soft decision feedback signal corresponding to the extrinsic information for the I data stream, namely,

$$
y_{2I}(t) = m_I(t) + n_{2I}(t)/A
$$

where $n_2(T)$ is again modeled as a piecewise constant baseband noise process, namely,

$$
n_{2I}(t) = \sum_{k=-\infty}^{\infty} n_{2Ik}p(t - kT_s) \tag{14}
$$

with $\{n_{2Ik}\}$ being i.i.d. zero mean Gaussian RVs with variance σ^2. Now also phase shift the received signal by $\pi/2$ rad. to form the quadrature input

$$
\begin{aligned}
y_1(t;\theta_c - \pi/2) &= -\sqrt{P}m_I(t)\cos(\omega_c t + \theta_c) \\
&\quad +\sqrt{P}m_Q(t)\sin(\omega_c t + \theta_c) + n_{1Q}(t)
\end{aligned}
$$

where

$$
n_{1Q}(t) = \sqrt{2}\left[N_{c1}(t)\sin(\omega_c t + \theta_c) + N_{s1}(t)\cos(\omega_c t + \theta_c)\right]
$$

and multiply this by a normalized version of the extrinsic information for the Q data stream, namely,

$$
y_{2Q}(t) = m_Q(t) + n_{2Q}(t)/A
$$

where, analogous to (14),

$$
n_{2Q}(t) = \sum_{k=-\infty}^{\infty} n_{2Qk}p(t - kT_s)
$$

and the sequence $\{n_{2Qk}\}$ is assumed to have the same properties as the sequence $\{n_{2Ik}\}$. Furthermore, it is reasonable to assume the two sequences independent of each other. The results of the above-mentioned multiplications are given by

$$
\begin{aligned}
u_I(t;\theta_c) &= \sqrt{P}\sin(\omega_c t + \theta_c) \\
&\quad +\sqrt{P}m_I(t)m_Q(t)\cos(\omega_c t + \theta_c) \\
&\quad +\left(\sqrt{P}/A\right)m_I(t)n_{2I}(t)\sin(\omega_c t + \theta_c) \\
&\quad +\left(\sqrt{P}/A\right)m_Q(t)n_{2I}(t)\cos(\omega_c t + \theta_c) \\
&\quad +\sqrt{2}m_I(t)\left[N_{c1}(t)\cos(\omega_c t + \theta_c)\right. \\
&\quad \left.-N_{s1}(t)\sin(\omega_c t + \theta_c)\right] \\
&\quad +\left(\sqrt{2}/A\right)n_{2I}(t)\left[N_{c1}(t)\cos(\omega_c t + \theta_c)\right. \\
&\quad \left.-N_{s1}(t)\sin(\omega_c t + \theta_c)\right]
\end{aligned}
$$

$$
\begin{aligned}
u_Q(t;\theta_c) &= \sqrt{P}\sin(\omega_c t + \theta_c) \\
&\quad -\sqrt{P}m_I(t)m_Q(t)\cos(\omega_c t + \theta_c) \\
&\quad -\left(\sqrt{P}/A\right)m_I(t)n_{2Q}(t)\cos(\omega_c t + \theta_c) \\
&\quad +\left(\sqrt{P}/A\right)m_Q(t)n_{2Q}(t)\sin(\omega_c t + \theta_c) \\
&\quad +\sqrt{2}m_Q(t)\left[N_{c1}(t)\sin(\omega_c t + \theta_c)\right. \\
&\quad \left.+N_{s1}(t)\cos(\omega_c t + \theta_c)\right] \\
&\quad +\left(\sqrt{2}/A\right)n_{2Q}(t)\left[N_{c1}(t)\sin(\omega_c t + \theta_c)\right. \\
&\quad \left.+N_{s1}(t)\cos(\omega_c t + \theta_c)\right]
\end{aligned}
$$

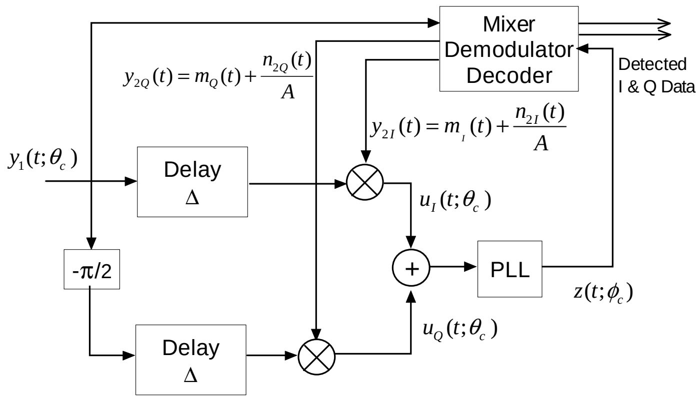


Fig. 3. QPSK receiver with Information-Reduced Carrier Phase Synchronizer Using Soft Decision Feedback

Adding $u_I(t;\theta_c)$ and $u_Q(t;\theta_c)$ eliminates the cross-modulation signal term and produces a signal that is again the sum of a pure sinusoidal tone at the carrier frequency plus a mixture of (signal×noise) and (noise×noise) terms. The signal $u(t;\theta_c)$ is input to a PLL which after demodulation by the reference signal in (4) gives

$$
\begin{aligned}
&z(t) = \sqrt{2P}\sin(\phi_c) \\
&+ \left(\sqrt{P/2}/A\right)[m_I(t)\,n_{2I}(t) + m_Q(t)\,n_{2Q}(t)]\sin(\phi_c) \\
&+ \left(\sqrt{P/2}/A\right)[m_Q(t)\,n_{2I}(t) - m_I(t)\,n_{2Q}(t)]\cos(\phi_c) \\
&+ [m_I(t)\,N_{c1}(t) + m_Q(t)\,N_{s1}(t)]\cos(\phi_c) \\
&+ [m_Q(t)\,N_{c1}(t) - m_I(t)\,N_{s1}(t)]\sin(\phi_c) \\
&+ (1/A)[n_{2I}(t)\,N_{c1}(t) + n_{2Q}(t)\,N_{s1}(t)]\cos(\phi_c) \\
&+ (1/A)[n_{2Q}(t)\,N_{c1}(t) - n_{2I}(t)\,N_{s1}(t)]\sin(\phi_c)
\end{aligned}
$$

As in the BPSK case, next we pass $z(t)$ through a matched filter (I&D in the case of square pulses) to produce (in the k^{th} interval) $(k+1)T_s \le t \le (k+2)T_s$ the piecewise constant error signal

$$
\begin{aligned}
&e_k = \int_{kT_s}^{(k+1)T_s} z(t)\,dt = T_s\sqrt{2P}\sin(\phi_c) \\
&+ \left(\sqrt{P/2}/A\right)T_s[d_{Ik}n_{2Ik} + d_{Qk}n_{2Qk}]\sin(\phi_c) \\
&+ \left(\sqrt{P/2}/A\right)T_s[d_{Qk}n_{2Ik} - d_{Ik}n_{2Qk}]\cos(\phi_c) \\
&+ [d_{Ik}N_{c1k} + d_{Qk}N_{s1k}]\cos(\phi_c) \\
&+ [d_{Qk}N_{c1k} - d_{Ik}N_{s1k}]\sin(\phi_c) \\
&+ (1/A)[n_{2Ik}N_{c1k} + n_{2Qk}N_{s1k}]\cos(\phi_c) \\
&+ (1/A)[n_{2Qk}N_{c1k} - n_{2Ik}N_{s1k}]\sin(\phi_c) \\
&= T_s\sqrt{2P}\sin(\phi_c) + v(k,\phi_c)
\end{aligned}
\tag{15}
$$

where N_{c1k},N_{s1k} are defined in (6).

Once again we must compute the slope of the S-curve and the equivalent noise PSD. From (15), the slope of the S-curve is immediately given by

$$K_g = T_s\sqrt{2P}. \tag{16}$$

The variance of the additive noise $v(k,0)$ is readily determined to be [3]

$$\sigma_v^2 = N_0 T_s\left[1 + (1+R_d)\,\sigma^2/A^2\right] \tag{17}$$

and thus from (8), the equivalent noise PSD is

$$N_e = 2T_s\sigma_v^2 = 2N_0T_s^2\left[1 + (1+R_d)\,\sigma^2/A^2\right] \tag{18}$$

Finally, applying (16) and (17) to (9), the mean-square phase error in the loop becomes

$$\sigma_{\phi_c}^2 = \frac{N_e B_L}{K_g^2} = \frac{N_o B_L}{P}\left[1 + (1+R_d)\frac{\sigma^2}{A^2}\right] \triangleq \frac{1}{\rho S_L}$$

[3]The additional noise variance factor $(1+R_d)$ comes from the presence of a quadrature signal×noise term that is absent in the BPSK case.

or equivalently, the "squaring loss" is given by

$$S_L = \left[1 + (1+R_d)\,\sigma^2/A^2\right]^{-1}$$

which applying $A/\sigma^2 = 1/2$ simplifies to

$$S_L^{IR} = [1 + (1+R_d)\,(2/A)]^{-1} \tag{19}$$

Comparing (19) with (11) we immediately observe the additional penalty (dependent now on symbol SNR: R_d) in carrier tracking performance using a PLL for QPSK relative to BPSK. For small symbol SNRs, this penalty becomes insignificant. Furthermore, note that because of the creation of a pure tone by the soft decision feedback, thus allowing the use of a PLL, there are no fourth order (signal×noise) or (noise×noise) products in the loop as in the conventional QPSK Costas loop or information-reduced carrier synchronization loop with hard decision feedback. Thus, the "squaring loss" penalty is inherently smaller than the "quadrupling loss" penalty associated with the above-mentioned loops.

As was the case for BPSK, it is also possible to construct an alternative digital implementation that once again would yield the same performance as its piecewise continuous analog counterpart discussed above.

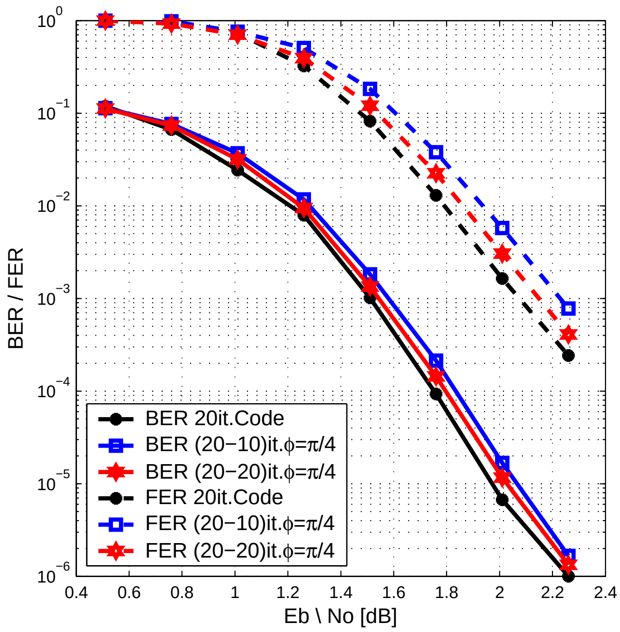
VI. Iterative Processing and Numerical Results

We have evaluated the performance of the all-digital BPSK baseband approach described in Section IV via joint decoding with an rate-1/2 (1944, 972) irregular low-density parity-check (LDPC) code developed in [20]. After a complex rotation to resolve phase ambiguity (discussed below), the signals z_{ck} and z_{sk} are multiplied by the decoder output y_{2k} to form u_{ck} and u_{sk}. As described in previous sections and shown in [1], if the PLL input has a small fraction of total modulated symbols in a block successfully removed then it can begin to produce a reasonable phase estimate, even at relatively low SNRs. We have found that the estimation/decoding process can be successfully started by assigning $y_{2k} = z_{sk}$ (Subsequent iterations derive y_{2k} from the decoder as described by (12)). After this assignment, the PLL operates once across all symbols in a codeword. LDPC decoder log-likelihood ratio inputs are then produced by combining the updated PLL phase estimates with z_{ck} and z_{sk}

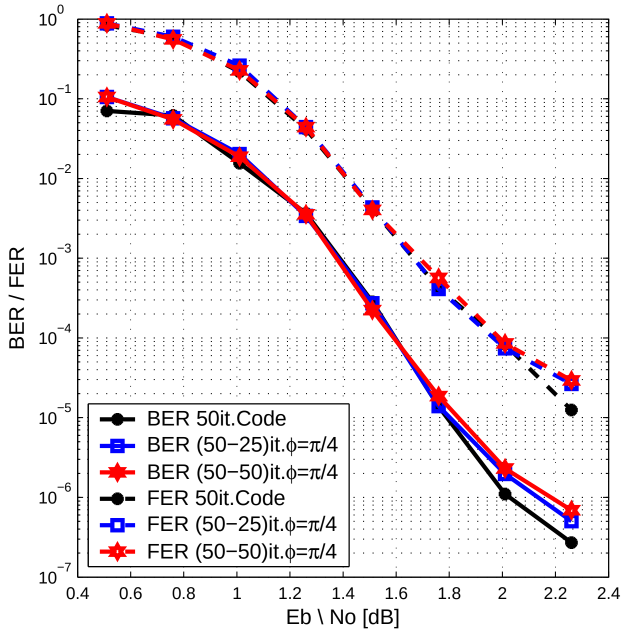
$$
\begin{aligned}
&Q_k = \tfrac{2}{\sigma_{llr}^2}\left(z_{sk}w_{ck} + z_{ck}w_{sk}\right) \\
&= \tfrac{2}{\sigma_{llr}^2}\left(\sqrt{P}T_s d_k cos(\phi_c) - N_{c1k}sin(\phi_c) - N_{s1k}cos(\phi_c)\right)
\end{aligned}
$$

where $\sigma_{llr}^2 = PT_s^2/(2E_s/N_o)$.

An "extrinsic" LLR feedback mechanism was employed in which prior LDPC inputs are subtracted from current outputs before new inputs (from the most recent PLL update) are added. Also, state information in the decoder (in particular the most recent extrinsic information arriving from check-nodes) is preserved between LDPC-to-PLL-to-LDPC iterates. The accumulator in Fig. 2 implements the first order transfer function



(a)



(b)

Fig. 4. IRCS BER/FER performance with a maximum of (a) 20 and (b) 50 LDPC iterations.

$$H(z) = \frac{K_p + K_i z^{-1}}{1 - z^{-1}}$$

where $K_p = 10^{-6}$ and $K_i = 10^{-8}$ were selected.

The bit and frame error rate performance of the system are shown in Fig. 4 for different update schedules between the decoder and the PLL circuit and for two cases of phase error ϕ_c. Simulations with $\phi_c = 0$ and $\phi_c = \pi/4$ represent cases of minimum and maximum initial phase error. Cases with $\phi_c = 0$ are not shown in the figure since they always achieve the same performance as the stand-alone code. All other initial phase offsets have error rate performance that lies between these two cases. The total number of LDPC iterations was set to either 20 or 50. The set of curves labeled (20-20) and (50-50) shows the error rate performance when a decoder iteration is followed by a PLL update. An alternative schedule (20-10) and (50-25) where a PLL update is done after two decoder iterations is also shown.

A plot of loop SNR, $1/\sigma^2_{\phi_c}$ (the inverse of the measured average squared estimation error), as a function of the number of iterations where $\phi_c = 0$ and $\phi_c = \pi/4$ is shown in Fig. 5 in conjunction with (50-25) scheduling (up to 50 LDPC iterations, with PLL updates on every other iteration). Fig. 5(b) explicitly shows the difference in loop SNR between cases with initial offsets of $\phi_c = 0$ and $\phi_c = \pi/4$ radians. Note that steady state operation is reached after 40 iterations when $\phi_c = 0$ and SNR varies between 1 and 2 dB. Up to 10 more iterations are required when the initial error is $\phi_c = \pi/4$. Note that even with these additional iterations, the loop SNR does not quite reach the levels observed in the in $\phi_c = 0$ case. This marginal degradation in loop-SNR explains the small (< 0.1 dB) performance loss in BER/FER performance in Fig. 4(b). Fig. 5 also shows that in the case of only 20 iterations (for either initial phase offset) the PLL has not reached steady state. The associated loss BER/FER in performance is shown in Fig. 4(a) where at a frame error rate of 10^{-3} the (20-10) schedule loses 0.15 dB and the (20-20) schedule loses 0.07 dB.

Note the curious performance in Fig. 5(a) of the zero-phase offset case during the first 10 LDPC iterations. In this region the PLL's phase estimate (which is initially correct) is highly noised by poor initial LDPC soft-symbol estimates. Both LDPC soft-symbol estimates and loop-SNR begin to improve dramatically after the 10th iteration.

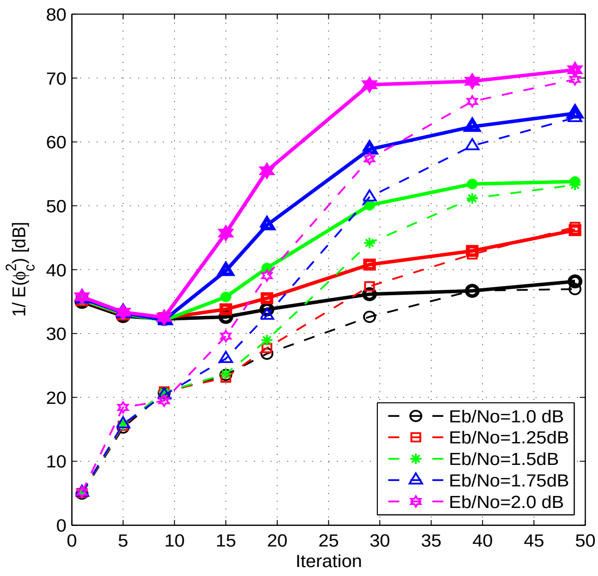
We conclude this section by noting that phase ambiguity (for offsets greater than $\pm\pi/2$ can be resolved by first measuring the average power across a single codeblock of the signals z_c and z_s. If the sine component (z_c) has average power greater than the cosine component (z_s), then these two components are swapped. This procedure may leave (or induce) a remaining error of π radians. To resolve this ambiguity we run a single PLL pass followed by several (up to 4) LDPC iterations. The orientation that produces the maximum number of satisfied odd-degree check equations is selected and the decoding procedure is reinitialized [4]. Similar techniques are proposed in [10], [11].

VII. Conclusions

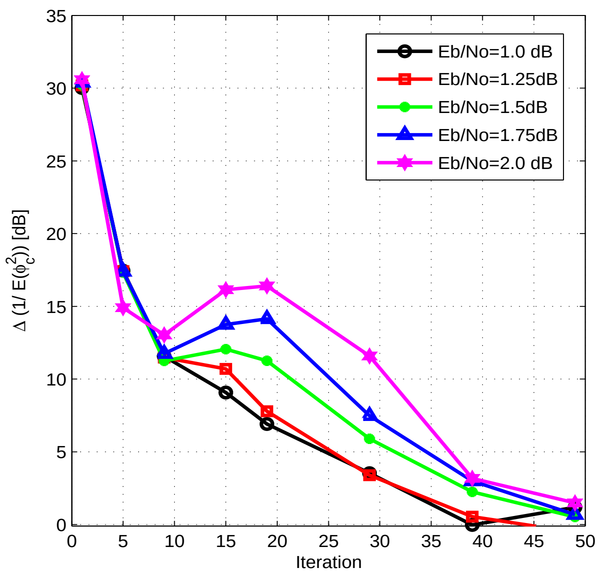
We have demonstrated a means for improving the carrier synchronization function for iterative decoded BPSK using information derived from the decoder extrinsics to remove the modulation (information-reduction) prior to the carrier tracking operation. The motivation for doing this is to

[4]Even degree checks remain satisfied under a rotation of all inputs by π.

overcome the penalty in noisy reference loss attributed to the large squaring loss at low SNRs that is characteristic of the traditional BPSK carrier sync loops such as the Costas-type loop. In comparison to the information-reduced carrier synchronization loop with hard decision feedback as proposed in [1] and [2], the scheme described in this work makes use of soft decision extrinsic information and does not require estimating the decoder error probability. This occurs as a consequence of the assumption here of a fixed carrier synchronization structure, i.e., a PLL, whose design does not change with knowledge obtained from the decoder. While in the soft decision feedback case considered here such a structure would only be asymptotically optimum (in the MAP motivation sense) at high SNR, it nevertheless provides a simple yet performance efficient carrier synchronization loop in SNR regions of interest for coded applications.



(a) Loop SNR for $\phi_c = 0$ (solid) and $\phi_c = \pi/4$ (dashed).



(b) Loop SNR Difference.

Fig. 5. IRCS Loop SNR Performance.

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