UCLA Graduate School of Engineering - Electrical Engineering Department

Communication Systems Laboratory

Universal Serially Concatenated Trellis Coded Modulation for MIMO Channels

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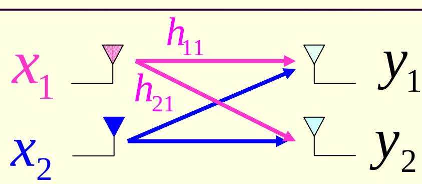
GLOBECOM 2006,
San Francisco

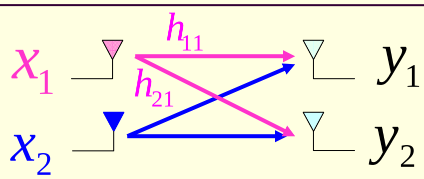
2×2 space-time channel



$$\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} z_1 \\ z_2 \end{bmatrix}$$

z_i : i.i.d. complex additive white Gaussian noise (AWGN) at receive antenna *i*.

2×2 space-time channel



$$\underbrace{\begin{bmatrix} y_1 \\ y_2 \end{bmatrix}}_{Y} = \underbrace{\begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix}}_{H} \underbrace{\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}}_{X} + \underbrace{\begin{bmatrix} z_1 \\ z_2 \end{bmatrix}}_{Z}$$

$$Y = HX + Z$$

λ_i's = eigenvalues of HH^T.

Capacity (Mutual Information) of a Single Cnannel

- The normalized 2x2 channel can be represented as

$$H_{eq} = \begin{bmatrix} \sqrt{\frac{1}{1+\kappa}} & 0 \\ 0 & \sqrt{\frac{\kappa}{1+\kappa}} \end{bmatrix} \begin{bmatrix} \cos\phi & \sin\phi \cdot e^{j\theta} \\ -\sin\phi & \cos\phi \cdot e^{j\theta} \end{bmatrix}, \text{ where } \kappa = \frac{\lambda_2}{\lambda_1} \text{ and } \lambda_1 + \lambda_2 = 1$$

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- The Mutual Information (MI) is the highest rate theoretically possible for a fixed transmit power spectrum.

$$\mathrm{MI}(H, E_s) = \log_2 \det\left(\mathrm{I} + \frac{\mathrm{E_s}}{\mathrm{N_o}} HH^{\dagger} \right) = \log_2 \left(1 + \frac{\mathrm{E_s}}{\mathrm{N_o}} \lambda_1 \right)\left(1 + \frac{\mathrm{E_s}}{\mathrm{N_o}} \lambda_2 \right)$$

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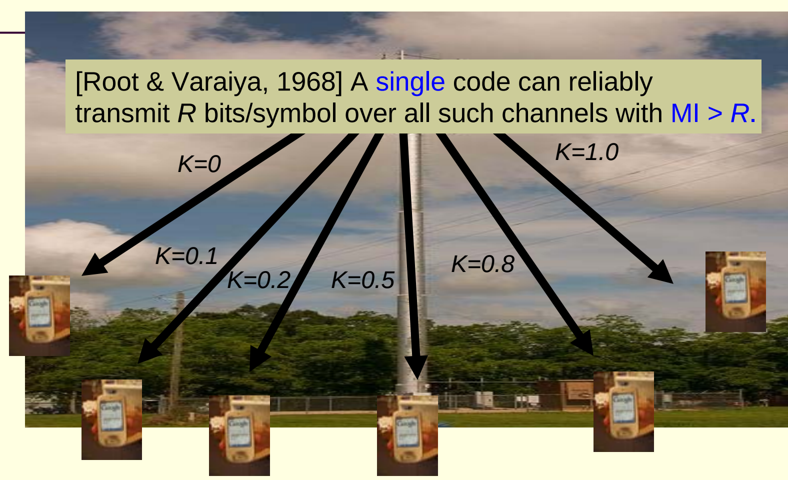
- MI only depends on the eigenvalue skew *K and* E_s/N_0.

Universal Coding for MIMO Channels

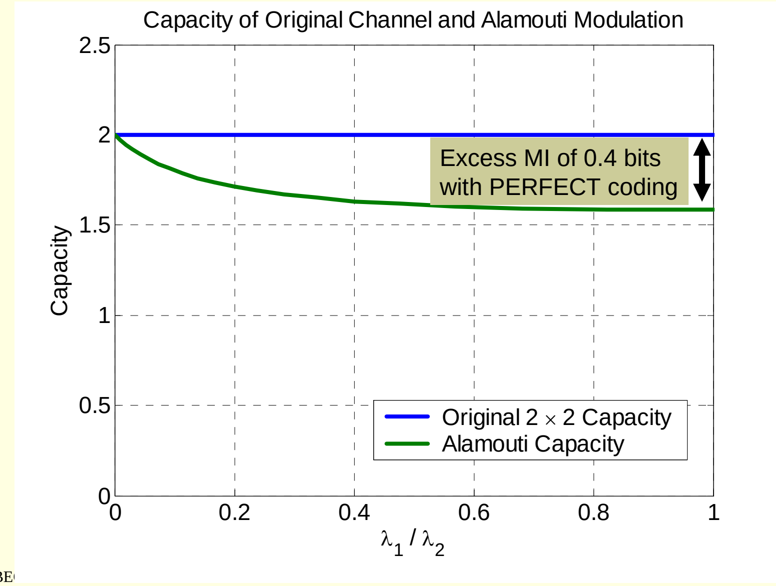
Universal Codes work whenever MI is sufficient.

[Root & Varaiya, 1968] A single code can reliably transmit *R* bits/symbol over all such channels with MI > *R*.

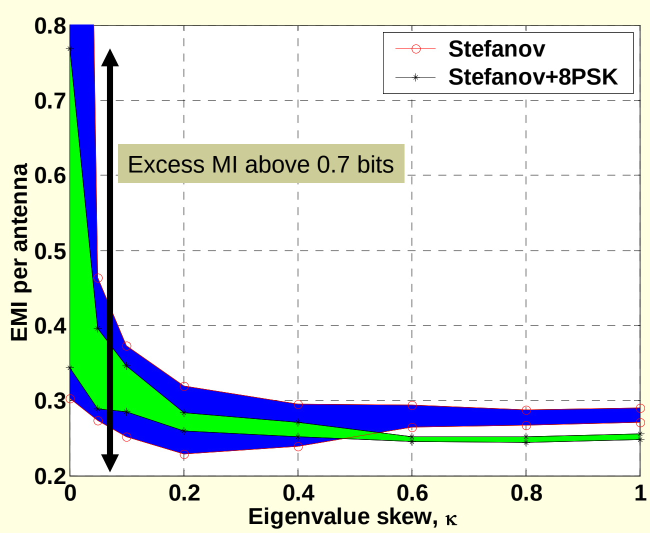


All six channels have the same quality,
the same mutual information,
but they have distinct eigenvalue skews K.

Alamouti is not universal for 2x2 systems (suffers on unitary channels)

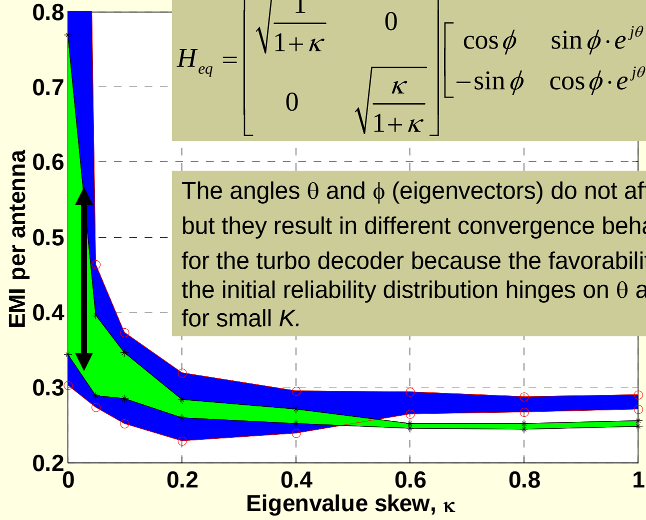


Current Space-Time Turbo Codes aren't Universal (suffer on singular channels)



Notice Variation for a fixed Eigenvalue Skew

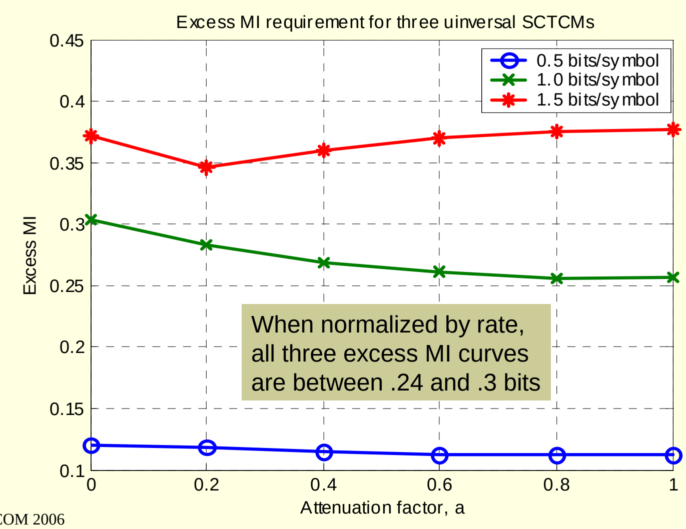
$$H_{eq} = \begin{bmatrix} \sqrt{\frac{1}{1+\kappa}} & 0 \\ 0 & \sqrt{\frac{\kappa}{1+\kappa}} \end{bmatrix} \begin{bmatrix} \cos\phi & \sin\phi \cdot e^{j\theta} \\ -\sin\phi & \cos\phi \cdot e^{j\theta} \end{bmatrix}$$



The angles θ and ϕ (eigenvectors) do not affect MI, but they result in different convergence behavior for the turbo decoder because the favorability of the initial reliability distribution hinges on θ and ϕ for small *K.*

We had universal behavior for single antennas [ICC 03].

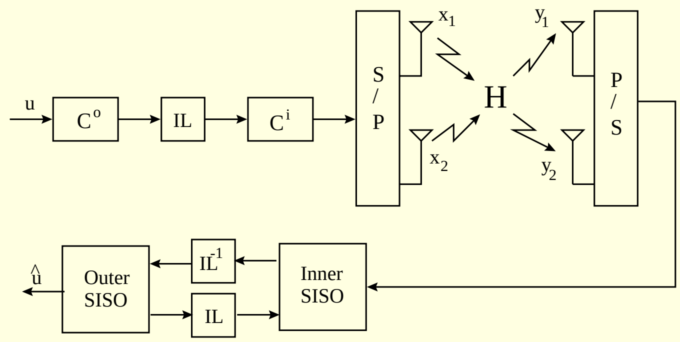
- Universal codes for three rates over period-2 periodic fading.



When normalized by rate, all three excess MI curves are between .24 and .3 bits

Space-Time SCTCM System

- Output symbols of the SCTCM are de-multiplexed to the antennas.
- The inner SISO is based on the *collapsed trellis* which combines 2 trellis stages together into a super-trellis.

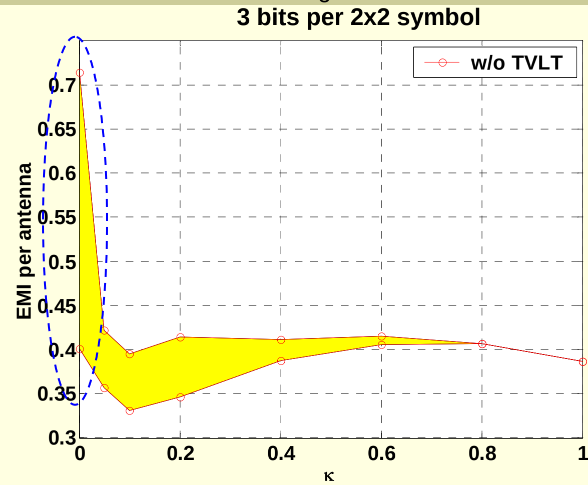


Design of Universal SCTCM for periodic fading

- Universal SCTCM for PFC considers:
  a. Constituent code complexity
  b. Outer code
  c. Inner code
  d. Constellation and labeling
  e. Interleaver design
- De-multiplexed space-time SCTCM working on diagonal channels with $\kappa = |q|^2$, and $\phi = \theta = 0$ is equivalent to SCTCM working on the [1 q] periodic fading.
- The universal SCTCMs for the [1 q] periodic fading deliver consistent EMI over κ for $\phi = \theta = 0$, but there is still the issue of non-diagonal channels.

EMI depends on ϕ and θ for turbo codes.

Our universal periodic turbo code for 1.5 bits per symbol
used as a space time turbo code for 3 bits per 2x2 symbol
has the same variation for near-singular channels as the Stefanov code.



Time-Varying Linear Transformation

- First proposed by Wei Shi *et al.*, TVLT rotates the channel to different angles, hoping that the dependence on the angles can be "averaged out".
- In the current work, we generalized the TVLT concept to improve the averaging effect.

Time-Varying Linear Transformation

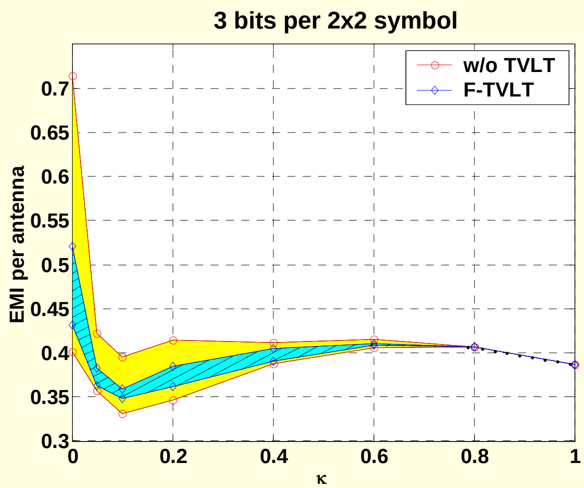
- First proposed by Wei Shi *et al.*, TVLT rotates the channel to different angles, hoping that the dependence on the angles can be "averaged out".
- In the current work, we generalized the TVLT concept to improve the averaging effect.
- A time-varying unitary matrix, Q_t, is multiplied to the signal vector before it is transmitted.

$$Q_t(\alpha,\beta,\gamma)=\begin{bmatrix}\cos\alpha & \sin\alpha\cdot e^{j\gamma}\\ -\sin\alpha\cdot e^{j\beta} & \cos\alpha\cdot e^{j(\beta+\gamma)}\end{bmatrix}$$

Sweep α, β, and γ uniformly over the interval $[0, 2\pi]$.

EMI after TVLT

- EMI difference over eigenvectors can be largely mitigated, but not completely eliminated,



Why can't TVLT remove all variation?

$$H = \begin{bmatrix} \sqrt{\lambda_1} & 0 \\ 0 & \sqrt{\lambda_2} \end{bmatrix} \begin{bmatrix} \cos\phi & \sin\phi \\ -\sin\phi & \cos\phi \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & e^{j\theta} \end{bmatrix}$$

$$= \Lambda M(\phi) D(\theta)$$

- Multiplying by $Q_t(\alpha,\beta,\gamma)$ changes ϕ and θ :

$$\tilde{H} = \Lambda M(\tilde{\phi}) D(\tilde{\theta})$$

where $\tilde{\theta} = \sigma(\alpha,\beta) + \gamma$, σ is a function of α and β

$$\tilde{\phi} = \frac{1}{2}\cos^{-1}\left[\cos(2\phi)\cos(2\alpha) + \sin(2\phi)\sin(2\alpha)\cos(\theta+\beta)\right]$$

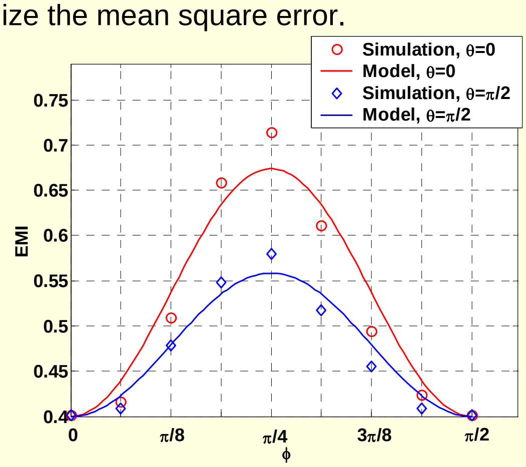
- For fixed α and β, $\tilde{\theta} \sim u(0,2\pi)$ if $\gamma \sim u(0,2\pi)$.
- $\tilde{\phi}$ is not uniform over $[0,2\pi)$ without CSI at TX.

Modeling the variation of EMI over ϕ

- From simulation results,

$$\mathrm{EMI}(\phi,\theta) \approx \mathrm{EMI}_{\min} + K_1\left[1-\cos(4\phi)\right]\left[K_2 + \cos(2\theta)\right]$$

where EMI_{min}, K_1, K_2 only depend on κ and are found to minimize the mean square error.



Analysis shows residual variation is expected.

- Assume that the EMI is the average of the instantaneous EMIs and α, β, and γ are uniformly distributed.
- The average EMI with TVLT is given by

$$\mathrm{EMI}^*(\phi,\theta) \approx \frac{1}{\mathrm{N}_\alpha N_\beta N_\gamma} \sum_\alpha \sum_\beta \sum_\gamma \mathrm{EMI}(\tilde{\phi},\tilde{\theta})$$
$$\approx \mathrm{EMI}_{\min} + \frac{K_1 K_2}{4}\left[5-\cos\left(4\phi\right)\right]$$

- Dependence on θ is eliminated but the average EMI still depends on ϕ, although with a smaller variance.

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$$\approx \mathrm{EMI}_{\min} + \frac{K_1 K_2}{4}\left[5-\cos\left(4\phi\right)\right]$$

- A relaxed sufficient condition for the values of α, β, and γ where the above equation holds is:

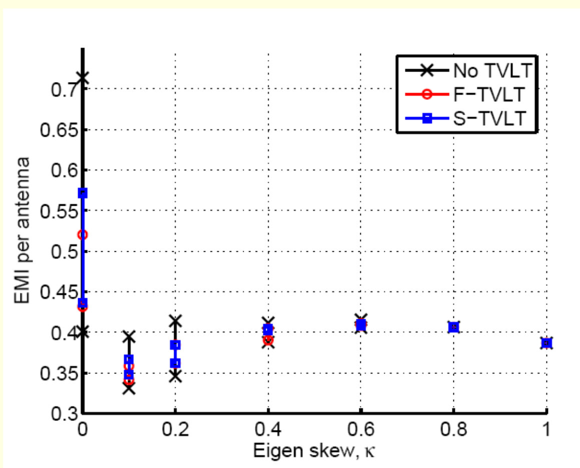
$$\sum_\alpha \cos(4\alpha)=0,\ \sum_\alpha \sin(4\alpha)=0,\ \sum_\beta \cos(2\beta+2\theta)=0,\ \sum_\gamma \cos(2\gamma)=0.$$

TVLT Granularity

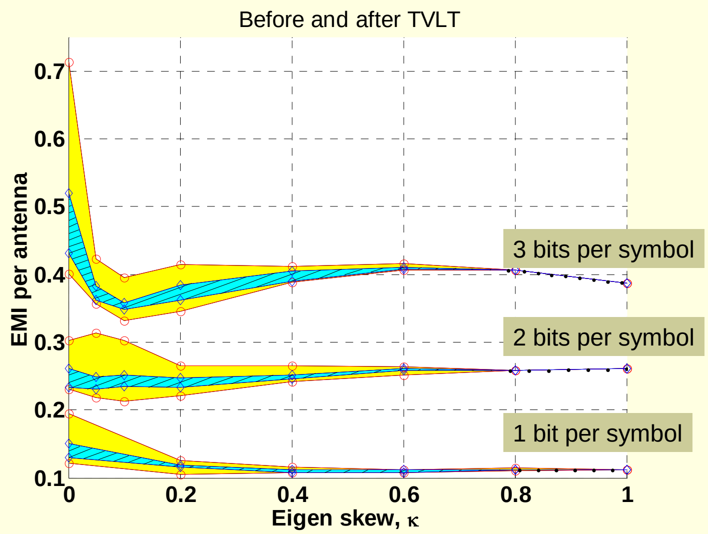
- **Fine-sampled TVLT (F-TVLT):** Use the minimum step size for α, β, and γ as $\frac{2\pi}{\sqrt[3]{N/R}}$
  - For example, a 10,000 bit block with R=3.0 bits per (space-time) symbol has 3,333 symbols/ block. Then the step size is about $2\pi/15$.
- **Simplified TVLT (S-TVLT):** Under the cosine cost function model, a minimum of 8 TVLT matrices with $\alpha = [0\ \pi/4]$, $\beta = [0\ \pi/2]$, $\gamma = [0\ \pi/2]$ can achieve the same average performance as the uniform-(α,β,γ) scheme.
- Simulation results show that the EMI with S-TVLT is almost identical to that with F-TVLT.

Comparison of F-TVLT and S-TVLT

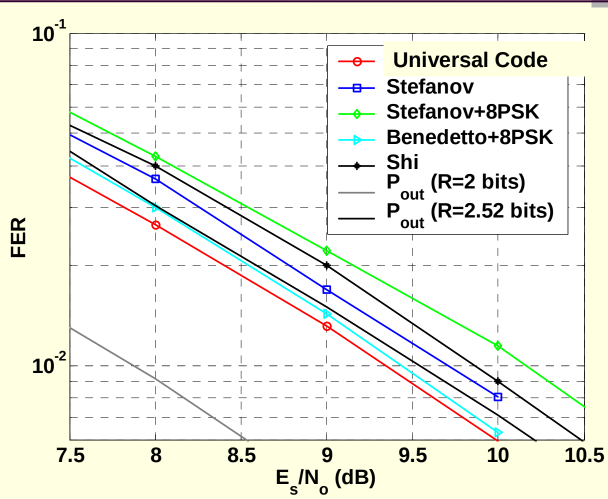
R=2.0 bits/s/Hz. Blocklength=10,000 bits, 12 iterations.



Universal SCTCMs at 3 rates with/without F-TVLT



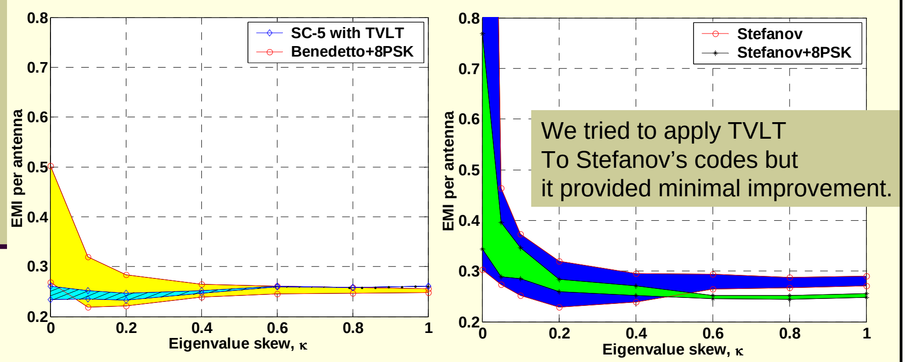
Rayleigh Fading Performance



- Our universal SC-TCM with F-TVLT is 1.5 dB away from P_{out}.
- Stefanov (PCCC-BICM); Benedetto (SCCC); Shi (PCTCM).

Channel-to-Channel Performance

- Average performance (Rayleigh fading) are similar but the universal SCTCM performs more consistently.
- Universal code requires approx. 0.26 bits of EMI for any quasi-static fading channel.



Conclusions

- The proposed universal SCTCMs of 1.0, 2.0 and 3.0 bits per channel use require a consistent normalized EMI of 0.11-0.18 bits. They are 1.1, 1.5, and 2.1 dB respectively from the outage probability at FER=10^{-2} on the Rayleigh fading channel.
- Universal SCTCMs for periodic fading on a de-multiplexed space-time scheme provides consistent EMI over eigenvalue skew.
- TVLT mitigates the EMI dependence of these codes on channel eigenvectors for the same eigenvalue skew.
- Universal SCTCMs perform well on the Rayleigh fading channel and are robust on any quasi-static fading channel.