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ST Microelectronics Partial Proposal for LDPCC as optional coding technique for IEEE 802.11 TGn High Troughput Standard

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Abstract

Optional enhancements to the IEEE IEEE 802.11 TGn High Troughput Standard are provided to describe 802.11n PHY operation in the key areas of the adopted coding technique.

1 Scope

The present document proposes the adoption of rate-compatible Low Density Parity Check Code (LDPCC) as an optional advanced coding technique for the IEEE 802.11 TGn High Throughput Standard. The proposal includes the description of the LDPCC matrices, the data formatting procedures and the achievable performance in AWGN and MIMO channels. The specifications described throughout this document are strictly related to the specifications endorsed in [1] (WWISE complete proposal), as optional advanced coding technique. However, the proposed rate-compatible parity-check matrix design strategy can be easily generalized to adapt to the parameters of different MIMO-OFDM systems.

2 Introduction

It has been demonstrated that advanced coding techniques are one of the most promising solution to achieve the required performance of 802.11n standard at signal-to-noise ratios (SNRs) compatible with low-cost RF front-end. Among the most advanced coding technique, LDPCC has been selected on the basis of the following considerations:

- Proposed LDPCCs are structured; structured LDPCCs, compared to randomly constructed codes, have been shown to yield near-optimum performance and at the same time allow an efficient decoder parallelization, thus ensuring small decoding latency.
- LDPCCs can significantly boost performance in MIMO-OFDM systems, which can be exploited to increase the coverage and/or the throughput of the system.
- LDPCC enables a good exploitation of the space-frequency diversity in fading channels.
- We also mention that LDPCCs do not require a channel interleaver because of the intrinsic randomization property embedded in the code structure.
- The iterative decoding process of LDPCC provides a coding gain dependent on the number of decoder iterations. As silicon technology improves every year, it can be expected that the achievable number of iterations, and the consequent coding gain, might go on improving in the next future.

The codeword length selection has been based on the following criteria:

- Performance factor: the code length has to be sufficient to provide a significant gain compared to 64 states Convolutional Code (CC), mandatory coding technique in both [1] and [2].
- The upper limit for the coded size length is determined by both latency and encoder/decoder complexity (which in their turn are depend on timing and cost constrains).
- The fine tuning of the codeword length has been done with the goal to minimize (or eliminate) the number of padding bits required to transmit an integer number of OFDM symbols (multiple of the number of transmit antennas Nt).
- More than one code size can be considered, provided that they are all multiple of a common given sub-matrix size *W*, which represents the achievable degree of parallelism, and also allow to minimize the number of required padding bits in the MIMO-OFDM system. This is possible if they are multiple of the number of OFDM sub-channels too.

The proposed LDPCC scheme fits in a MIMO-OFDM system using 54 OFDM data tones, as in [1]; but can be easily generalized for other parameters, like OFDM systems having 48 data tones, by changing the block sizes of the LDPCCs. Applying the aforementioned criteria, the three coded block sizes equal to 1994, 1296, 648 bits have been selected.
A key feature of the proposed LDPCC schemes is the rate-compatibility, i.e. the ability to support different code rates reusing without increasing the hardware complexity required to implement the lowest, or “mother”, code rate (1/2 in our case). This is obtained designing parity-check matrices using a row-combining procedure; this technique is explained in detail in section 3 of the document. This way, code rates 1/2, 2/3, 3/4 and 5/6 have been generated. Particular care was applied to the optimization of the higher code rates (3/4 and 5/6) as high-throughput is the main property of the target system [1].
Structured parity check matrices are proposed. It can be shown that the performance of different matrices does not change significantly, even for matrices built according to different methods, for a codeword length in the order of 2000 bits or shorter. Thus with this regard, the real key factor for selecting the parity check matrix design strategy is the complexity. It can be demonstrated that structured parity check matrices greatly simplify the decoder design; another desirable property is having a same codeword size for all the supported code rates.

3 LDPCC parity-check matrices design

The key feature of the considered LDPCCs is that combining rows of the parity-check matrix (**H**) for the lowest rate code produces **H** for higher rates. This is equivalent to replacing a group of check nodes with a single check node that sums all the edges coming into each of the original check nodes. Direct consequences are that all code rates have the same block length and the same variable node degree distribution, a desirable property for LDPCCs.
The LDPCC encoder is systematic, i.e. encodes an information block of size k, I=$(i_0,i_1,\dots\ i_{(k-1)})$ into a codeword **C** of size n **C**=$(i_0,i_1,\dots\ i_{(k-1)},\ p_0,\ p_1,\dots,\ p_{(n-k-1)})$ by adding n-k parity bits obtained so that $\boldsymbol{H}\cdot\boldsymbol{c}^T=\boldsymbol{0}$.
k and n are reported in table 1, according to the selected code rate and size.

An example is shown in Fig. 1 where replacing each pair of nodes with a new single node converts a rate-1/2 code to a rate-3/4 code. This is equivalent to summing pairs of rows in the parity check matrix **H** if the check nodes don't have any common neighbors, as is the case with the codes presented here. This structure changes the rate without changing the block length or the basic architecture of the decoder. Furthermore, the variable degree distribution also does not change as the rate changes. A single variable node degree distribution can be employed that works well for all the different code rates.

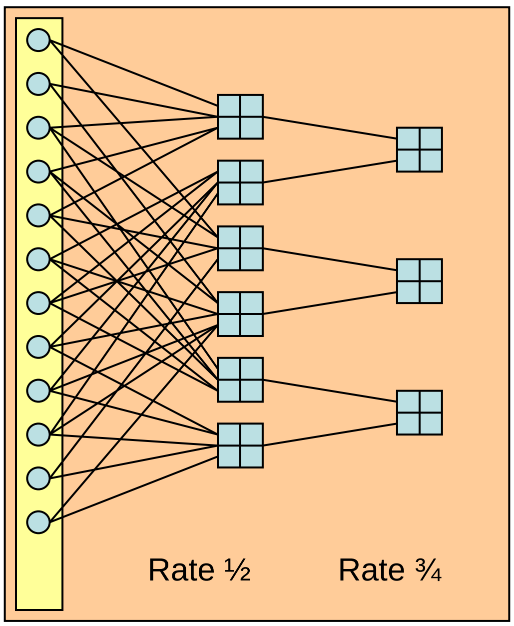

Figure 1 - Proposed Constant Block Length Multiple Rate Structure.

The mother parity check matrix used to generate all the code rates is rate 1/2 (i.e. number of rows n-k = n/2, n being the number of columns). This belongs to the class of the so-called structured parity check matrices (compared to "random" generated codes). More specifically, the matrix can be partitioned into square sub-blocks of size WxW where W=27, which are either cyclic permutation of the diagonal identity matrix (matrices S_i in fig. 3) or null sub-matrices. A few of them may contain two (cyclically shifted) diagonals.
In addition, in order to enable a simple encoding process the parity-check matrices include a block lower triangular structure at the right-hand side. This allows a linear-complexity encoder (based on back substitution).
More precisely, the parity-check matrix is of the type:

$$H_{(n-k)\times n} = [A_{(n-k)\times k}\ B_{(n-k)\times(n-k)}]$$

where B is block lower triangular, i.e. has the structure reported in fig. 2.

$$B_{(n-k)\times(n-k)} = \begin{pmatrix} b_{11} & 0 & \cdots & & 0 \\ b_{21} & b_{22} & 0 & & \\ & b_{32} & \ddots & \ddots & \vdots \\ \vdots & & \ddots & \ddots & 0 \\ & & & \ddots & 0 \\ b_{(n-k)1} & & \cdots & & b_{(n-k)(n-k)} \end{pmatrix}$$
, b_{ij} =circulant permutation matrix; (*i*,*j*) sub-matrix indices.

Figure 2 – lower triangular part of the parity check matrix

In order to generate variable rate LDPCCs, the following criteria apply:
a) Row-combining of rows which do not have a '1' in the same position (this implies a same resulting variable node degree distribution for all the code rates).
b) The selection of rows to be summed preserves the lower triangular structure of the right-hand side of the matrix through all the rates.
The last block at bottom right $b_{(n-k)(n-k)}$ is the double diagonal matrix, S_d in fig. 3.
Taking into account above-mentioned criteria, the code rates are obtained as follows:

- Code rate 2/3
row *i* (please notice that rows are enumerated from top to bottom) is generated by summing row *i* of **H** of rate 1/2 code and the row with index *i+M/2* where *M* is the total number of check equations in rate 1/2 code, for all the rows where *i < M/3*

- Code rate 3/4.
row *i* is generated by summing row *i* of **H** of rate 1/2 code and the row with index *i+M/2* where *M* is the total number of **H** rows (i.e. check equations) in rate 1/2 code.

- Code rate 5/6.
Rows are summed three at a time as: row *i* plus the row with index *i+M/3* plus the row with index *i+2M/3.*

On top of this the codes are designed to avoid length 4 cycles and also to have a good performance in the error floor region by conditioning the graph using the algorithms explained in [3] and [4].

Following figure 3 illustrates some examples (for a 10x10 matrix) of the basic circular permutation matrix S_i., clarifying how cyclic shift indexes *i* are to be interpreted.

$$S_0=\begin{bmatrix} 1 &&&&&&&&& \\ &1&&&&&&&& \\ &&1&&&&&&& \\ &&&1&&&&&& \\ &&&&1&&&&& \\ &&&&&1&&&& \\ &&&&&&1&&& \\ &&&&&&&1&& \\ &&&&&&&&1& \\ &&&&&&&&&1 \end{bmatrix} \quad S_3=\begin{bmatrix} &&&1&&&&&& \\ &&&&1&&&&& \\ &&&&&1&&&& \\ &&&&&&1&&& \\ &&&&&&&1&& \\ &&&&&&&&1& \\ &&&&&&&&&1 \\ 1&&&&&&&&& \\ &1&&&&&&&& \\ &&1&&&&&&& \end{bmatrix} \quad S_7=\begin{bmatrix} &&&&&&&1&& \\ &&&&&&&&1& \\ &&&&&&&&&1 \\ 1&&&&&&&&& \\ &1&&&&&&&& \\ &&1&&&&&&& \\ &&&1&&&&&& \\ &&&&1&&&&& \\ &&&&&1&&&& \\ &&&&&&1&&& \end{bmatrix} \quad S_d=\begin{bmatrix} 1 &&&&&&&&& \\ 1&1&&&&&&&& \\ &1&1&&&&&&& \\ &&1&1&&&&&& \\ &&&1&1&&&&& \\ &&&&1&1&&&& \\ &&&&&1&1&&& \\ &&&&&&1&1&& \\ &&&&&&&1&1& \\ &&&&&&&&1&1 \end{bmatrix}$$

Figure 3 – Structure of the cyclic permutations of the identity matrix. Matrix S_i is produced by cyclically shifting the columns of the identity matrix to the right *i* places.

3.1 LDPCC Code Rates and Block Sizes

The supported code rates – information and coded block sizes are reported in **Error! Reference source not found.**:

Table 1 — LDPCC parameters for 1944 – 1296 - 648 coded size (bits)

Coding rate (R)	LDPC n=1944: information bits (k)	LDPC n=1296 information bits (k)	LDPCn= 648 information bits (k)
1/2	972	648	324
2/3	1296	864	432
3/4	1458	972	486
5/6	1620	1080	540

4 Performance in AWGN

This paragraph reports the achievable performance of the proposed code in AWGN channel. The number of performed iteration is 12 for the LDPCC 1944. This value has been identified as the minimum for justifying the adoption of LDPCC. As a comparison, for shorter codeword lengths (1296 and 648) the paragraph includes also the performance achievable with more iterations. The decoder adopts the Sum Product Algorithm.

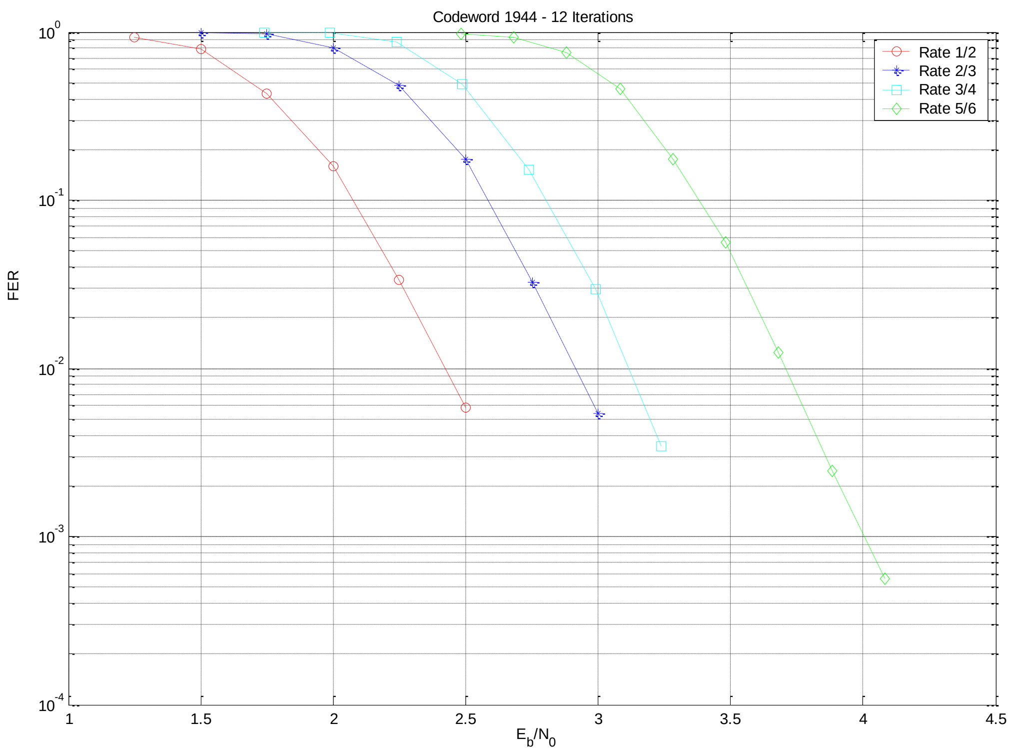

Figure 4 - Performance of LDPCC 1944 at different rates and 12 iterations.

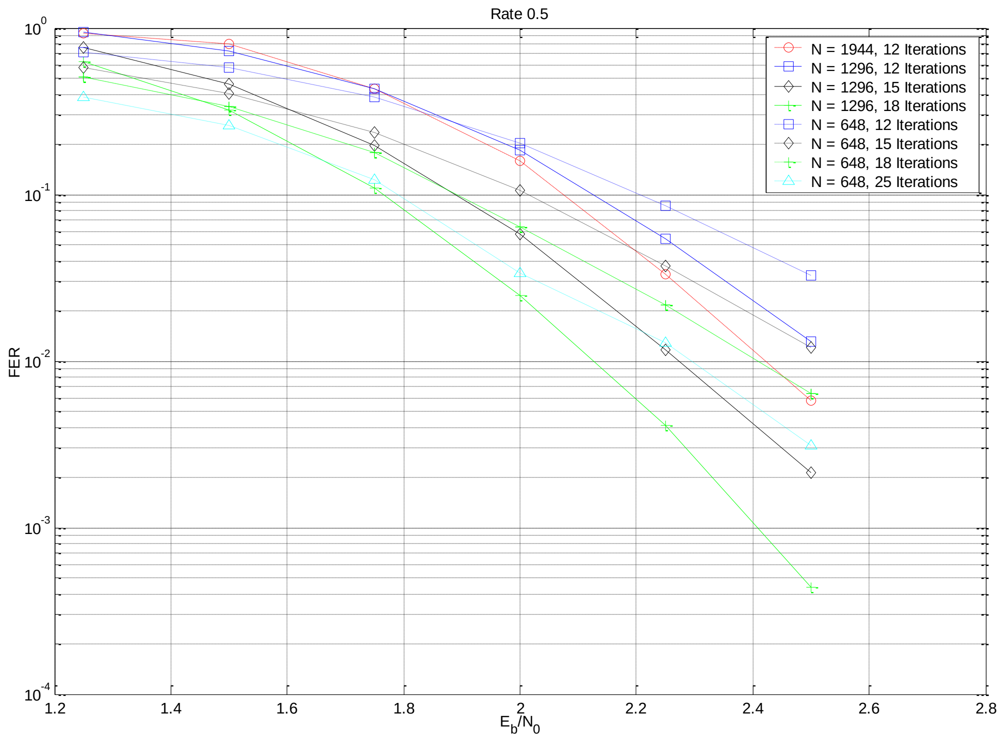

Figure 5 - Performance of LDPCC 1944, 1296 and 648 at rate ½ and different number of 12 iterations.

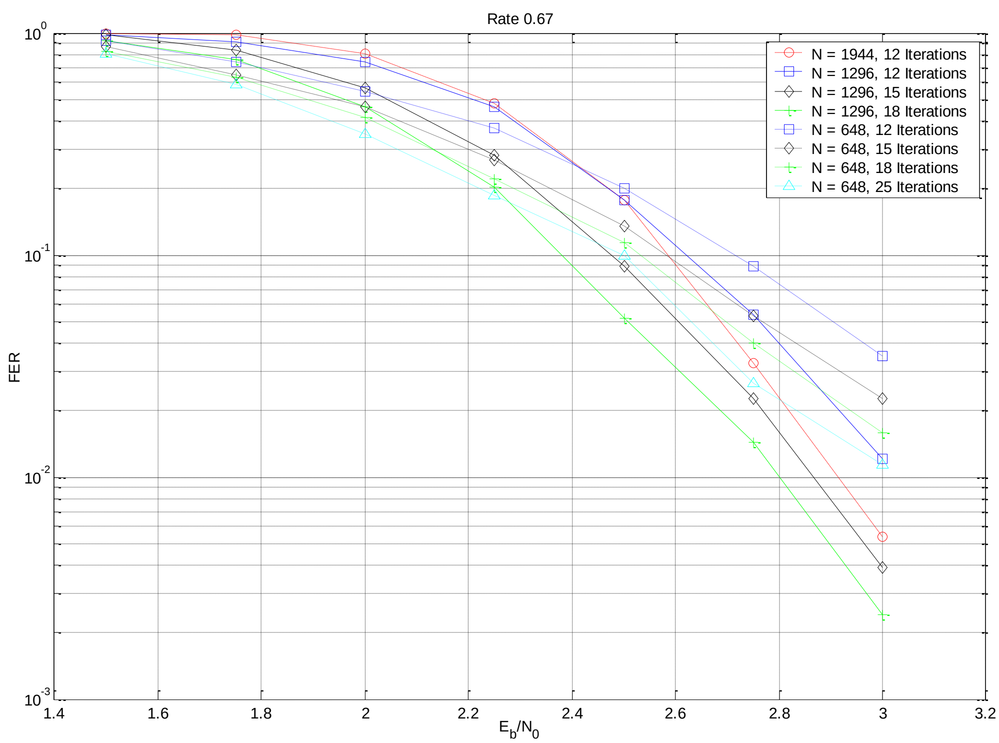

Figure 6 - Performance of LDPCC 1944, 1296 and 648 at rate 2/3 and different number of 12 iterations.

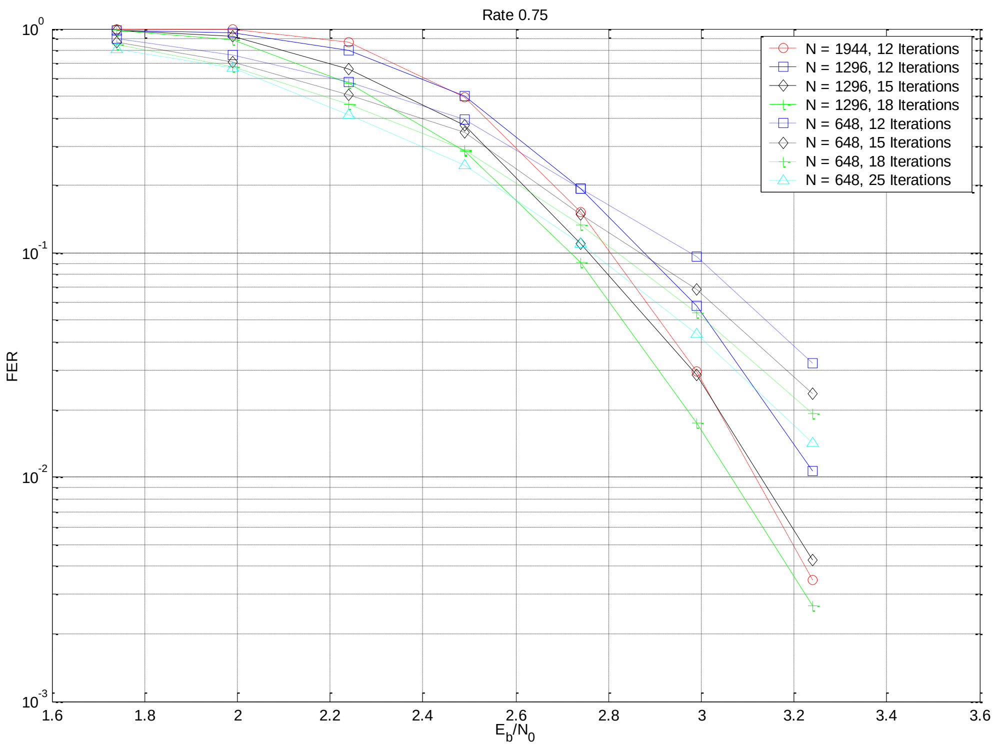

Figure 7 - Performance of LDPCC 1944, 1296 and 648 at rate 3/4 and different number of 12 iterations.

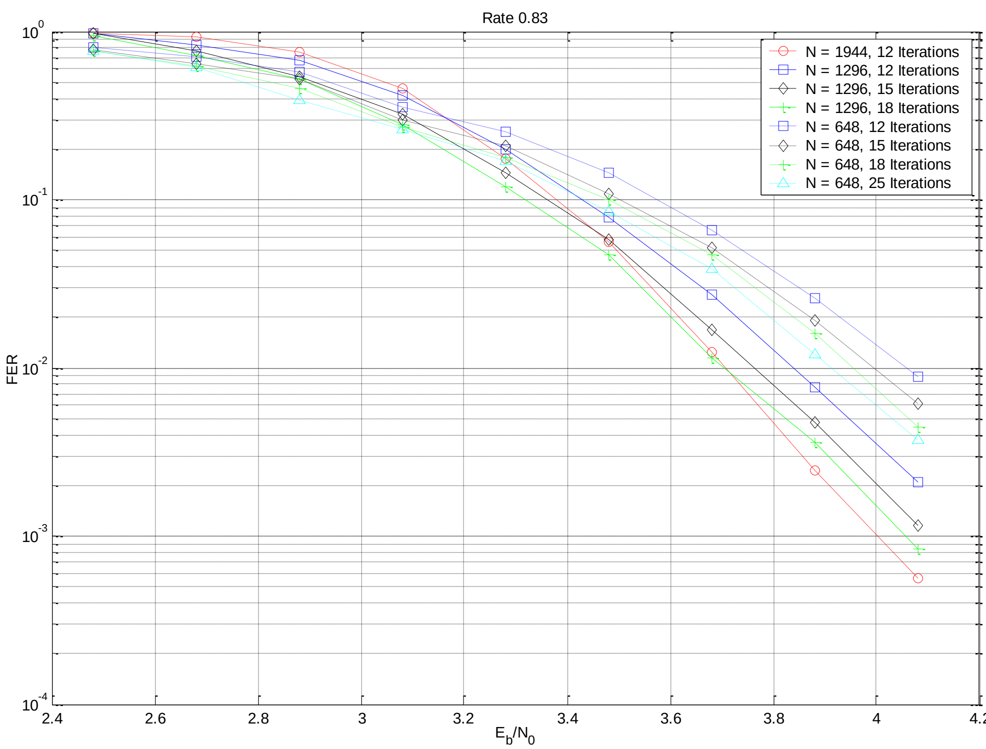

Figure 8 - Performance of LDPCC 1944, 1296 and 648 at rate 5/6 and different number of 12 iterations.

5 Performance in MIMO fading channel

The simulation setup is the following.

MIMO configurations 2x2, 2x3

Carrier Freq. 5 GHz

BW 20 MHz

Modes 16QAM rate 1/2 16QAM rate 3/4 64QAM rate 2/3 64QAM rate 3/4 64QAM rate 5/6

Packet size 1000Byte

Detector MMSE

Detector soft outputs Max-log-MAP (i.e. reduced complexity log-MAP)

Antenna separation 0.5 wavelength

PER/SNR Averaged over the whole simulation length (target: 100 independent packet errors down to 10-2 PER)

Channel estimation and synchronization: real

Channel models: as for [5]

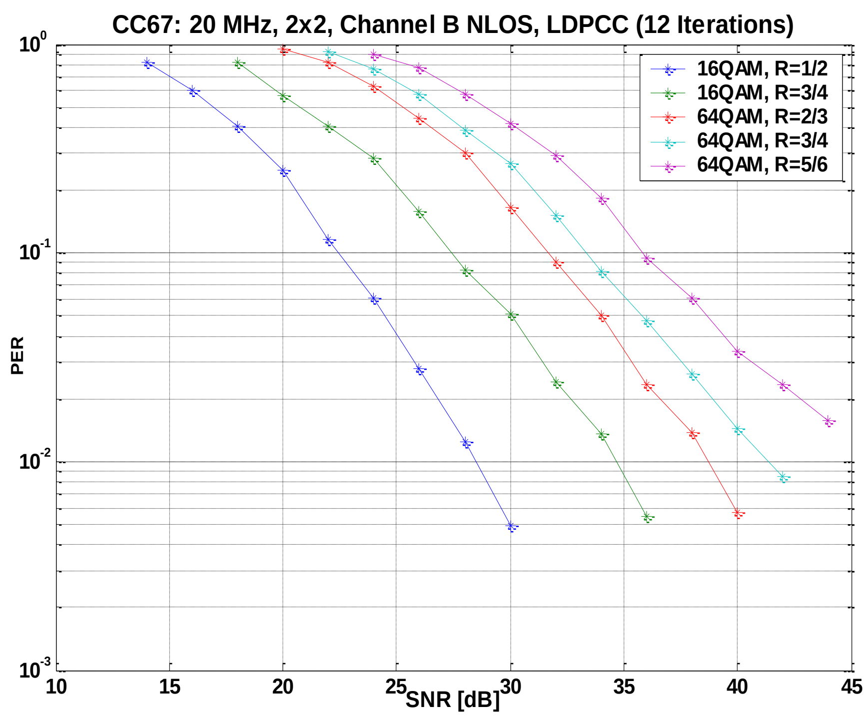

Figure 9 – 2x2 channel B - Performance of LDPCC 1944, 12 iterations, for various code rates and modulation orders.

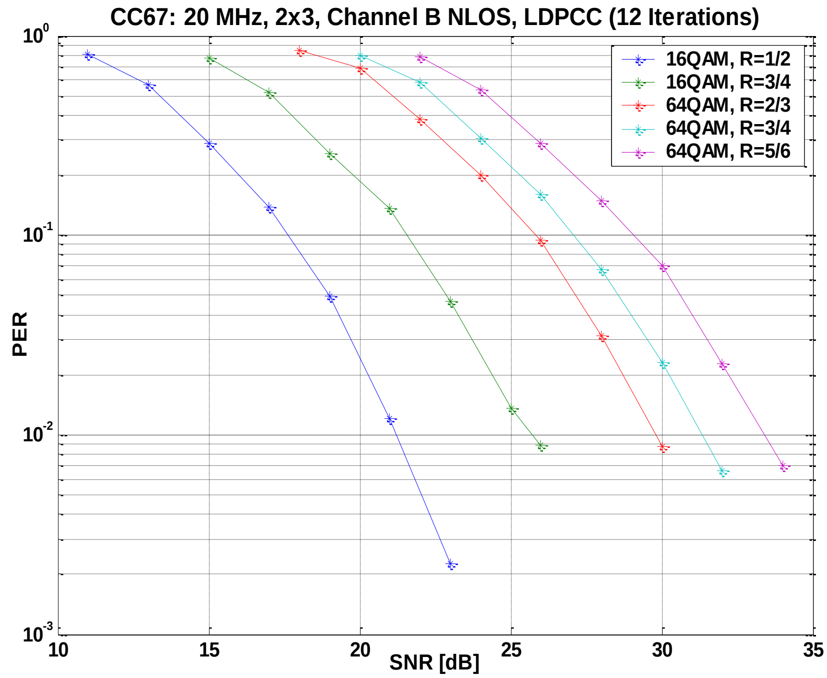

Figure 10 – 2x3 channel B - Performance of LDPCC 1944, 12 iterations, for various code rates and modulation orders.

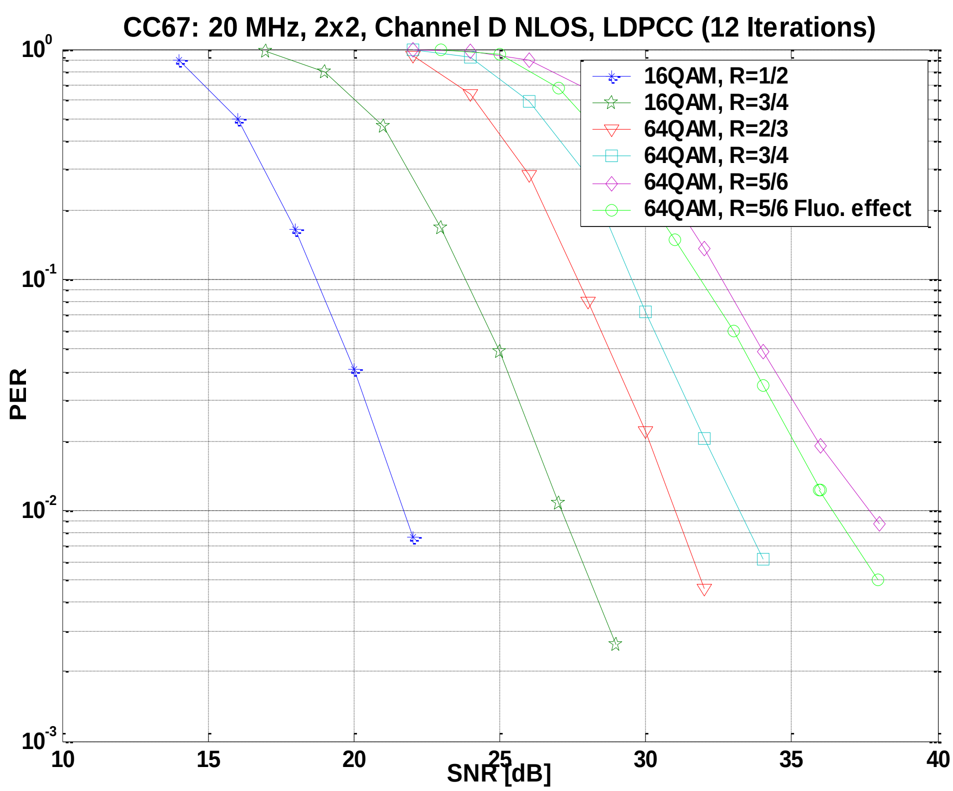

Figure 11 – 2x2 channel D - Performance of LDPCC 1944, 12 iterations, for various code rates and modulation orders.

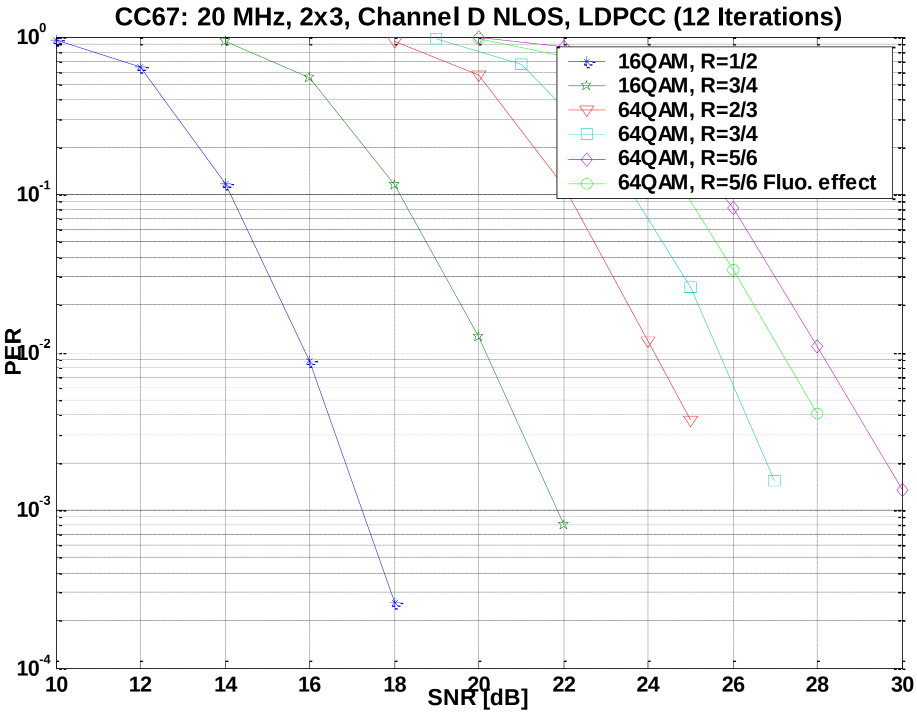

Figure 12 – 2x3 channel D - Performance of LDPCC 1944, 12 iterations, for various code rates and modulation orders.

6 References

[1] IEEE Std. 802.11a-1999, “Part 11: Wireless LAN Medium Access Control (MAC) and Physical Layer (PHY) specification: high speed physical layer in the 5 GHz band”, IEE-SA Standards Board (1999-09-16)

[2] IEEE 802.11/04-0886-00-000n, “WWiSE group PHY and MAC specification,” M. Singh, B. Edwards et al.

[3] Tian T., Jones C., Villasenor J. D. and Wesel R. D., "Selective Avoidance of Cycles in Irregular LDPC Code Construction," IEEE Transactions on Communications, August 2004.

[4] Ramamoorthy A. and Wesel R. D., "Construction of Short Block Length Irregular LDPC Codes,", in Proc. IEEE ICC 2004, Paris, France, June 2004.

[5] IEEE 802.11-03/940r1, "IEEE P802.11, Wireless LANs - TGn Channel Models" - January 9, 2004

[6] J. Ha and S.W. McLaughlin, “Analysis and design of punctured LDPCCs over Gaussian channel with erasures,” Proc. Int. Symposium Inform. Theory, Lausanne, Switzerland, June 2002.

[7] T. Tian, C. Jones, and J. Villasenor. “Rate-compatible low-density parity-check codes,” in ISIT 2004, Chicago, July 2004.

[8] M. M. Mansour and N. R. Shanbhag, “Low power VLSI decoder architectures for LDPC codes,” in 2002 International Low Power Electronics and Design, 2002, pp. 284-289.

[9] M. Yang and W. E. Ryan. “Lowering the Error-Rate Floors of Moderate-Length High-Rate Irergular LDPC Codes,” in IEEE International Symposium on Information Theory, Yokohama, Japan, June 2003.

Annex A - Matrix descriptors

Tables from 2 to 13 describe the “matrix prototype”, i.e. a structure where the basic elements are squared sub-matrices whose size is 27x27. Each sub matrix is either a null matrix or a cyclic permutation of the identity matrix (or the lower bidiagonal matrix at the right hand bottom of the matrix). For sake of simplicity only non-zero submatrices have been reported
The last line in all the tables, indicated as "st", indicates the staircase (or bi-diagonal) structure (S_d in fig. 2).
Tables follow the following format: Row number - Column number - Cyclic shift to the right (of columns).
Rows are enumerated from top to bottom, columns from left to right.

Table 2 – code size 1944 – Description of the matrix prototype of the rate 1/2 parity check matrix

Row	Col	Shift
0	3	9
0	7	15
0	16	5
0	22	23
0	32	6
0	35	12
0	36	23
1	2	12
1	6	16
1	10	19
1	26	12
1	31	23
1	36	5
1	37	24
2	2	9
2	9	15
2	16	2
2	29	15
2	37	8
2	37	9
2	38	2
3	1	11
3	5	15
3	14	17
3	22	7
3	36	12
3	38	19
3	39	25
4	4	15
4	8	3
4	10	15
4	26	16
4	34	6
4	39	6
4	40	10
5	4	20
5	10	24
5	11	17
5	23	4
5	33	25
5	40	24
5	41	15
6	1	14
6	5	17
6	13	0
6	28	14
6	35	16
6	41	24
6	42	5
7	1	7
7	9	7
7	11	3
7	21	12
7	30	18
7	42	15
7	43	21
8	2	9
8	7	25
8	10	4
8	18	15
8	34	15
8	43	18
8	44	18
9	4	13
9	9	5
9	18	6
9	20	3
9	39	8
9	44	14
9	45	15
10	0	3
10	7	11
10	11	10
10	23	21
10	33	17
10	45	13
10	46	12
11	1	11
11	9	9
11	11	15
11	29	6
11	38	11
11	46	9
11	47	5
12	5	16
12	5	22
12	10	9

12	24	0
12	45	5
12	47	7
12	48	10
13	3	13
13	10	0
13	13	20
13	20	23
13	42	7
13	48	21
13	49	14
14	3	8
14	7	1
14	11	3
14	25	24
14	46	3
14	49	3
14	50	7
15	0	8
15	9	0
15	11	5
15	19	26
15	31	8

15	50	5
15	51	2
16	4	4
16	8	6
16	15	4
16	19	11
16	43	20
16	51	5
16	52	16
17	1	10
17	5	24
17	10	13
17	27	3
17	32	26
17	52	7
17	53	24
18	4	10
18	8	8
18	12	10
18	30	21
18	48	7
18	53	16
18	54	17

19	4	23
19	6	15
19	14	14
19	28	4
19	50	5
19	54	13
19	55	11
20	3	9
20	6	18
20	15	16
20	25	5
20	40	0
20	55	12
20	56	8
21	0	4
21	6	20
21	11	6
21	24	9
21	55	20
21	56	24
21	57	12
22	2	10
22	8	12

22	12	12
22	27	4
22	53	3
22	57	0
22	58	19
23	0	2
23	9	17
23	17	17
23	21	9
23	49	14
23	58	22
23	59	23
24	1	16
24	7	5
24	17	7
24	29	12
24	56	19
24	59	23
24	60	4
25	0	8
25	8	13
25	18	1
25	28	25

25	44	1
25	60	26
25	61	13
26	4	1
26	9	1
26	21	10
26	26	12
26	52	3
26	61	6
26	62	18
27	3	21
27	5	8
27	22	13
27	27	17
27	59	21
27	62	21
27	63	12
28	1	13
28	8	26
28	20	18
28	32	11
28	58	21
28	63	23

28	64	3
29	2	16
29	5	6
29	19	10
29	25	17
29	54	20
29	64	16
29	65	8
30	3	16
30	7	15
30	16	1
30	31	0
30	41	13
30	65	15
30	66	17
31	0	20
31	6	8
31	15	4
31	34	1
31	51	22
31	66	4
31	67	8
32	2	11

32	7	17
32	13	3
32	33	12
32	57	20
32	67	7
32	68	3
33	0	19
33	6	22
33	14	8
33	30	14
33	35	18
33	68	16
33	69	0
34	3	6
34	6	0
34	17	16
34	24	19
34	47	12
34	69	1
34	70	21
35	2	22
35	8	22
35	12	26

35	23	1
35	70	5
35	71	st

Table 3 – code size 1944 – Description of the matrix prototype of the rate 2/3 parity check matrix

Row	Col	Shift
0	5	16
0	5	22
0	10	9
0	24	0
0	45	5
0	47	7
0	48	10
1	3	13
1	10	0
1	13	20
1	20	23
1	42	7
1	48	21
1	49	14
2	3	8
2	7	1
2	11	3
2	25	24
2	46	3
2	49	3
2	50	7
3	0	8
3	9	0
3	11	5
3	19	26
3	31	8
3	50	5
3	51	2
4	4	4
4	8	6
4	15	4
4	19	11
4	43	20
4	51	5
4	52	16
5	1	10
5	5	24
5	10	13
5	27	3
5	32	26
5	52	7
5	53	24
6	3	9
6	4	10
6	7	15
6	8	8
6	12	10
6	16	5
6	22	23
6	30	21
6	32	6
6	35	12
6	36	23
6	48	7
6	53	16
6	54	17
7	2	12
7	4	23
7	6	15
7	6	16
7	10	19
7	14	14
7	26	12
7	28	4
7	31	23
7	36	5
7	37	24
7	50	5
7	54	13
7	55	11
8	2	9
8	3	9
8	6	18
8	9	15
8	15	16
8	16	2
8	25	5
8	29	15
8	37	8
8	37	9
8	38	2
8	40	0
8	55	12

8	56	8
9	0	4
9	1	11
9	5	15
9	6	20
9	11	6
9	14	17
9	22	7
9	24	9
9	36	12
9	38	19
9	39	25
9	55	20
9	56	24
9	57	12
10	2	10
10	4	15
10	8	3
10	8	12
10	10	15
10	12	12
10	26	16

10	27	4
10	34	6
10	39	6
10	40	10
10	53	3
10	57	0
10	58	19
11	0	2
11	4	20
11	9	17
11	10	24
11	11	17
11	17	17
11	21	9
11	23	4
11	33	25
11	40	24
11	41	15
11	49	14
11	58	22
11	59	23
12	1	14

12	1	16
12	5	17
12	7	5
12	13	0
12	17	7
12	28	14
12	29	12
12	35	16
12	41	24
12	42	5
12	56	19
12	59	23
12	60	4
13	0	8
13	1	7
13	8	13
13	9	7
13	11	3
13	18	1
13	21	12
13	28	25
13	30	18

13	42	15
13	43	21
13	44	1
13	60	26
13	61	13
14	2	9
14	4	1
14	7	25
14	9	1
14	10	4
14	18	15
14	21	10
14	26	12
14	34	15
14	43	18
14	44	18
14	52	3
14	61	6
14	62	18
15	3	21
15	4	13
15	5	8

15	9	5
15	18	6
15	20	3
15	22	13
15	27	17
15	39	8
15	44	14
15	45	15
15	59	21
15	62	21
15	63	12
16	0	3
16	1	13
16	7	11
16	8	26
16	11	10
16	20	18
16	23	21
16	32	11
16	33	17
16	45	13
16	46	12

16	58	21
16	63	23
16	64	3
17	1	11
17	2	16
17	5	6
17	9	9
17	11	15
17	19	10
17	25	17
17	29	6
17	38	11
17	46	9
17	47	5
17	54	20
17	64	16
17	65	8
18	3	16
18	7	15
18	16	1
18	31	0
18	41	13

18	65	15
18	66	17
19	0	20
19	6	8
19	15	4
19	34	1
19	51	22
19	66	4
19	67	8
20	2	11
20	7	17
20	13	3
20	33	12
20	57	20
20	67	7
20	68	3
21	0	19
21	6	22
21	14	8
21	30	14
21	35	18
21	68	16

21	69	0
22	3	6
22	6	0
22	17	16
22	24	19
22	47	12
22	69	1
22	70	21
23	2	22
23	8	22
23	12	26
23	23	1
23	70	5
23	71	st

1

2

3

Table 4 – code size 1944 – Description of the matrix prototype of the rate 3/4 parity check matrix

Row	**Col**	**Shift**
0	3	9
0	4	10
0	7	15
0	8	8
0	12	10
0	16	5
0	22	23
0	30	21
0	32	6
0	35	12
0	36	23
0	48	7
0	53	16
0	54	17
1	2	12
1	4	23
1	6	15
1	6	16
1	10	19
1	14	14
1	26	12
1	28	4
1	31	23
1	36	5
1	37	24
1	50	5
1	54	13
1	55	11
2	2	9
2	3	9
2	6	18
2	9	15
2	15	16
2	16	2
2	25	5
2	29	15
2	37	8
2	37	9
2	38	2
2	40	0
2	55	12
2	56	8
3	0	4
3	1	11
3	5	15
3	6	20
3	11	6
3	14	17
3	22	7
3	24	9
3	36	12
3	38	19
3	39	25
3	55	20
3	56	24
3	57	12
4	2	10
4	4	15
4	8	3
4	8	12
4	10	15
4	12	12
4	26	16
4	27	4
4	34	6
4	39	6
4	40	10
4	53	3
4	57	0
4	58	19
5	0	2
5	4	20
5	9	17
5	10	24
5	11	17
5	17	17
5	21	9
5	23	4
5	33	25
5	40	24
5	41	15
5	49	14
5	58	22

5	59	23
6	1	14
6	1	16
6	5	17
6	7	5
6	13	0
6	17	7
6	28	14
6	29	12
6	35	16
6	41	24
6	42	5
6	56	19
6	59	23
6	60	4
7	0	8
7	1	7
7	8	13
7	9	7
7	11	3
7	18	1
7	21	12

7	28	25
7	30	18
7	42	15
7	43	21
7	44	1
7	60	26
7	61	13
8	2	9
8	4	1
8	7	25
8	9	1
8	10	4
8	18	15
8	21	10
8	26	12
8	34	15
8	43	18
8	44	18
8	52	3
8	61	6
8	62	18
9	3	21

9	4	13
9	5	8
9	9	5
9	18	6
9	20	3
9	22	13
9	27	17
9	39	8
9	44	14
9	45	15
9	59	21
9	62	21
9	63	12
10	0	3
10	1	13
10	7	11
10	8	26
10	11	10
10	20	18
10	23	21
10	32	11
10	33	17

10	45	13
10	46	12
10	58	21
10	63	23
10	64	3
11	1	11
11	2	16
11	5	6
11	9	9
11	11	15
11	19	10
11	25	17
11	29	6
11	38	11
11	46	9
11	47	5
11	54	20
11	64	16
11	65	8
12	3	16
12	5	16
12	5	22

12	7	15
12	10	9
12	16	1
12	24	0
12	31	0
12	41	13
12	45	5
12	47	7
12	48	10
12	65	15
12	66	17
13	0	20
13	3	13
13	6	8
13	10	0
13	13	20
13	15	4
13	20	23
13	34	1
13	42	7
13	48	21
13	49	14

13	51	22
13	66	4
13	67	8
14	2	11
14	3	8
14	7	1
14	7	17
14	11	3
14	13	3
14	25	24
14	33	12
14	46	3
14	49	3
14	50	7
14	57	20
14	67	7
14	68	3
15	0	8
15	0	19
15	6	22
15	9	0
15	11	5

15	14	8
15	19	26
15	30	14
15	31	8
15	35	18
15	50	5
15	51	2
15	68	16
15	69	0
16	3	6
16	4	4
16	6	0
16	8	6
16	15	4
16	17	16
16	19	11
16	24	19
16	43	20
16	47	12
16	51	5
16	52	16
16	69	1

16	70	21
17	1	10
17	2	22
17	5	24
17	8	22
17	10	13
17	12	26
17	23	1
17	27	3
17	32	26
17	52	7
17	53	24
17	70	5
17	71	st

1

2

3

4

5

Table 5 – code size 1944 – Description of the matrix prototype of the rate 5/6 parity check matrix

Row	Col	Shift
0	1	16
0	3	9
0	5	16
0	5	22
0	7	5
0	7	15
0	10	9
0	16	5
0	17	7
0	22	23
0	24	0
0	29	12
0	32	6
0	35	12
0	36	23
0	45	5
0	47	7
0	48	10
0	56	19
0	59	23
0	60	4
1	0	8
1	2	12
1	3	13
1	6	16
1	8	13
1	10	0
1	10	19
1	13	20
1	18	1
1	20	23
1	26	12
1	28	25
1	31	23
1	36	5
1	37	24
1	42	7
1	44	1
1	48	21
1	49	14
1	60	26
1	61	13
2	2	9
2	3	8
2	4	1
2	7	1
2	9	1
2	9	15
2	11	3
2	16	2
2	21	10
2	25	24
2	26	12
2	29	15
2	37	8
2	37	9
2	38	2
2	46	3
2	49	3
2	50	7
2	52	3
2	61	6
2	62	18
3	0	8
3	1	11
3	3	21
3	5	8
3	5	15
3	9	0
3	11	5
3	14	17
3	19	26
3	22	7
3	22	13
3	27	17
3	31	8
3	36	12
3	38	19
3	39	25
3	50	5
3	51	2
3	59	21
3	62	21

3	63	12
4	1	13
4	4	4
4	4	15
4	8	3
4	8	6
4	8	26
4	10	15
4	15	4
4	19	11
4	20	18
4	26	16
4	32	11
4	34	6
4	39	6
4	40	10
4	43	20
4	51	5
4	52	16
4	58	21
4	63	23
4	64	3

5	1	10
5	2	16
5	4	20
5	5	6
5	5	24
5	10	13
5	10	24
5	11	17
5	19	10
5	23	4
5	25	17
5	27	3
5	32	26
5	33	25
5	40	24
5	41	15
5	52	7
5	53	24
5	54	20
5	64	16
5	65	8
6	1	14

6	3	16
6	4	10
6	5	17
6	7	15
6	8	8
6	12	10
6	13	0
6	16	1
6	28	14
6	30	21
6	31	0
6	35	16
6	41	13
6	41	24
6	42	5
6	48	7
6	53	16
6	54	17
6	65	15
6	66	17
7	0	20
7	1	7

7	4	23
7	6	8
7	6	15
7	9	7
7	11	3
7	14	14
7	15	4
7	21	12
7	28	4
7	30	18
7	34	1
7	42	15
7	43	21
7	50	5
7	51	22
7	54	13
7	55	11
7	66	4
7	67	8
8	2	9
8	2	11
8	3	9

8	6	18
8	7	17
8	7	25
8	10	4
8	13	3
8	15	16
8	18	15
8	25	5
8	33	12
8	34	15
8	40	0
8	43	18
8	44	18
8	55	12
8	56	8
8	57	20
8	67	7
8	68	3
9	0	4
9	0	19
9	4	13
9	6	20

9	6	22
9	9	5
9	11	6
9	14	8
9	18	6
9	20	3
9	24	9
9	30	14
9	35	18
9	39	8
9	44	14
9	45	15
9	55	20
9	56	24
9	57	12
9	68	16
9	69	0
10	0	3
10	2	10
10	3	6
10	6	0
10	7	11

10	8	12
10	11	10
10	12	12
10	17	16
10	23	21
10	24	19
10	27	4
10	33	17
10	45	13
10	46	12
10	47	12
10	53	3
10	57	0
10	58	19
10	69	1
10	70	21
11	0	2
11	1	11
11	2	22
11	8	22
11	9	9
11	9	17

11	11	15
11	12	26
11	17	17
11	21	9
11	23	1
11	29	6
11	38	11
11	46	9
11	47	5
11	49	14
11	58	22
11	59	23
11	70	5
11	71	st

1

Table 6 – Description of the matrix prototype of the rate ½ parity check matrix with length 1296

Row	Col	Shift
0	0	5
0	4	16
0	6	4
0	15	19
0	23	11
0	23	17
0	24	23
1	0	8
1	5	21
1	7	1
1	18	11
1	21	26
1	24	23
1	25	21
2	2	4
2	3	6
2	7	18
2	16	5
2	22	19
2	25	3
2	26	17
3	0	17
3	3	0
3	7	23
3	17	1
3	22	15
3	26	5
3	27	18
4	2	23
4	4	19
4	8	1
4	12	19
4	26	21
4	27	12
4	28	14
5	3	3
5	4	21
5	7	5
5	15	13
5	20	21
5	28	5
5	29	1
6	2	7
6	5	20
6	9	3
6	18	7
6	25	22
6	29	1
6	30	3
7	2	14
7	4	16
7	11	17
7	19	7
7	20	14
7	30	12
7	31	16
8	0	17
8	3	18
8	7	17
8	13	16
8	21	14
8	31	18
8	32	13
9	1	9
9	4	22
9	10	1
9	17	23
9	32	3
9	32	12
9	33	1
10	1	4
10	6	13
10	7	13
10	12	4
10	29	22
10	33	5
10	34	24
11	0	7
11	3	15
11	7	20
11	13	2
11	28	22
11	34	19
11	35	4

12	3	8
12	4	17
12	10	26
12	19	21
12	33	10
12	35	6
12	36	0
13	1	4
13	5	15
13	9	20
13	14	10
13	35	20
13	36	7
13	37	22
14	0	19
14	6	21
14	11	17
14	14	9
14	36	22
14	37	17
14	38	23
15	0	21

15	5	9
15	8	9
15	16	21
15	31	19
15	38	20
15	39	4
16	3	20
16	4	7
16	13	19
16	20	21
16	30	25
16	39	26
16	40	3
17	2	5
17	5	6
17	14	0
17	23	13
17	39	20
17	40	16
17	41	8
18	1	6
18	6	26

18	10	9
18	21	26
18	38	24
18	41	15
18	42	17
19	1	7
19	6	25
19	12	8
19	19	23
19	34	21
19	42	4
19	43	8
20	1	12
20	5	7
20	15	16
20	18	1
20	37	26
20	43	7
20	44	3
21	2	13
21	5	18
21	11	1

21	22	15
21	24	15
21	44	16
21	45	0
22	1	17
22	6	17
22	9	0
22	17	17
22	27	21
22	45	1
22	46	21
23	2	5
23	6	25
23	8	1
23	16	26
23	46	5
23	47	

Table 7 – Description of the matrix prototype of the rate 2/3 parity check matrixwith codeword length 1296

Row	Col	Shift
0	0	17
0	3	18
0	7	17
0	13	16
0	21	14
0	31	18
0	32	13
1	1	9
1	4	22
1	10	1
1	17	23
1	32	3
1	32	12
1	33	1
2	1	4
2	6	13
2	7	13
2	12	4
2	29	22
2	33	5
2	34	24
3	0	7
3	3	15
3	7	20
3	13	2
3	28	22
3	34	19
3	35	4
4	0	5
4	3	8
4	4	16
4	4	17
4	6	4
4	10	26
4	15	19
4	19	21
4	23	11
4	23	17
4	24	23
4	33	10
4	35	6
4	36	0
5	0	8
5	1	4
5	5	15
5	5	21
5	7	1
5	9	20
5	14	10
5	18	11
5	21	26
5	24	23
5	25	21
5	35	20
5	36	7
5	37	22
6	0	19
6	2	4
6	3	6
6	6	21
6	7	18
6	11	17
6	14	9
6	16	5
6	22	19
6	25	3
6	26	17
6	36	22
6	37	17
6	38	23
7	0	17
7	0	21
7	3	0
7	5	9
7	7	23
7	8	9
7	16	21
7	17	1
7	22	15
7	26	5
7	27	18
7	31	19
7	38	20
7	39	4

8	2	23
8	3	20
8	4	7
8	4	19
8	8	1
8	12	19
8	13	19
8	20	21
8	26	21
8	27	12
8	28	14
8	30	25
8	39	26
8	40	3
9	2	5
9	3	3
9	4	21
9	5	6
9	7	5
9	14	0
9	15	13
9	20	21

9	23	13
9	28	5
9	29	1
9	39	20
9	40	16
9	41	8
10	1	6
10	2	7
10	5	20
10	6	26
10	9	3
10	10	9
10	18	7
10	21	26
10	25	22
10	29	1
10	30	3
10	38	24
10	41	15
10	42	17
11	1	7
11	2	14

11	4	16
11	6	25
11	11	17
11	12	8
11	19	7
11	19	23
11	20	14
11	30	12
11	31	16
11	34	21
11	42	4
11	43	8
12	1	12
12	5	7
12	15	16
12	18	1
12	37	26
12	43	7
12	44	3
13	2	13
13	5	18
13	11	1

13	22	15
13	24	15
13	44	16
13	45	0
14	1	17
14	6	17
14	9	0
14	17	17
14	27	21
14	45	1
14	46	21
15	2	5
15	6	25
15	8	1
15	16	26
15	46	5
15	47	st

1
2

Table 8 – Description of the matrix prototype of the rate 3/4 parity check matrixwith codeword length 1296

Row	**Col**	**Shift**
0	0	5
0	3	8
0	4	16
0	4	17
0	6	4
0	10	26
0	15	19
0	19	21
0	23	11
0	23	17
0	24	23
0	33	10
0	35	6
0	36	0
1	0	8
1	1	4
1	5	15
1	5	21
1	7	1
1	9	20
1	14	10
1	18	11
1	21	26
1	24	23
1	25	21
1	35	20
1	36	7
1	37	22
2	0	19
2	2	4
2	3	6
2	6	21
2	7	18
2	11	17
2	14	9
2	16	5
2	22	19
2	25	3
2	26	17
2	36	22
2	37	17
2	38	23
3	0	17
3	0	21
3	3	0
3	5	9
3	7	23
3	8	9
3	16	21
3	17	1
3	22	15
3	26	5
3	27	18
3	31	19
3	38	20
3	39	4
4	2	23
4	3	20
4	4	7
4	4	19
4	8	1
4	12	19
4	13	19
4	20	21
4	26	21
4	27	12
4	28	14
4	30	25
4	39	26
4	40	3
5	2	5
5	3	3
5	4	21
5	5	6
5	7	5
5	14	0
5	15	13
5	20	21
5	23	13
5	28	5
5	29	1
5	39	20
5	40	16
5	41	8

6	1	6
6	2	7
6	5	20
6	6	26
6	9	3
6	10	9
6	18	7
6	21	26
6	25	22
6	29	1
6	30	3
6	38	24
6	41	15
6	42	17
7	1	7
7	2	14
7	4	16
7	6	25
7	11	17
7	12	8
7	19	7
7	19	23

7	20	14
7	30	12
7	31	16
7	34	21
7	42	4
7	43	8
8	0	17
8	1	12
8	3	18
8	5	7
8	7	17
8	13	16
8	15	16
8	18	1
8	21	14
8	31	18
8	32	13
8	37	26
8	43	7
8	44	3
9	1	9
9	2	13

9	4	22
9	5	18
9	10	1
9	11	1
9	17	23
9	22	15
9	24	15
9	32	3
9	32	12
9	33	1
9	44	16
9	45	0
10	1	4
10	1	17
10	6	13
10	6	17
10	7	13
10	9	0
10	12	4
10	17	17
10	27	21
10	29	22

10	33	5
10	34	24
10	45	1
10	46	21
11	0	7
11	2	5
11	3	15
11	6	25
11	7	20
11	8	1
11	13	2
11	16	26
11	28	22
11	34	19
11	35	4
11	46	5
11	47	st

1
2

Table 9 – Description of the matrix prototype of the rate 5/6 parity check matrixwith codeword length 1296

Row	Col	Shift
0	0	5
0	0	17
0	3	18
0	3	20
0	4	7
0	4	16
0	6	4
0	7	17
0	13	16
0	13	19
0	15	19
0	20	21
0	21	14
0	23	11
0	23	17
0	24	23
0	30	25
0	31	18
0	32	13
0	39	26
0	40	3
1	0	8
1	1	9
1	2	5
1	4	22
1	5	6
1	5	21
1	7	1
1	10	1
1	14	0
1	17	23
1	18	11
1	21	26
1	23	13
1	24	23
1	25	21
1	32	3
1	32	12
1	33	1
1	39	20
1	40	16
1	41	8
2	1	4
2	1	6
2	2	4
2	3	6
2	6	13
2	6	26
2	7	13
2	7	18
2	10	9
2	12	4
2	16	5
2	21	26
2	22	19
2	25	3
2	26	17
2	29	22
2	33	5
2	34	24
2	38	24
2	41	15
2	42	17
3	0	7
3	0	17
3	1	7
3	3	0
3	3	15
3	6	25
3	7	20
3	7	23
3	12	8
3	13	2
3	17	1
3	19	23
3	22	15
3	26	5
3	27	18
3	28	22
3	34	19
3	34	21
3	35	4
3	42	4
3	43	8

4	1	12
4	2	23
4	3	8
4	4	17
4	4	19
4	5	7
4	8	1
4	10	26
4	12	19
4	15	16
4	18	1
4	19	21
4	26	21
4	27	12
4	28	14
4	33	10
4	35	6
4	36	0
4	37	26
4	43	7
4	44	3
5	1	4

5	2	13
5	3	3
5	4	21
5	5	15
5	5	18
5	7	5
5	9	20
5	11	1
5	14	10
5	15	13
5	20	21
5	22	15
5	24	15
5	28	5
5	29	1
5	35	20
5	36	7
5	37	22
5	44	16
5	45	0
6	0	19
6	1	17

6	2	7
6	5	20
6	6	17
6	6	21
6	9	0
6	9	3
6	11	17
6	14	9
6	17	17
6	18	7
6	25	22
6	27	21
6	29	1
6	30	3
6	36	22
6	37	17
6	38	23
6	45	1
6	46	21
7	0	21
7	2	5
7	2	14

7	4	16
7	5	9
7	6	25
7	8	1
7	8	9
7	11	17
7	16	21
7	16	26
7	19	7
7	20	14
7	30	12
7	31	16
7	31	19
7	38	20
7	39	4
7	46	5
7	47	st

1

2

3

4

5

Table 10 – Description of the matrix prototype of the rate 1/2 parity check matrixwith codeword length 648

Row	Col	Shift
0	1	6
0	1	8
0	4	7
0	7	22
0	11	5
0	11	20
0	12	15
1	1	12
1	3	6
1	3	20
1	9	8
1	12	9
1	12	26
1	13	1
2	1	2
2	2	10
2	5	26
2	8	0
2	10	1
2	13	15
2	14	24
3	0	18
3	3	4
3	5	0
3	8	8
3	13	9
3	14	1
3	15	8
4	0	5
4	2	19
4	3	6
4	6	19
4	10	26
4	15	17
4	16	7
5	1	18
5	2	7
5	3	10
5	9	26
5	14	1
5	16	23
5	17	13
6	0	3
6	2	24
6	4	11
6	6	20
6	16	26
6	17	6
6	18	3
7	0	24
7	2	3
7	3	9
7	7	0
7	15	12
7	18	16
7	19	8
8	0	7
8	1	4
8	7	1
8	11	4
8	18	7
8	19	7
8	20	3
9	0	2
9	2	24
9	4	6
9	9	15
9	19	24
9	20	16
9	21	0
10	1	3
10	2	16
10	6	5
10	10	2
10	17	12
10	21	1
10	22	21
11	0	25
11	3	8
11	5	15
11	8	2
11	22	5
11	23	st

Table 11 – Description of the matrix prototype of the rate 2/3 parity check matrixwith codeword length 648

Row	**Col**	**Shift**
0	0	5
0	2	19
0	3	6
0	6	19
0	10	26
0	15	17
0	16	7
1	1	18
1	2	7
1	3	10
1	9	26
1	14	1
1	16	23
1	17	13
2	0	3
2	1	6
2	1	8
2	2	24
2	4	7
2	4	11
2	6	20
2	7	22
2	11	5
2	11	20
2	12	15
2	16	26
2	17	6
2	18	3
3	0	24
3	1	12
3	2	3
3	3	6
3	3	9
3	3	20
3	7	0
3	9	8
3	12	9
3	12	26
3	13	1
3	15	12
3	18	16
3	19	8
4	0	7
4	1	2
4	1	4
4	2	10
4	5	26
4	7	1
4	8	0
4	10	1
4	11	4
4	13	15
4	14	24
4	18	7
4	19	7
4	20	3
5	0	2
5	0	18
5	2	24
5	3	4
5	4	6
5	5	0
5	8	8
5	9	15
5	13	9
5	14	1
5	15	8
5	19	24
5	20	16
5	21	0
6	1	3
6	2	16
6	6	5
6	10	2
6	17	12
6	21	1
6	22	21
7	0	25
7	3	8
7	5	15
7	8	2
7	22	5
7	23	st

Table 12 – Description of the matrix prototype of the rate 3/4 parity check matrixwith codeword length 648

Row	Col	Shift
0	0	3
0	1	6
0	1	8
0	2	24
0	4	7
0	4	11
0	6	20
0	7	22
0	11	5
0	11	20
0	12	15
0	16	26
0	17	6
0	18	3
1	0	24
1	1	12
1	2	3
1	3	6
1	3	9
1	3	20
1	7	0
1	9	8
1	12	9
1	12	26
1	13	1
1	15	12
1	18	16
1	19	8
2	0	7
2	1	2
2	1	4
2	2	10
2	5	26
2	7	1
2	8	0
2	10	1
2	11	4
2	13	15
2	14	24
2	18	7
2	19	7
2	20	3
3	0	2
3	0	18
3	2	24
3	3	4
3	4	6
3	5	0
3	8	8
3	9	15
3	13	9
3	14	1
3	15	8
3	19	24
3	20	16
3	21	0
4	0	5
4	1	3
4	2	16
4	2	19
4	3	6
4	6	5
4	6	19
4	10	2
4	10	26
4	15	17
4	16	7
4	17	12
4	21	1
4	22	21
5	0	25
5	1	18
5	2	7
5	3	8
5	3	10
5	5	15
5	8	2
5	9	26
5	14	1
5	16	23
5	17	13
5	22	5
5	23	st

Table 13 – Description of the matrix prototype of the rate 5/6 parity check matrixwith codeword length 648

Row	Col	Shift
0	0	5
0	0	7
0	1	4
0	1	6
0	1	8
0	2	19
0	3	6
0	4	7
0	6	19
0	7	1
0	7	22
0	10	26
0	11	4
0	11	5
0	11	20
0	12	15
0	15	17
0	16	7
0	18	7
0	19	7
0	20	3
1	0	2
1	1	12
1	1	18
1	2	7
1	2	24
1	3	6
1	3	10
1	3	20
1	4	6
1	9	8
1	9	15
1	9	26
1	12	9
1	12	26
1	13	1
1	14	1
1	16	23
1	17	13
1	19	24
1	20	16
1	21	0
2	0	3
2	1	2
2	1	3
2	2	10
2	2	16
2	2	24
2	4	11
2	5	26
2	6	5
2	6	20
2	8	0
2	10	1
2	10	2
2	13	15
2	14	24
2	16	26
2	17	6
2	17	12
2	18	3
2	21	1
2	22	21
3	0	18
3	0	24
3	0	25
3	2	3
3	3	4
3	3	8
3	3	9
3	5	0
3	5	15
3	7	0
3	8	2
3	8	8
3	13	9
3	14	1
3	15	8
3	15	12
3	18	16
3	19	8
3	22	5
3	23	st