Stopping-Set Structures up to Weight Five of Low-Density Parity-Check Codes

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Abstract

This manuscript contains the list of all stopping-set structures of LDPC codes up to weight five. Stopping sets are the failure sets for an iterative message-passing decoder over the binary erasure channel (BEC). Avoding low weight stopping-set sub-codes in an LDPC code construction is equivalent to increasing the minimum distance of the code.

I. Introduction

Low-density parity-check (LDPC) codes are a class of linear block codes with sparse parity-check matrices that were discovered by Gallager [1] and later shown to approach capacity [2]. LDPC code design techniques can be categorized as being *algebraic* or *non-algebraic*. Algebraic techniques design the code analytically as in [3] and others. Non-algebraic techniques construct codes by optimizing some of the properties of the code's underlying graph such as *girth, degree distribution, cycle extrinsic message degree* etc. [4], [5], [6]. The constructed graphs correspond to binary LDPC codes with edges indicating binary 1s in the corresponding parity-check matrices. Non-zero labels chosen from a finite Galois field GF(q) with q $>$ 2 in the place of the 1s of the constructed parity-check matrices yield non-binary LDPC (NB-LDPC) codes [7].

This manuscript presents an exhaustive list of all stopping-set structures of LDPC codes up to weight five whose parity-check matrices do not contain variable nodes of degree one and have girth of at least six. A similar work in [8] catalogs the list of some trapping sets of LDPC codes. The manuscript is organized as follows: Section II reviews background and notation. Section III presents all the stopping-set structures up to weight five.

II. Background and Notation

An LDPC code is defined by a sparse parity-check matrix H containing m rows and n columns, whose entries belong to a finite field GF(q). The rate of the code is given by $r \geq \frac{n-m}{n}$, with equality if H is full rank. A bipartite undirected Tanner graph $G(V, C, E)$ is an alternative representation for H. The columns of H form the variable-node set V and the rows of H form the check-node set C. The edge set E has elements $e_{ij} : H_{ji} > 0$ which connect elements from V with elements from C. The number of edges connected to a node is its degree. The girth g of a graph is the length of its shortest cycle(s).

A *stopping set* [9] S is a set of a variable nodes whose b neighboring check nodes each have at least two edges connected to S. With no variable nodes of degree 1, a stopping set is always a cycle or an interconnection of cycles [5]. The sub-matrix H_S corresponding to the stopping set is of size $b \times a$, where a is the *weight* of the stopping set. A *sub-code* is a stopping set that has a non-trivial null-space associated with its sub-matrix. We categorize a stopping set as wide, square, or tall depending upon the structure of its sub-matrix as follows:

1) A *wide stopping set* has $a > b$ and is always a sub-code since a wide sub-matrix H_s always has a non-trivial null-space. These stopping sets are *"unavoidable"* sub-codes as even labeling in the case of NB-LDPC codes cannot prevent them from being sub-codes.
2) A stopping set is *square* if $a = b$ and *tall* if $a < b$. These stopping sets are sub-codes only if their sub-matrices are rank-deficient. They can be prevented from being sub-codes through proper labeling in the case of NB-LDPC codes, while for binary LDPC codes they should avoided during the construction of codes.

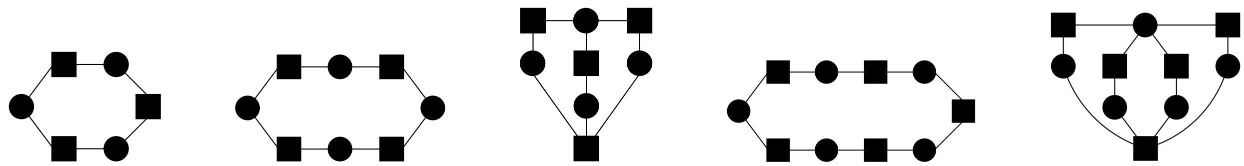
The weight of the smallest stopping-set sub-code in H is the minimum symbol distance. The NB-LDPC code labeling scheme introduced in [10] for $(2, d_c)$ NB-LDPC codes prevented square stopping sets (cycles) from being sub-codes and maximized the minimum distance of wide sub-codes yielding an improved performance. The labeling scheme of [6] uses this catalog in order to construct high-rate, irregular NB-LDPC codes with a low error-floor region.

III. Stopping-Set Structures up to Weight Five of LDPC Codes

This section catalogs *all* stopping-set structures up to weight five in LDPC codes. The catalog facilitates the identification of stopping sets through a simple algorithm [6], and can thus be used to construct good codes by avoiding them. [6] also presents the combinatorial arguments used in obtaining this list although a general theory of number of possible structures of any weight remains incomplete as far as our knowledge goes.

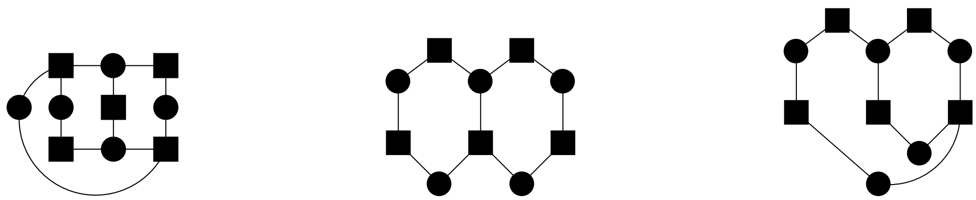
We denote square, wide and tall stopping sets as ss, sw and st respectively. In all figures, circles represent variable nodes and squares represent check nodes. We will specify the variable node degrees in any structure by a list of degrees. For example the list 3322 denotes a structure with two variable nodes of degree 3 and two variable nodes of degree 2.

A. Square Stopping-Set Structures of LDPC Codes up to Weight Five



(a) $ss1$ - 222 (b) $ss2$ - 2222 (c) $ss3$ - 3222 (d) $ss4$ - 22222 (e) $ss5$ - 42222

Fig. 1. Five square stopping-set structures



(a) $ss6$ - 33222 (b) $ss7$ - 32222 (c) $ss8$ - 32222

Fig. 2. Three square stopping-set structures

B. The Only Wide Stopping-Set Structure of Weight Five

The only wide structure, shown in Fig. 3, has to be avoided during an LDPC code construction for the code to have a minimum distance of at least six symbols [6] since wide stopping sets are sub-codes for any labeling.

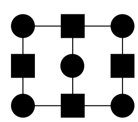
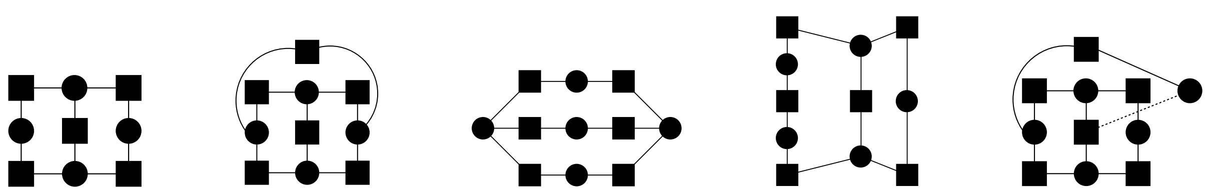


Fig. 3. The only wide stopping-set structure - $sw1$ - 22222

C. Tall Stopping-Set Structures up to Weight Five

The tall structures all have at least two variable nodes that are not of degree 2 [6]. Dotted lines represent connections that are not in the same plane.



(a) $st1$ - 3322 (b) $st2$ - 3333 (c) $st3$ - 33222 (d) $st4$ - 33222 (e) $st5$ - 33322

Fig. 4. Five tall stopping-set structures

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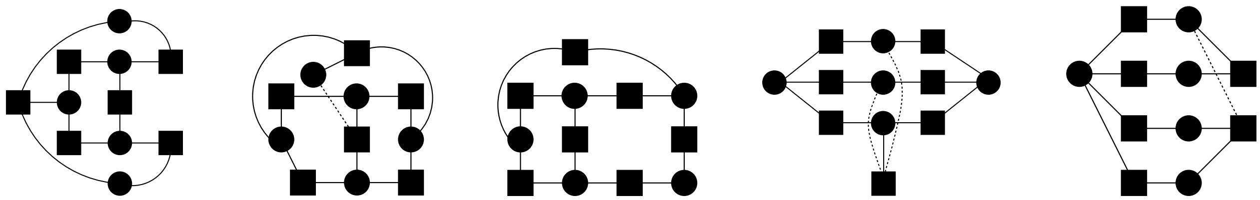
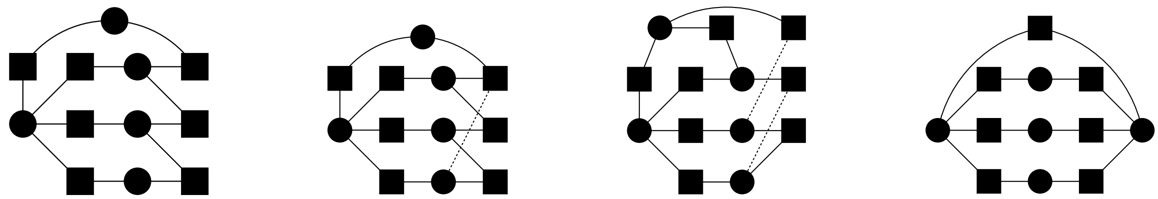


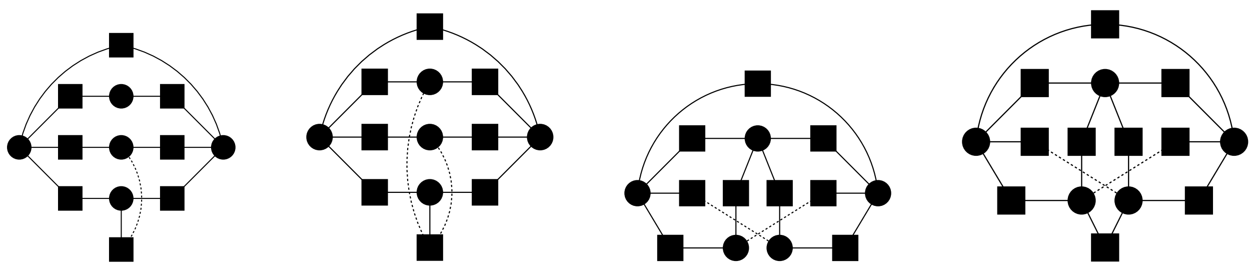
(a) $st6$ - 33322 (b) $st7$ - 33332 (c) $st8$ - 33332 (d) $st9$ - 33333 (e) $st10$ - 43222

Fig. 5. Five tall stopping-set structures



(a) $st11$ - 43322 (b) $st12$ - 43332 (c) $st13$ - 43333 (d) $st14$ - 44222

Fig. 6. Four tall stopping-set structures



(a) $st15$ - 44332 (b) $st16$ - 44333 (c) $st17$ - 44433 (d) $st18$ - 44444

Fig. 7. Four tall stopping-set structures

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