Feedback Systems using Non-Binary LDPC Codes with a Limited Number of Transmissions

Kasra Vakilinia, Adam R. Williamson, Sudarsan V. S. Ranganathan, Dariush Divsalar*, and Richard D. Wesel
Department of Electrical Engineering, University of California, Los Angeles, Los Angeles, California 90095
*Jet Propulsion Laboratory, California Institute of Technology, Pasadena, California 91109

***Abstract*—One advantage of incremental transmissions with feedback in point-to-point memoryless channels is a reduction in average blocklength required to approach capacity. This paper optimizes the size of each incremental transmission for non-binary (NB) LDPC codes to maximize throughput in VLFT and two-phase VLF settings. The optimization problem uses approximation based on the inverse-Gaussian p.d.f. of the blocklength required for successful decoding. By using the optimized incremental transmission lengths (with an average blocklength of less than 500 bits), NB-LDPC codes for VLFT limited to 5 transmissions achieve a throughput greater than 96% of that obtained by an unlimited-transmission VLFT scheme with the same average blocklength. With a similar average blocklength, a two-phase VLF system limited to five transmissions (with optimized lengths) using NB-LDPC codes achieves greater than 90% of the capacity of the 2dB binary-input AWGN channel. Two-phase VLF does not match the throughput of VLFT, but it is more practical than VLFT because it does not assume noiseless transmitter confirmation.**

I. Introduction

The classical results from [1] show that feedback does not increase the asymptotic capacity of memoryless channels. Polyanskiy et al. [2] and Chen et al. [3] illustrate that by using feedback, one can approach capacity in a small number of channel uses (low latency). Polyanskiy et al. [2] introduce variable-length coding with termination (VLFT) and without termination (VLF) which theoretically approach capacity with average block lengths on the order of a few hundred bits. Without feedback, similar performance requires a capacity-approaching coding technique such as LDPC with blocklengths of several thousand bits.

In VLFT, the receiver provides full noiseless feedback to the transmitter. The transmitter sends additional incremental information over the channel until it determines that the receiver has correctly decoded. Termination, the "T" in VLFT, occurs when the transmitter sends a noiseless transmitter confirmation (NTC) to terminate the transmission. The NTC is sent through a noiseless channel separate from the primary communication channel. The transmitter's knowledge of the decoder state through feedback and the NTC together facilitate zero probability of error in VLFT.

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In contrast to VLFT, in the VLF framework, the *receiver* determines when to stop the transmission and informs the transmitter via noiseless feedback. The stopping criterion that the receiver uses can be a CRC, a reliability metric about the decoded codeword as in [4], or a confirmation message from the transmitter through the primary communication channel as in [5]. This last scheme, with each communication phase followed by a confirmation phase, is called "two-phase" VLF.

Vakilinia et al. [6] show throughput versus latency for various non-binary (NB) LDPC codes in VLFT with unlimited transmissions. Additional incremental bits are selected one at a time, and decoding is attempted after each received bit.

This paper considers a more practical scenario in which the number of transmissions is limited. The size of the incremental messages in each transmission significantly affects the overall throughput of the system. We describe methods to select the size of each incremental message to maximize the throughput in VLFT and two-phase VLF schemes.

We approximate the probability density of the blocklengths that support successful decoding and the corresponding instantaneous rates via inverse-Gaussian and Gaussian densities respectively. These approximations enable us to formulate an analytical throughput maximization problem that facilitates optimization of the sizes of the incremental transmissions.

We compare the throughput achieved by our optimization applied to NB-LDPC codes with convolutional codes whose blocklengths were optimized by a coordinate-descent algorithm in [7] for information blocks of $k = 12, 24$, and 36 $GF(256)$ symbols ($k = 96, 192,$ and 288 bits) with $m = 5$ transmissions in VLFT and two-phase VLF settings.

The paper proceeds as follows: Sec. II provides an overview of the VLFT system with NB-LDPC codes and the inverse-Gaussian approximation for the histogram of the cumulative blocklengths. Sec. III presents the optimization techniques to select the size of each incremental transmission in VLFT. Sec. IV gives an overview of the two-phase VLF scheme and optimizes the cumulative blocklengths in each decoding attempt. Sec.V compares the throughput and the expected latency of NB-LDPC and convolutional codes in VLFT and two-phase VLF settings. Sec. VI concludes the paper.

II. VLFT with Non-Binary LDPC Codes

In [6], Vakilinia et al. use NB-LDPC codes in a VLFT system with 1-bit increments. After the initial transmission, the transmitter sends one bit at a time until the decoder decodes

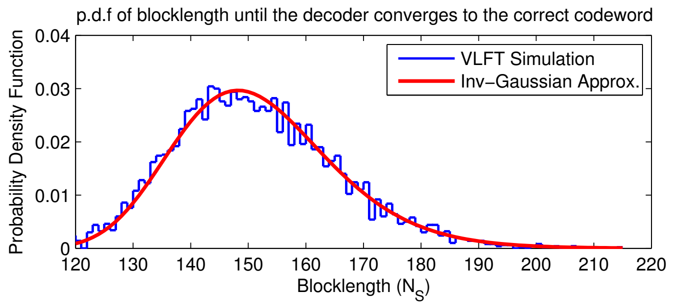

Fig. 1: Histogram and inverse-Gaussian approximation for blocklength to decode correctly in the VLFT setting for SNR 2.0 dB, $k = 96$ bits and initial blocklength of $N_0 = 120$ bits. The corresponding initial coding rate is $R_0 = \frac{k}{N_0} = 0.8$.

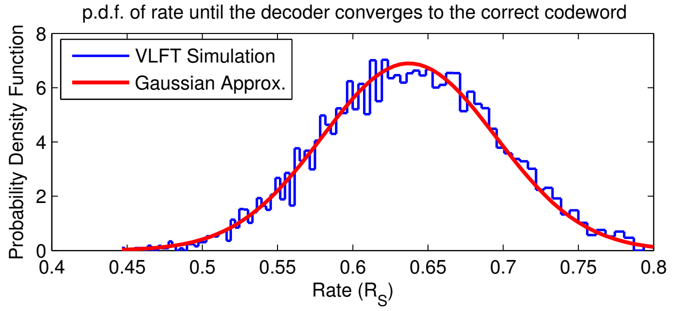

Fig. 2: Histogram of $R_S = \frac{k}{N_S}$ computed from Fig. 1 of $R_S = \frac{k}{N_S}$ and Gaussian fit with $\mu_S = 0.63$ and $\sigma_S^2 = 0.057$.

correctly. In [6], to maximize throughput the initial code-rate of the NB-LDPC code is chosen so that almost no codeword is successfully decoded in the initial transmission. Thus, the histogram of the number of additional increments required to decode correctly does not have a spike at zero.

For the system of [6], the "VLFT simulation" plot in Fig. 1 shows the empirical probability mass function (p.m.f.) of the total blocklength (N_S), which includes the size of the initial block and each of the incremental redundancies, required for the NB-LDPC code to decode to the transmitted codeword. The "VLFT simulation" plot in Fig. 2 shows the empirical histogram of the instantaneous rate $\left(R_S = \frac{k}{N_S}\right)$ at which decoding is successful. Fig. 2 shows that R_S is well-approximated by the Gaussian distribution

$$f_{R_S}(r) = \frac{1}{\sqrt{2\pi\sigma_S^2}} e^{-\frac{(r-\mu_S)^2}{2\sigma_S^2}} \tag{1}$$

with mean $\mu_S = E(R_S)$ and variance $\sigma_S^2 = \text{var}(R_S)$. This is consistent with the Gaussian approximation of information density in [8].

Note that μ_S is *not* the expected throughput but rather the average of the instantaneous rates supported by the channel. The cumulative distribution function (c.d.f.) of N_S is $F_{N_S}(n) = P(N_S \leq n)$, and we have

$$F_{N_S}(n) = P\left(\frac{k}{R_S} \leq n\right) = P\left(R_S \geq \frac{k}{n}\right) = 1 - F_{R_S}(\frac{k}{n}). \tag{2}$$

Taking the derivative of F_{N_S} using the Gaussian approximation of F_{R_S} produces the following "inverse-Gaussian" approximation for p.d.f. of N_S:

$$f_{N_S}(n) = \frac{k}{n^2\sqrt{2\pi\sigma_S^2}} e^{\frac{-\left(\frac{k}{n}-\mu_S\right)^2}{2\sigma_S^2}}. \tag{3}$$

As shown in Fig. 1, this p.d.f closely approximates the empirical distribution of N_S. For $N_1 < N_2$, the probability of the decoding attempt being successful at blocklength N_2 but not at N_1 using this approximation is

$$\int_{N_1}^{N_2} f_{N_S}(n)dn = \int_{N_1}^{N_2} \frac{k}{n^2\sqrt{2\pi\sigma_S^2}} e^{\frac{-\left(\frac{k}{n}-\mu_S\right)^2}{2\sigma_S^2}} dn \tag{4}$$

$$= Q\left(\frac{\frac{k}{N_2}-\mu_S}{\sigma_S}\right) - Q\left(\frac{\frac{k}{N_1}-\mu_S}{\sigma_S}\right). \tag{5}$$

The increase in blocklength from N_1 to N_2 reduces the rate from $\frac{k}{N_1}$ to $\frac{k}{N_2}$. Note that (5) gives the probability that the channel supports rate $\frac{k}{N_2}$ while not supporting the higher rate $\frac{k}{N_1}$. The Q functions in (5) are due to the normally-distributed success rate (R_S) at $\frac{k}{N_1}$ and $\frac{k}{N_2}$.

III. VLFT WITH LIMITED NUMBER OF TRANSMISSIONS

In this section, we use the p.d.f of N_S from (3) to find the optimal blocklengths $\{N_1, N_2, \ldots, N_m\}$ which maximize the throughput when at most m transmissions can be accumulated at the receiver. If decoding is not successful after the m^{th} decoding attempt, the accumulated transmissions are forgotten and transmission starts over with a new transmission of the first block of N_1 symbols. Define the throughput as $R_T = \frac{E(K)}{E(N)}$, where $E(K)$ is the effective number of *information* bits transferred correctly over the channel in one accumulation cycle (AC) and $E(N)$ represents the expected number of channel uses in one AC.

The expression for $E(N)$ is

$$E(N) = N_1 Q\left(\frac{\frac{k}{N_1}-\mu_S}{\sigma_S}\right) \tag{6}$$

$$+\sum_{i=2}^{m} N_i \left[Q\left(\frac{\frac{k}{N_i}-\mu_S}{\sigma_S}\right) - Q\left(\frac{\frac{k}{N_{i-1}}-\mu_S}{\sigma_S}\right)\right] \tag{7}$$

$$+N_m\left[1 - Q\left(\frac{\frac{k}{N_m}-\mu_S}{\sigma_S}\right)\right] \tag{8}$$

The right hand side of (6) shows the expected latency for successful decoding on the first attempt in the AC. $Q\left(\frac{\frac{k}{N_1}-\mu_S}{\sigma_S}\right)$ is the probability decoding successfully with the initial block of N_1; or alternatively, the probability that the channel supports the highest rate $\frac{k}{N_1}$. Similarly, the terms in (7) account for the latency of decoding that is first successful at total blocklength N_i (at the i^{th} decoding attempt). Finally, the probability of not being able to decode even at N_m is $1-Q\left(\frac{\frac{k}{N_m}-\mu_S}{\sigma_S}\right)$ which is shown in (8). Even when the decoding has not been successful at N_m, the channel has been used for N_m channel symbols.

The expected number of successfully transferred information bits $E(K)$ is

$$E(K) = kQ\left(\frac{\frac{k}{N_m} - \mu_S}{\sigma_S}\right), \tag{9}$$

where $Q\left(\frac{\frac{k}{N_m}-\mu_S}{\sigma_S}\right)$ is the probability of successful decoding at some point in the AC. Note that $E(K)$ depends only upon N_m. In fact, $E(K) \approx k$ and thus not sensitive to the choice of N_m for large values of N_m.

We optimize $\{N_1, N_2, \ldots, N_m\}$ to maximize $R_T = \frac{E(K)}{E(N)}$. The order of complexity for the exhaustive search (ES) algorithm is $O\binom{N_{max}-N_0}{m}$, where N_{max} is the maximum allowable overall blocklength for an AC. Since $E(K) \approx k$, maximization of R_T is equivalent to minimization of $E(N)$.

Over the range of possible N_1 values, $\{N_2, \ldots, N_m\}$ are optimized to minimize $E(N)$ for each fixed value of N_1 by setting derivatives to zero as follows:

$$\left\{N_2, \ldots, N_m : \frac{\partial E(N)}{\partial N_i} = 0,\ \forall i = 1, \ldots, m-1\right\}. \tag{10}$$

For each $i \in \{2, \ldots, m\}$, the optimal value of N_i is found by setting $\frac{\partial E(N)}{\partial N_{i-1}} = 0$, yielding a sequence of relatively simple computations. In other words, we select the N_i that makes our previous choice of N_{i-1} optimal in retrospect. Thus, to find N_2 we compute the derivative

$$\frac{\partial E(N)}{\partial N_1} = Q\left(\frac{\frac{k}{N_1} - \mu_S}{\sigma_S}\right) + (N_1 - N_2)Q'\left(\frac{\frac{k}{N_1} - \mu_S}{\sigma_S}\right) = 0 \tag{11}$$

and solve for N_2 as

$$N_2 = \frac{Q\left(\frac{\frac{k}{N_1}-\mu_S}{\sigma_S}\right) + N_1 Q'\left(\frac{\frac{k}{N_1}-\mu_S}{\sigma_S}\right)}{Q'\left(\frac{\frac{k}{N_1}-\mu_S}{\sigma_S}\right)} \tag{12}$$

where $Q'\left(\frac{\frac{k}{N_i}-\mu_S}{\sigma_S}\right) = \frac{\partial Q\left(\frac{\frac{k}{N_i}-\mu_S}{\sigma_S}\right)}{\partial N_i} = \frac{k}{N_i^2 \sigma_S}\frac{1}{\sqrt{2\pi}} e^{\frac{\left(\frac{k}{N_i}-\mu_S\right)^2}{2\sigma_S^2}}$.

For $i > 2$, $\frac{\partial E(N)}{\partial N_{i-1}} = 0$ depends only on $\{N_{i-2}, N_{i-1}, N_i\}$ as follows:

$$\frac{\partial E(N)}{\partial N_{i-1}} = Q\left(\frac{\frac{k}{N_{i-1}} - \mu}{\sigma}\right) + (N_{i-1} - N_i)Q'\left(\frac{\frac{k}{N_{i-1}} - \mu}{\sigma}\right) - Q\left(\frac{\frac{k}{N_{i-2}} - \mu}{\sigma}\right).$$

Thus we can solve for N_i as

$$N_i = \frac{Q\left(\frac{\frac{k}{N_{i-1}}-\mu}{\sigma}\right) + N_{i-1}Q'\left(\frac{\frac{k}{N_{i-1}}-\mu}{\sigma}\right) - Q\left(\frac{\frac{k}{N_{i-2}}-\mu}{\sigma}\right)}{Q'\left(\frac{\frac{k}{N_{i-1}}-\mu}{\sigma}\right)}. \tag{13}$$

Table I shows the optimized throughput (R_T) and the expected latency, $\lambda = k/R_T$, for various m. The optimized blocklengths and R_T obtained from the sequential differential approximation (SDA) algorithm are very close to the true optimized values obtained by exhaustive search (ES).

TABLE I: Optimized blocklengths, maximum R_T, and minimum λ obtained from ES and SDA for k = 96 bits for VLFT.

Alg.	m	$\{N_1, N_2, \ldots, N_m\}$	R_T	λ
ES, SDA	2	158 , 188	0.566	169.6
ES	3	150, 167, **194**	0.58638	163.71
SDA	3	150, 167, **195**	0.58635	163.72
ES	4	146, 158, 172, **198**	0.59709	160.77
SDA	4	146, 158, 172, **197**	0.59707	160.78
ES, SDA	5	143, 153, 163, 176, 201	0.603	159.2
ES, SDA	6	140, 149, 157, 166, 179, 204	0.608	157.9
ES, SDA	7	139, 147, 154, 161, 170, 182, 206	0.611	157.1

TABLE II: Throughput and latency for various m in VLFT.

m	2	3	5	10	15	20	∞
λ	167.1	163.7	159	155.5	154.3	153.7	151.9
R_T	0.566	0.586	0.603	0.618	0.622	0.624	0.632

For $m = 2, 5, 6,$ and 7, the optimized blocklengths for both approaches are the same. For $m = 3$ and 4 the blocklengths differ only in the value of N_m (shown in bold) and only by one bit. This small difference in N_m causes a negligible difference in the maximum throughput R_T and minimum latency $\lambda = \frac{k}{R_T}$.

Table II shows the optimum R_T and λ for various m using SDA. Since the complexity of ES is exponential in m, it is infeasible to obtain a globally optimal solution for $m > 7$. whereas SDA, with complexity $O(N_{max} - N_0)$, can find a solution within seconds even for large m.

IV. Two-phase VLF

Now we consider the two-phase VLF model in which the transmitter uses the primary communication channel to confirm whether the receiver has decoded to the correct codeword. As in [5] the two-phase incremental redundancy scheme has each *communication* phase followed by a *confirmation* phase.

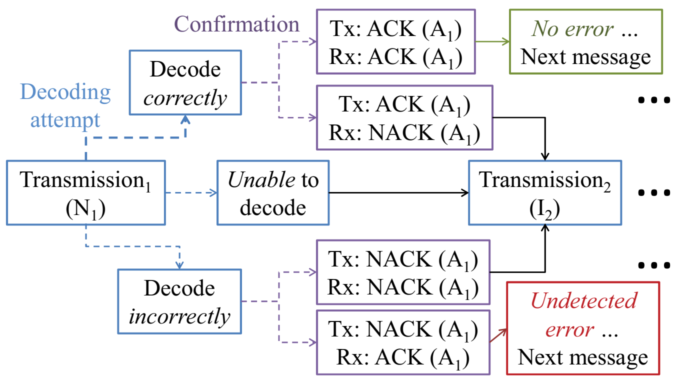

Fig. 3: Two-phase VLF block diagram

Fig. 3 shows a block diagram for the two-phase communication scheme. Starting at the left, a message block of size N_1 is transmitted. If the decoder decodes correctly, the transmitter sends a coded forward "ACK" on the same noisy channel to confirm the successful decoding. If the receiver decodes incorrectly, the transmitter sends a coded forward NACK. The ACKs and NACKs are repetition codes of length A_1 symbols.

Since the receiver and transmitter run the same decoder on the same received channel symbols, if the decoder does not converge to any codeword with N_1 symbols, the transmitter skips the unnecessary confirmation phase and immediately transmits the second increment (I_2). The transmitter terminates transmission when either the receiver decodes both the message and the following ACK correctly or when the receiver decodes both the message the subsequent NACK incorrectly. The latter scenario is the only case that results in an error, and this error is undetected by the receiver. The blocklengths of each transmission and confirmation message are selected to guarantee a probability of undetected error of at most ϵ. If the message is not decoded correctly even after m transmissions (and the NACKs are correctly received), the receiver deletes all received symbols and a new AC begins with the transmitter sending the original block of N_1 symbols again.

Since the transmitter sends NACK only when a wrong codeword is decoded, it is crucial to differentiate between *erroneous* decoding and failure to converge to a codeword. Fig. 4 shows the histogram of the required cumulative number of symbols until the receiver stops converging to an incorrect codeword (N_E). Note that Fig. 4 is conditioned on the decoder initially decoding to a wrong codeword at $N_0 = 120$. The probability that the decoder decodes incorrectly at N_0 is α, and $\alpha = 0.165$ for the experiment that produced the histogram in Fig. 4 . For blocklengths larger than N_E, the decoder either decodes correctly or fails to converge to any codeword. This is a different condition than correct decoding, the distribution is well-modeled by an inverse Gaussian as in Fig. 1. Fig. 5 shows the histogram of $R_E = \frac{k}{N_E}$, the instantaneous rate at which the decoder stops decoding to the wrong codeword, and the corresponding Gaussian approximation.

To optimize the blocklengths in the two-phase VLF setting, we use the probability distributions of N_S, R_S, N_E and R_E from Figs. 1, 2, 4, and 5. The optimization problem is to maximize $R_T = \frac{E(K)}{E(N)}$ under the constraint that the probability of error is smaller than ϵ. Thus

$$\sum_{i=1}^{m} P_i^{EE} < \epsilon\,, \tag{14}$$

where P_i^{EE} represents the probability the receiver decodes both the message and the NACK erroneously.

The exact expression for $E(K)$ is

$$E(K) = k\left(Q\left(\frac{\frac{k}{N_m} - \mu_S}{\sigma_S}\right) - \sum_{i=1}^{m} P_i^{EE}\right)\,, \tag{15}$$

As in Sec. II we assume $E(K) \approx k$. For $E(N)$,

$$E(N) = \sum_{i=1}^{m} (N_i + A_i)\left[P_i^{SS} + P_i^{EE}\right] + A_i\left[P_i^{SE} + P_i^{ES}\right] \tag{16}$$

$$+ N_m\left(1 - Q\left(\frac{\frac{k}{N_m} - \mu_S}{\sigma_S}\right) + P_m^{SE}\right)\,, \tag{17}$$

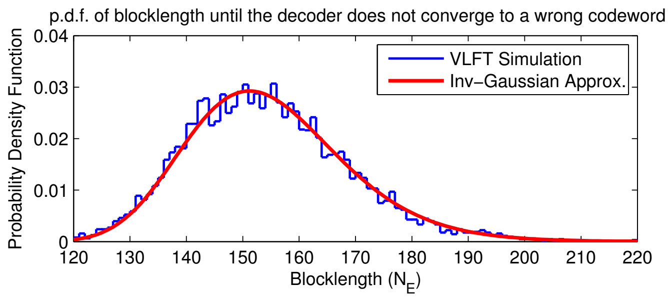

Fig. 4: Empirical histogram and inverse-Gaussian fit for the cumulative blocklength (N_E) to stop decoding incorrectly.

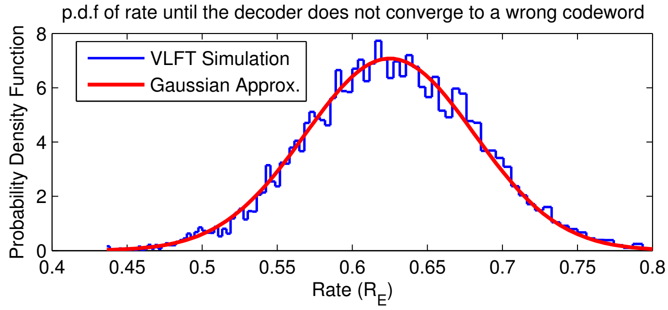

Fig. 5: Empirical histogram and Gaussian approximation with $\mu_E = 0.626$ and $\sigma_E^2 = 0.056$ of R_E in VLFT setting.

where P_i^{SS} is the probability the receiver decodes both message and ACK successfully. P_i^{SS} and P_i^{EE} are the stopping conditions for an AC. The term multiplying N_m in (17) is the probability that an AC ends without satisfying either of these stopping conditions. P_i^{SE} is the probability of decoding the message successfully but decoding the ACK as a NACK. Conversely, P_i^{ES} is the probability of decoding the message erroneously but decoding the NACK successfully. These probabilities are computed as follows:

$$P_i^{SS} = \left[Q\left(\frac{\frac{k}{N_i} - \mu_S}{\sigma_S}\right) - Q\left(\frac{\frac{k}{N_{i-1}} - \mu_S}{\sigma_S}\right)\right]\left[1 - Q\left(\frac{\sqrt{A_i}}{\sigma_c}\right)\right] \tag{18}$$

$$P_i^{EE} = \left[\alpha\left(1 - Q\left(\frac{\frac{k}{N_i} - \mu_E}{\sigma_E}\right)\right)\right]\left[Q\left(\frac{\sqrt{A_i}}{\sigma_c}\right)\right] \tag{19}$$

$$P_i^{SE} = \left[Q\left(\frac{\frac{k}{N_i} - \mu_S}{\sigma_S}\right) - Q\left(\frac{\frac{k}{N_{i-1}} - \mu_S}{\sigma_S}\right)\right]\left[Q\left(\frac{\sqrt{A_i}}{\sigma_c}\right)\right] \tag{20}$$

$$P^{ES} = \left[\alpha\left(1 - Q\left(\frac{\frac{k}{N_i} - \mu_E}{\sigma_E}\right)\right)\right]\left[\left(1 - Q\left(\frac{\sqrt{A_i}}{\sigma_c}\right)\right)\right]. \tag{21}$$

In (18) the probability of decoding correctly at N_i and not at N_{i-1} is $Q\left(\frac{\frac{k}{N_i} - \mu_S}{\sigma_S}\right) - Q\left(\frac{\frac{k}{N_{i-1}} - \mu_S}{\sigma_S}\right)$ and $Q\left(\frac{\sqrt{A_1}}{\sigma_c}\right)$ is the probability that the ACK is decoded as a NACK, where σ_c is the standard deviation of the channel noise. In (19), $\alpha\left[1 - Q\left(\frac{\frac{k}{N_i} - \mu_E}{\sigma_E}\right)\right]$ is the probability of decoding erroneously at N_i.

TABLE III: Optimized $\{N_1,\dots,N_m\}$ for m=5 two-phase VLF using SDA and ES with $\{A_1,\dots,A_5\}=\{5,4,3,3,3\}$.

Alg.	k	$\{N_1,N_2,\dots,N_5\}$	ϵ
SDA	96	145, 156, 167, 180, 202	1.2E-3
SDA	96	146, 158, 171, 188, 230	9.4E-4
ES	96	146, 158, 170, 184, 211	9.9E-4

To optimize the blocklengths for two-phase VLF we used both ES and SDA approaches from Sec. III for fixed values of $\{A_1,\dots,A_m\}$. Table III shows two sets of $\{N_1,\dots,N_m\}$ obtained for different N_1 in SDA with $\epsilon\approx 10^{-3}$. The optimized $\{N_1,\dots,N_m\}$ with $\epsilon\le 10^{-3}$ from ES is close to the SDA optimized blocklengths. The optimized blocklengths from SDA can also be used as optimization limits for ES algorithm and significantly reduce the ES optimization space.

V. Results

Table IV summarizes the blocklengths that maximize the throughput in the two-phase VLF setting with ϵ=10^{-3}, for both NB-LDPC codes and tail-biting convolutional codes. Blocklengths for the NB-LDPC codes are obtained from Eqns. (15-17) and blocklengths for the convolutional codes are based on the coordinate-descent algorithm in [7] using the assumption of rate-compatible sphere-packing. The rate-1/3 convolutional codes (CC) have octal generator polynomials $(117,127,155)$ for the 64-state code and $(2325,2731,3747)$ for the 1024-state code. Table IV also shows the percentage of BI-AWGN capacity obtained in the two-phase VLF setting with $m=5$ transmissions. For $k=192$ and 288, the NB-LDPC code obtains throughputs greater than 90% of BI-AWGN capacity with a latency (λ) less than 500 bits.

Fig. 6 shows R_T versus λ for NB-LDPC and convolutional codes (CC) in VLFT and two-phase VLF settings. In VLFT with unlimited number of transmissions (1-bit increments), CCs with ML decoders perform very well at short latencies of up to 200 bits. VLFT schemes have throughputs greater than the capacity at short blocklengths because of the NTC. VLFT with NB-LDPC codes outperforms CCs at larger latencies because the codeword error rate of CCs increases once the blocklength exceeds twice the traceback depth.

Fig. 6 also shows the throughput obtained in the two-phase VLF setting for NB-LDPC codes, 64-state and 1024-state tail-biting convolutional codes with m=5 and ϵ=10^{-3}. As the blocklength increases, as mentioned in [9], the performance of the codes in VLF gets closer to the performance in VLFT.

VI. Conclusion

This paper uses the inverse-Gaussian approximation for the blocklenth of first successful decoding to optimize the size of each incremental transmission for non-binary (NB) LDPC codes to maximize throughput in VLFT and two-phase VLF settings. In the 300-500 symbol average block length regime, this paper reports the best VLFT and VLF throughputs yet. VLFT throughputs are higher than VLF, but VLF is more practical because it does not require NTC. For two-phase VLF with m=5, NB-LDPC codes with optimized blocklengths achieve greater than 90% of the capacity of the 2dB BI-AWGN channel in the 300-500 symbol range of average blocklength.

TABLE IV: Optimized $\{N_1,\dots,N_m\}$ for two-phase VLF with m=5 at SNR 2dB, and simulation results for R_T and λ. $\{A_1,\dots,A_5\}=\{5,4,3,3,3\}$ for NB-LDPC. For the CCs, $A_i=6$, 8, and 9 $\forall i$ for $k=96$, 192, and 288 bits, respectively.

Code	k	$\{N_1,N_2,\dots,N_5\}$	λ	R_T	%
NB-LDPC	96	146, 158, 170, 184, 211	170.4	0.563	87.7
1024-CC	96	138, 153, 166, 180, 204	168.6	0.569	88.6
NB-LDPC	192	301, 322, 344, 369, 408	330.5	0.581	90.5
1024-CC	192	287, 309, 331, 352, 384	349.4	0.549	85.4
NB-LDPC	288	459, 487, 518, 550, 597	495.7	0.581	90.5
1024-CC	288	416, 441, 463, 488, 532	599.6	0.480	74.8

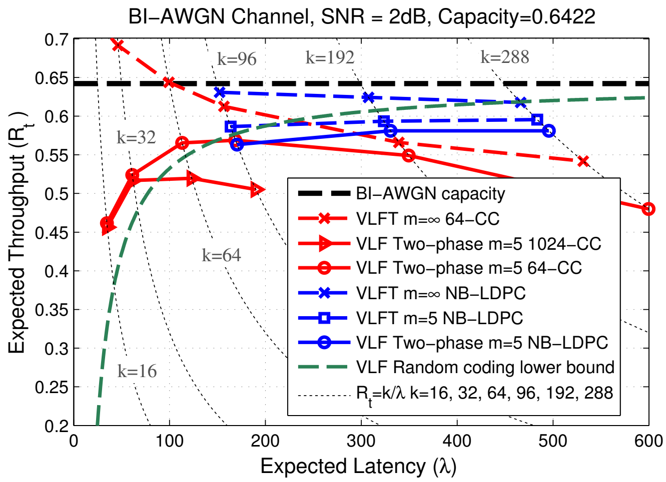

Fig. 6: R_T vs. λ for NB-LDPC, 64/1024-state CCs for VLFT with $m=\infty$ and $m=5$, and two-phase VLF with $m=5$.

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