LDPC Absorbing Sets, the Null Space of the Cycle Consistency Matrix, and Tanner's Constructions

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***Abstract*—Dolecek et al. introduced the cycle consistency condition, which is a necessary condition for cycles – and thus the absorbing sets that contain them – to be present in separable circulant-based (SCB) LDPC codes. This paper introduces a cycle consistency matrix (CCM) for each possible absorbing set in an SCB LDPC code. The CCM efficiently enforces the cycle consistency condition for all cycles in a specified absorbing set by spanning its associated binary cycle space. Under certain conditions, a CCM not having full column rank is a necessary and sufficient condition for the LDPC code to contain the absorbing set associated with that CCM. This paper uses the CCM approach to carefully analyze LDPC codes based on the Tanner construction for $r = 4$ rows of sub-matrices (i.e., Tanner-construction LDPC codes with column weight 4).**

I. Introduction

Low-density parity-check (LDPC) codes were first introduced by Gallager [1] and are known to have a provable capacity-approaching property under iterative decoding [2]. Moderate blocklength, high-rate LDPC codes often exhibit a flattening of the FER curve called the error floor in the range of low frame error rate (FER). Suboptimality of message passing in a graph with cycles is at least one of causes of the error floor. Prior work also indicates that certain sub-graphs called trapping sets [3], and, in particular, a subset of trapping sets called absorbing sets [4] are a primary cause of the error floor. Absorbing sets are trapping sets that are stable under bit-flipping decoding. The absorbing set spectrum enumerates absorbing sets by their cardinality and type.

LDPC codes achieving low FERs and having structure that is compatible with high-throughput hardware architectures are often demanded in high-speed, highly-reliable systems, such as data storage devices. This paper focuses on a class of regular LDPC codes, known as separable, circulant-based (SCB) codes. SCB codes retain standard properties of quasi-cyclic LDPC codes such as girth, code structure, and compatibility with existing high-throughput hardware implementations. The construction of SCB codes is based on a suitably derived submatrix of a common mother matrix. This code family includes common quasi-cyclic codes such as array-based LDPC codes [5], and several other constructions [6], [7].

Given that presence of absorbing sets (or trapping sets) in the graphical representation of the code directly influences code performance in the low FER region, several recent works have proposed methods to improve the absorbing set (trapping set) spectrum. The approach in [8] introduces additional check nodes while the method in [9] increases the girth to eliminate small trapping sets for some codes. The algorithm in [10] constructs quasi-cyclic codes from Latin squares so that the graphical representation of the code (a.k.a. Tanner graph) does not contain certain trapping sets. In related work [11], [12], we develop a new approach based on carefully selecting the rows/columns of the (SCB) mother matrix to improve the error floor. This method provably improves the absorbing set spectrum by structurally eliminating dominant absorbing sets, and does so without compromising certain code properties.

This paper introduces a cycle consistency matrix (CCM) for each possible absorbing set in an SCB LDPC code. For an absorbing set to be present in an SCB LDPC code, it is a necessary condition for the associated CCM not to have full column-rank. Under certain conditions, a CCM not having full column rank is also a sufficient condition for the LDPC code to contain the absorbing set associated with that CCM. This paper uses the CCM approach to carefully analyze LDPC codes based on the Tanner et al. construction [6] for $r = 4$ rows of sub-matrices. For the highest-rate tanner constructions, this paper shows that $(4, 4)$ absorbing sets always exist. Simulation results verify the existence of these absorbing codes for example codes in this class.

Section II introduces separable circulant-based (SCB) codes and the cycle consistency matrix (CCM). Section III identifies the CCMs for the smallest possible absorbing sets of the Tanner-construction codes with $r = 4$. Section III then shows that the smallest possible absorbing sets always exist for the highest-rate Tanner-construction codes. Through the collected error profiles of some high-rate Tanner-construction codes, Section IV provides simulation results demonstrating the existence of such absorbing sets. Section V delivers the conclusions.

II. Definition and Preliminaries

This section defines separable, circulant-based (SCB) codes and the cycle consistency matrix (CCM) associated with absorbing sets in SCB codes.

A. Circulant-based LDPC codes

Circulant-base LDPC codes are composed of circulant matrices and form a subset of (r, c) regular LDPC codes, where r is the variable node degree and c is the check node degree. The structure of these codes is particularly compatible with high-throughput hardware applications [13].

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The parity-check matrix of circulant-based LDPC codes can be described as follows:

$$H_{p,f}^{r,c} = \begin{bmatrix} \sigma^{f(0,0)} & \sigma^{f(0,1)} & \sigma^{f(0,2)} & \dots & \sigma^{f(0,c-1)} \\ \sigma^{f(1,0)} & \sigma^{f(1,1)} & \sigma^{f(1,2)} & \dots & \sigma^{f(1,c-1)} \\ \sigma^{f(2,0)} & \sigma^{f(2,1)} & \sigma^{f(2,2)} & \dots & \sigma^{f(2,c-1)} \\ \vdots & \vdots & \vdots & \dots & \vdots \\ \sigma^{f(r-1,0)} & \sigma^{f(r-1,1)} & \sigma^{f(r-1,2)} & \dots & \sigma^{f(r-1,c-1)} \end{bmatrix},$$

where σ is the following $p \times p$ circulant matrix:

$$\sigma = \begin{bmatrix} 0 & 0 & \dots & 0 & 1 \\ 1 & 0 & \dots & 0 & 0 \\ 0 & 1 & \dots & 0 & 0 \\ \vdots & \vdots & \dots & \vdots & \vdots \\ 0 & 0 & \dots & 1 & 0 \end{bmatrix}.$$

A column (row) group is a column (row) of circulant matrices. Each variable node has a label (j,k) with $j \in \{0,...,c-1\}$ that is the index of the corresponding column group and $k \in \{0,...,p-1\}$ identifies the specific column within the group. Similarly, each check node has a label (i,l) where $i \in \{0,...,r-1\}$ and $l \in \{0,...,p-1\}$.

This paper focuses on separable, circulant-based (SCB) codes, which are defined as follows:

Definition 1 (Separable, Circulant-Based (SCB) Code): An SCB code is a circulant-based LDPC code with a parity-check matrix $H_{p,f}^{r,c}$ in which $f(i,j)$ is separable, i.e., $f(i,j) = a(i) \cdot b(j)$. ■

Parity check matrices of SCB codes with a specified circulant matrix can be viewed as originating from a common SCB mother matrix $H_{p,f_m}^{p,p}$ with $f_m(i,j) = i \cdot j$. The functions $a(i)$ and $b(j)$ effectively specify which rows and columns of the mother matrix are selected for the resultant SCB matrix. The ranges of $a(i)$ and $b(j)$ are both $\{0,\dots,p-1\}$.

SCB codes include, for example, the constructions in [6], [5] and [7]. The girth is guaranteed to be at least 6 by placing a constraint on the submatrix exponent value $f(i,j)$ [4].

The SCB structure imposes certain conditions [4] on the variable and check nodes:

Bit Consistency: The neighboring check nodes of a variable node must have distinct row-group (i) labels.

Check Consistency: The neighboring variable nodes of a check node must have distinct column-group (j) labels.

Pattern Consistency: (As shown in [4]) Since every entry in an SCB parity check matrix with the value 1 satisfies $k+ij = l \mod p$, if two variable nodes (j_1,k_1) and (j_2,k_2) share a check node in row group i, they must satisfy:

$$k_1 + ij_1 = k_2 + ij_2 \mod p. \quad (1)$$

Note that the converse also holds: If (1) is satisfied, then the two variable nodes (j_1,k_1) and (j_2,k_2) share a check node in row group i of the SCB mother matrix.

Cycle consistency: As shown in [4], the equations of the form (1) for any length-$2t$ cycle in an SCB mother matrix, which involves t variable nodes with column-group labels j_1 through j_t and t check nodes with row-group labels i_1 through i_t, show that the cycle must satisfy:

$$\sum_{m=1}^{t} i_m (j_{(m+1) \mod t} - j_m) = 0 \bmod p. \quad (2)$$

After reviewing absorbing sets [4] below, (2) is used to construct a necessary matrix equation for an absorbing set to exist based on the cycles contained in that absorbing set.

B. Absorbing sets and the Cycle Consistency Matrix

An LDPC code with parity-check matrix H is often viewed as a bipartite (Tanner) graph $G_H = (V,F,E)$, where the set V represents the variable nodes, the set F represents the check nodes, and E corresponds to the edges between variable and check nodes.

For a variable node subset $V_{\text{as}} \subset V$, analogous to G_H, let $G_{\text{as}} = (V_{\text{as}}, F_{\text{as}}, E_{\text{as}})$ be the bipartite graph of the edges E_{as} between the variable nodes V_{as} and their neighboring check nodes F_{as}. Let $o(V_{\text{as}}) \subset F_{\text{as}}$ be the neighbors of V_{as} with odd degree (unsatisfied check nodes) in G_{as} and $e(V_{\text{as}}) \subset F_{\text{as}}$ be the neighbors of V_{as} with even degree in G_{as} (satisfied check nodes).

Definition 2 (Absorbing Set): An (a,b) absorbing set is a set $V_{\text{as}} \subset V$ with $|V_{\text{as}}| = a$ and $|o(V_{\text{as}})| = b$, where each node in V_{as} has strictly fewer neighbors in $o(V_{\text{as}})$ than $e(V_{\text{as}})$. ■

Suppose there are n variable nodes in the absorbing set. Let $j_1,\dots,j_n$ be the column-group labels of these n nodes in the SCB mother matrix. Define $u_m = j_m - j_1, m = 2,...,n$ and $\mathbf{u} = [u_2,...,u_n]$. For each cycle in the absorbing set, by replacing the difference of j's with the difference of u's and manipulating the expression, (2) may be written as

$$\sum_{m=2}^{t} (i_{m-1} - i_m) u_m = 0 \mod p, \quad (3)$$

where $2t$ is the length of that cycle. Note that the sequence $\{i_1, i_2, \dots, i_m\}$ will be different for different cycles reflecting the particular cycle trajectories.

Every cycle in the absorbing set satisfies an equation of the form (3). Taken together, these equations produce a matrix equation: $\mathbf{Mu} = 0 \mod p$, where $\mathbf{M}_{ym}$ is the coefficient of u_m in (3) for the yth cycle.

A key property of $\mathbf{M}$ is that $\mathbf{Mu} = 0 \mod p$ completely characterizes the requirement that every cycle in G_{as} satisfies (3). Even so, it is not necessary for $\mathbf{M}$ to include a row for every cycle in the absorbing set.

A cycle need not be included in $\mathbf{M}$ if it is a linear combination of cycles already included in $\mathbf{M}$. Thus the number of rows needed in $\mathbf{M}$ is the number of linearly independent cycles in G_{as}. Some definitions [14] from graph theory are necessary to establish the number of linearly independent cycles in G_{as} and hence how many rows are needed for $\mathbf{M}$.

Definition 3 (Incidence Matrix): For a graph with n vertices and q edges, the (unoriented) incidence matrix is an $n \times q$ matrix B with $B_{ij} = 1$ if vertex v_i and edge x_j are incident and 0 otherwise. ■

Note that since each edge is incident to exactly two vertices, each column of B has exactly two ones.

The incidence matrix of a graph is useful for identifying the cycles in the graph because every cycle has the property that the indicator vector $\mathbf{x}_c$ of the edges in the cycle satisfies $B\mathbf{x}_c = 0 \mod 2$. In fact, the edges identified by the vector

$\mathbf{x}_c$ form a cycle (or a union of cycles) if and only if $B\mathbf{x}_c = 0 \mod 2$. This is formalized in the definition below.

Definition 4 (Binary Cycle Space): The binary cycle space of a graph is the null space of its incidence matrix over $GF(2)$. ■

For any absorbing-set bipartite graph G_{as} a graph can be constructed whose only vertices are V_{as} and two vertices are connected iff there is a check node that connects them. We call this graph the variable-node graph of the absorbing set. It is for the variable-node graph that we construct the incidence matrix B. If each satisfied check node in G_{as} has degree 2, then the incidence matrix B is simply the transpose of the submatrix of $H_{p,f}^{r,c}$ whose rows and columns correspond to F_{as} and V_{as} respectively.

The number of linearly independent cycles in an absorbing set, which is the dimension of its binary cycle space D_{bcs}, is the size of the null space of the incidence matrix B_{as}: $D_{\text{bcs}} = |E_{\text{as}}| - \text{rank}(B_{\text{as}})$. Knowing the number of linearly independent cycles allows us the specify an efficient $\mathbf{M}$, which we define to be a Cycle Consistency Matrix as follows:

Definition 5 (Cycle Consistency Matrix): A Cycle Consistency Matrix $\mathbf{M}$ of an absorbing-set graph G_{as} has $|V_{\text{as}}| - 1$ columns and D_{bcs} rows. The rows of $\mathbf{M}$ correspond to D_{bcs} linearly independent cycles in G_{as}. Each row has the coefficients of $\mathbf{u}$ in (3) for one of these cycles. ■

Note that $\mathbf{M} \cdot \mathbf{u} = 0 \mod p$ completely characterizes the requirement that every cycle in G_{as} satisfies (3).

Recall that the vector $\mathbf{u}$ contains difference information about the column groups involved in the absorbing set: the value of the first column group is subtracted from each of the others. The vector $\mathbf{u}$ cannot be an all-zero vector because the Check Consistency condition requires that variable nodes sharing a check node have distinct column groups, and a zero entry indicates that the variable node is in the first ($j = j_1$) column group. Thus a necessary condition for the existence of a given absorbing set is that its $\mathbf{M}$ does not have full column-rank in $GF(p)$.

Note that each variable node in a given absorbing set must be a part of at least one cycle whenever the variable node degree is at least 2. Equipped with these definitions, the next theorem gives necessary and sufficient conditions for the existences of absorbing sets in SCB codes.

Theorem 1: Given a proposed absorbing set graph $G_{\text{as}} = (V_{\text{as}}, F_{\text{as}}, E_{\text{as}})$, where every variable node is involved in at least one cycle, the column group labels of the variable nodes in V_{as} in the SCB mother matrix, and the row-group labels of the check nodes in F_{as} in the SCB mother matrix, the following are necessary conditions for the proposed absorbing set to exist in each daughter SCB LDPC code (with a parity check matrix H that includes the specified row and column groups of that SCB mother matrix): (1) A CCM for G_{as} does not have full column-rank; (2) Variable nodes in V_{as} satisfy the Bit Consistency condition and can form a difference vector $\mathbf{u}$ in the null space of the CCM; and (3) Each check node in F_{as} satisfies the Check Consistency condition. These conditions are also sufficient if the variable-node graph of this absorbing set is not extensible.

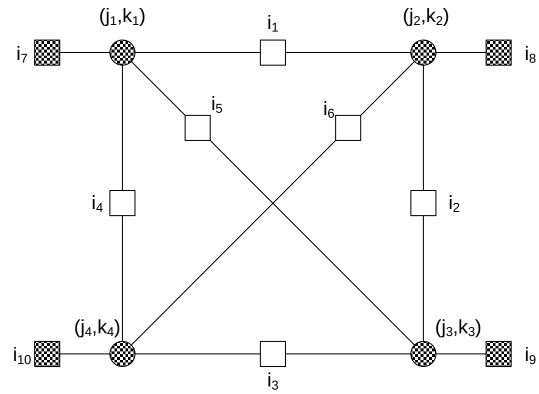

Fig. 1. Depiction of the (4,4) absorbing set configuration.

Proof: Each of the three conditions has already been shown to be a necessary condition for the existence of G_{as} in an SCB. If all of these three conditions are satisfied, all the cycles corresponding to the rows of the CCM exist in G_H because they can be constructed as follows: Conditions (1) and (2), ensure a sequence of variable node column groups $[j_1, j_2, ..., j_{|V_{as}|}]$ that form a vector $[u_2, ..., u_{|V_{as}|}]$ in the null space of the CCM. For any fixed k_1, we can compute $k_2, ..., k_{|V_{as}|}$ with pattern consistency in equation (1). We can also find any linear combination of these cycles exists in G_H as well.

These cycles cover every edge except the edges between variable nodes and degree-1 check nodes in G_{as}. If (1) a variable node's unsatisfied check node is the same as another variable node's unsatisfied check node, or (2) a variable node's unsatisfied check node is the same as one satisfied check node in the graph, or (3) two of the satisfied check nodes are the same, then there exist other independent cycles in the variable-node graph which extend the variable-node graph. Thus if the variable-node graph is not extensible, the above constructed solution fully describes the existence of the proposed absorbing set. This concludes that G_{as} is present in G_H. ■

III. Illustrative case study on Tanner's construction for $r = 4$

In this section we study certain high-rate quasi-cyclic (QC) codes under the Tanner construction [6]. Tanner Construction codes are a subset of SCB codes that can be described by the parity check matrix $H_{p,f}^{r,c}$, $f(i,j) = a^i \cdot b^j$, where a and b are elements in $GF(p)$ with multiplicative order $o(a) = r$ and $o(b) = c$. This paper focuses on the analysis of Tanner-construction codes with $r = 4$ (four row groups).

Section III-A constructs a CCM for the $(4,4)$ absorbing sets and shows that a singular CCM is sufficient to establish the presence of these absorbing sets in certain cases. Section III-B shows that the CCM will always be singular for certain Tanner-construction codes and further proves the existence of the $(4,4)$ absorbing sets in these codes.

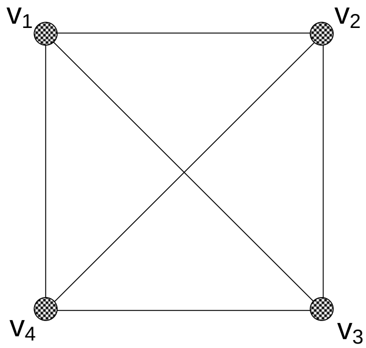

Fig. 2. Variable-node graph of (4,4) absorbing sets.

A. Constructing a CCM for (4,4) Absorbing Sets

From [4], the smallest possible absorbing sets for SCB codes with $r = 4$ are the $(4, 4)$ absorbing sets, shown in Figure 1. We can apply the CCM approach to establish necessary and sufficient conditions for the existence of these $(4, 4)$ absorbing sets.

Using the technique of Section II-B, we construct a variable-node graph shown in Figure 2 for the absorbing set graph in Figure 1. There are 5 cycles in the variable-node graph, but we only need a subset of them to build CCM. The incidence matrix of this graph is

$$B_{\text{as}} = \begin{bmatrix} 1 & 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 & 1 & 1 \end{bmatrix}, \tag{4}$$

and $\text{rank}(B_{\text{as}}) = 3$ under $GF(2)$. Thus the dimension of the binary cycle space is $D_{\text{bcs}} = |E_{\text{as}}| - \text{rank}(B_{\text{as}}) = 3$, which means that three linearly independent cycles span the binary cycle space.

We construct the CCM in (5) by selecting the following linearly independent cycles: $v_1 - v_2 - v_3, v_1 - v_2 - v_4, v_1 - v_3 - v_4$:

$$\mathbf{M} = \begin{bmatrix} i_1 - i_2 & i_2 - i_5 & 0 \\ i_1 - i_6 & 0 & i_6 - i_4 \\ 0 & i_5 - i_3 & i_3 - i_4 \end{bmatrix}. \tag{5}$$

Remark 1: In this absorbing set, since every satisfied check node has degree-2, the incidence matrix is the transpose of the submatrix of the parity-check matrix that only includes the variable nodes and satisfied check nodes:

$$\hat{H}_{\text{as}} = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 \end{bmatrix}. \tag{6}$$

Lemma 1: In the Tanner graph corresponding to quasi-cyclic LDPC code with the parity check matrix $H^{4,p}_{p,a(i)\cdot b(j)}$, the necessary and sufficient condition of the existence of $(4, 4)$ absorbing sets in Figure 1 is $\det \mathbf{M} = 0 \mod p$.

Proof: Since the variable-node graph of $(4, 4)$ absorbing sets is a fully connected graph, it is not extensible without introducing a length-4 cycle in the corresponding bipartite graph. Length-4 cycles are not possible because SCB codes have a girth of at least 6. Thus we can use Theorem 1 to prove the sufficient condition. ■

Here is an example of how the absorbing set of Lemma 1 is constructed. Because the $(4, 4)$ absorbing set as shown in Figure 1 exists, there is a non-zero vector $\mathbf{u}$ that satisfies $\mathbf{M} \cdot \mathbf{u} = 0 \mod p$. Thus $\det \mathbf{M} = 0 \mod p$. Moreover, if $\det \mathbf{M} = 0 \mod p$, there exists a non-zero solution to $\mathbf{M} \cdot \mathbf{u} = 0 \mod p$, where $\mathbf{u} = [u_2, u_3, u_4]^T$. Without loss of generality, suppose $u_2 \neq 0$. With check consistency, $i_1 - i_2 \neq 0, i_2 - i_5 \neq 0$, and $i_1 - i_2 \neq i_2 - i_5$. Thus $u_3 \neq 0$ and $u_2 \neq u_3$. Similarly $u_4 \neq 0$, $u_2 \neq u_4$, and $u_3 \neq u_4$. Then, for a fixed j_1, we can find j_2, j_3, and j_4 without contradicting bit consistency. For any specified k_1 value, we can find a $(4, 4)$ absorbing set in the code. Therefore $\det \mathbf{M} = 0 \mod p$ is a sufficient condition for the existence of $(4, 4)$ absorbing sets.

From [4], there is only one possible non-isomorphic check labeling for this configuration that satisfies bit consistency and check consistency: $(i_1, i_2, i_3, i_4, i_5, i_6)$ is (x, y, x, y, z, w), where $\{x, y, z, w\} \subset (a(0), a(1), a(2), a(3))$. Therefore the necessary and sufficient condition for the existence of $(4, 4)$ absorbing sets can be also formulated as

$$\det \mathbf{M} = (z - x)(w - y) + (z - y)(w - x) = 0 \mod p \tag{7}$$

B. Identifying the dominant absorbing sets of high-rate Tanner-construction codes

This section shows that for $r = 4$, the row-selection function $a(i)$ of the Tanner construction [6] will always introduce (4,4) absorbing sets for the case set forth in the following lemma:

Lemma 2: In the Tanner graph corresponding to the quasi-cyclic LDPC code with the parity-check matrix $H^{4,p-1}_{p,a(i)\cdot b(j)}$, where $a(i) = \alpha^i, o(\alpha) = 4, b(j) = \beta^j, o(\beta) = p - 1$, $(4, 4)$ absorbing sets as shown in Figure 1 always exist.

Proof: With the result of Lemma 1, the determinant of the CCM is shown in (7). Since $a(i) = \alpha^i, o(\alpha) = 4$, we can assign $x = 1, w = \alpha, y = \alpha^2, z = \alpha^3$. Then the determinant can be represented as

$$\begin{aligned} \det \mathbf{M} &= (\alpha^3 - 1)(\alpha - \alpha^2) + (\alpha^2 - \alpha^3)(1 - \alpha) \\ &= 2(1 - \alpha + \alpha^2 - \alpha^3) \\ &= 2(1 - \alpha)(1 + \alpha^2) . \end{aligned} \tag{8}$$

With the fact that $o(\alpha) = 4, \alpha^4 = 1$, we have $o(\alpha^2) = 2$ and the only possible $\alpha^2 = p - 1$ under $GF(p)$. Thus $\det \mathbf{M} = 0 \mod p$.

According to the analysis in the proof of Lemma 1, $\det \mathbf{M} = 0 \mod p$ implies that there exists a non-zero $\mathbf{u} = [u_2, u_3, u_4]^T$, where $u_2 \neq 0, u_3 \neq 0, u_4 \neq 0$ is required to satisfy the bit-consistency. Since $o(\beta) = p - 1$, the parity check matrix $H^{4,p-1}_{p,a(i)\cdot b(j)}$ includes almost all the column groups except the one with index 0. Thus by carefully selecting a non-zero j_1 such that $j_1 \neq p - u_2, j_1 \neq p - u_3, j_1 \neq p - u_4$, we can always find $j_2 \neq 0, j_3 \neq 0, j_4 \neq 0$, and the resulting $[j_1, j_2, j_3, j_4]$ will produce the $(4, 4)$ absorbing sets for any specific k_1 value. ■

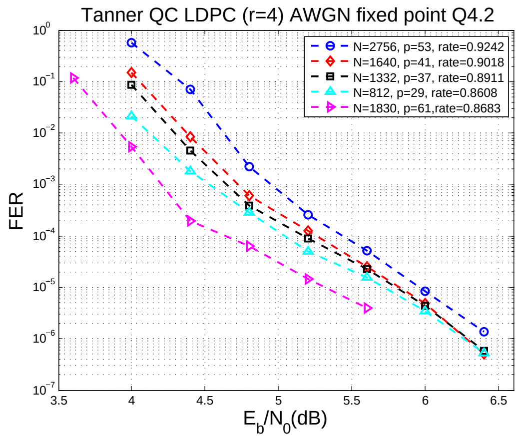

Fig. 3. Frame error rate of high-rate Tanner-construction codes.

Remark 2: Tanner constructions with $o(\beta) < p-1$ and $r=4$ sometimes can avoid all $(4,4)$ absorbing sets by either increasing the girth or making the intersection between null space of $\mathbf{M}$ and variable node space to be empty. However, $(4,4)$ absorbing sets still can exist in some constructions with $o(\beta) < p-1$ and $r=4$, and they dominate the error floor when they exist. For example, with $p=67, a(i)=11^i, b(j)=5^j, o(11)=4, o(5)=30$, the resulting code has dominant $(4,4)$ absorbing sets.

IV. Results

In this section we experimentally demonstrate the error floor of a group of high-rate Tanner-construction codes. Our simulations use 200 iterations and a $Q4.2$ fixed-point quantization with 4, resp. 2, bits to represent integer, resp. fractional, values. Our decoder employs the soft-xor algorithm [15].

Figure 3 shows simulation results for five Tanner-construction QC LDPC codes ($f(i,j) = a^i \cdot b^j$) with the parameters described in Table II. From Figure 3 and Table I, the high-rate Tanner-construction codes exhibit an error-floor because of the existence of $(4,4)$ and $(6,2)$ absorbing sets. This result is consistent with our analysis in Section III-B showing that $(4,4)$ absorbing sets always exist for these high-rate Tanner-construction LDPC codes. A Similar analysis can be applied to the $(6,2)$ absorbing sets for Tanner-construction codes and the codes in [7].

TABLE I
Error profiles for the high-rate Tanner-construction codes. From top to bottom, they are corresponding to (2756,2547), (1640,1479), (1332,1187), (812,699) and (1830,1589) codes respectively. N.E. is the number of collected errors.

SNR	n.e.	(4,4)	(6,2)	(7,4)	(8,0)	(8,2)
6.0dB	50	19	29	0	0	0
6.4dB	51	18	27	0	0	0
6.0dB	50	12	29	0	0	0
6.4dB	56	17	31	0	2	0
6.0dB	50	15	29	0	0	0
6.4dB	59	21	32	0	1	0
6.0dB	50	19	25	0	2	1
6.4dB	53	30	18	0	1	0
5.2dB	50	47	0	1	0	0
5.6dB	58	49	0	0	0	0

TABLE II
Parameters for Tanner-construction codes.

n	k	p	rate	a	$o(a)$	b	$o(b)$
2756	2547	53	0.9242	23	4	2	52
1640	1479	41	0.9018	9	4	6	40
1332	1187	37	0.8911	6	4	2	36
812	699	29	0.8608	12	4	2	28
1830	1589	61	0.8683	11	4	5	30

V. Conclusion

This paper introduces a cycle consistency matrix (CCM) in order to efficiently identify and avoid dominant absorbing sets. As an example of the CCM technique we analyze certain LDPC codes using the Tanner-construction for $r=4$ rows of sub-matrices. This CCM analysis shows the existence of (4,4) absorbing sets for these codes.

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