Non-Linear Turbo Codes for Interleaver-Division Multiple Access on the OR Channel.

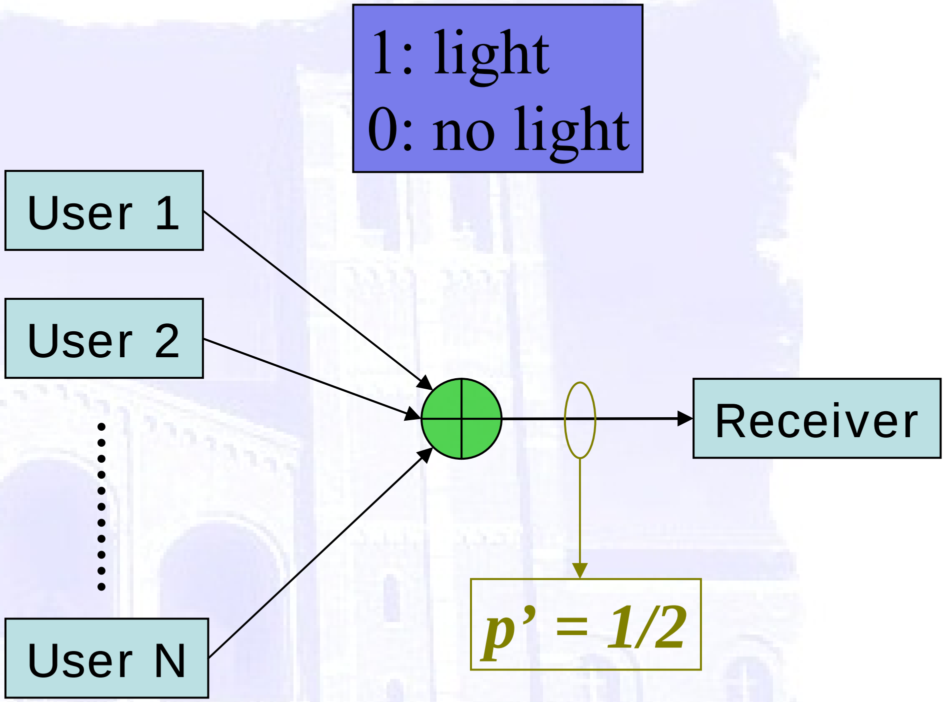
Miguel Griot,
Andres I. Vila Casado,
and Richard D. Wesel

Outline

- Uncoordinated multiple access to the OR channel.
  - Interleaver-Division Multiple Access (IDMA).
  - Single-user decoding: the Z-Channel.
- Parallel Concatenated Non-Linear Trellis Codes (PC-NLTC).
  - Proposed Structure.
  - BER bounding technique (uniform interleaver analysis for non-linear codes).
  - Limitations on the number of users.
- Results.
- Conclusions.

The OR Multiple Access Channel (OR-MAC)

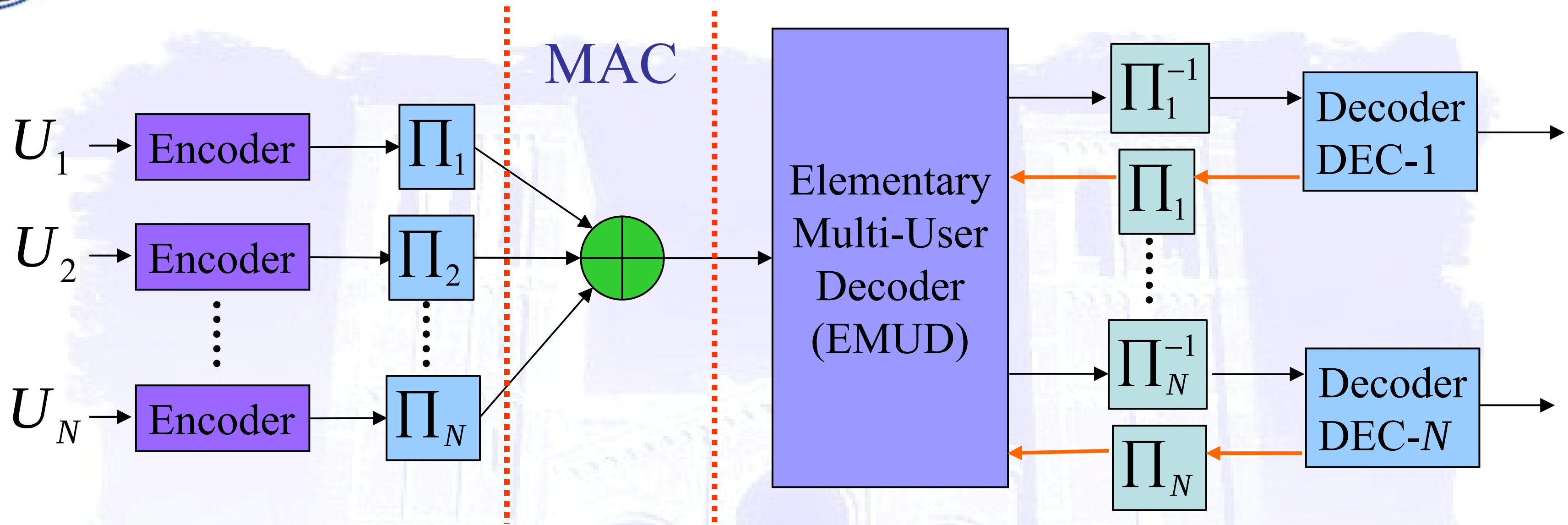
- Simple model for multiple-user optical channel with non-coherent combining.
- 0+X=X, 1+X=1
- N users, all transmitting with the same ones density p: P(*X*=1)=*p*, P*(X*=0*)*=*1-p.*



- Theoretically: Sum-rate = 1 (100% efficiency) can be achieved with a ones density in the transmission of

$$p(N) = 1 - (1/2)^{1/N} \approx \frac{\ln(2)}{N}$$

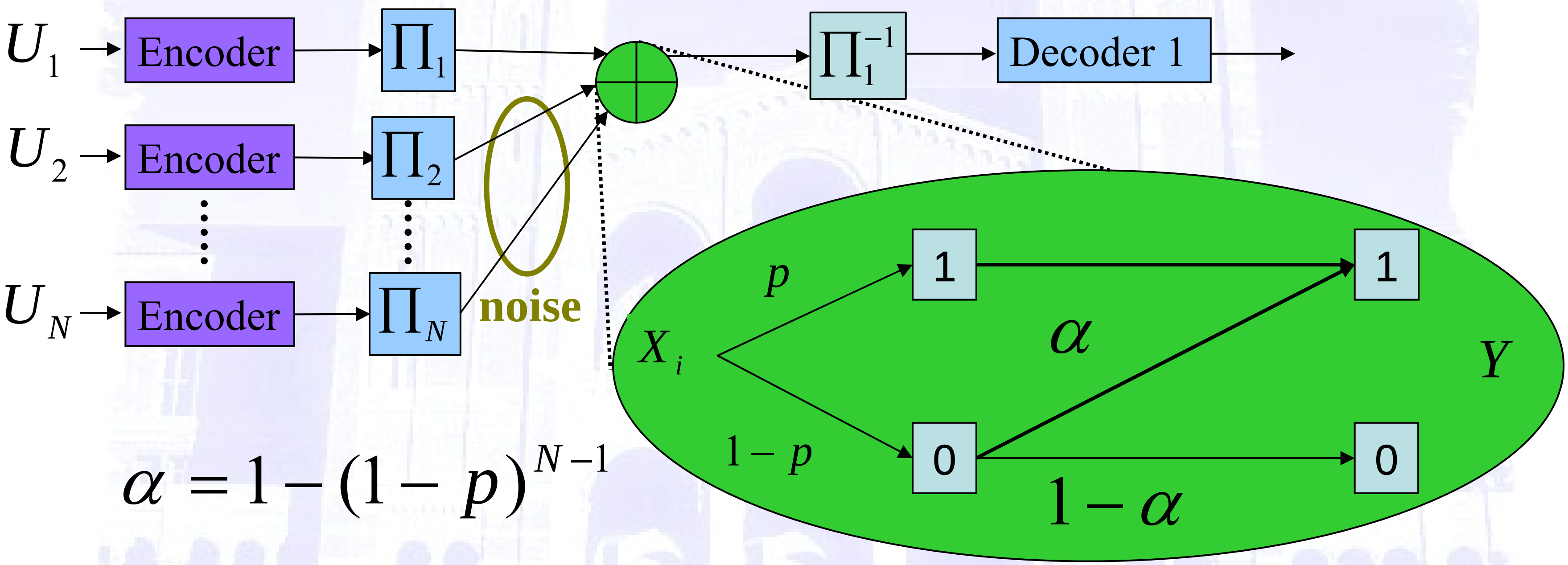
IDMA-Based Architecture



- [Ping *et al.*'06] for general MAC.
- With appropriately designed codes it can be applied over the OR-MAC.
- Joint Iterative decoding.
- For a large number of users joint decoding may not be too complex.

Single-user decoding: Z-Channel

- A practical alternative is to treat all but a desired user as noise.
- When treating other users as noise in an OR-MAC, each user “sees” a Z-Channel.

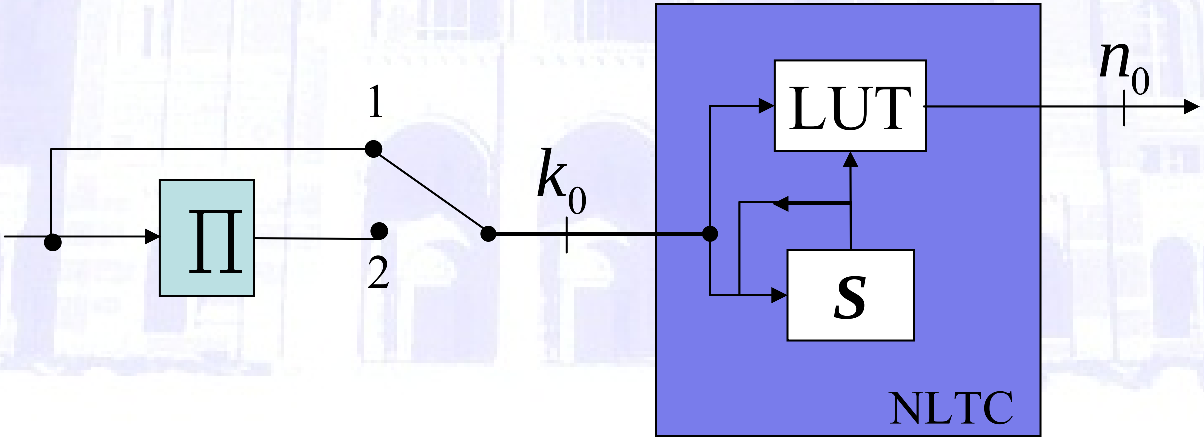


$$\alpha = 1-(1-p)^{N-1}$$

- The achievable sum-rate is lower bounded by ln(2) (around 70%), for any number of users.

Parallel Concatenated Non-Linear Trellis Codes

- Non-linear codes with controlled ones densities are required in this application.
- Previous work using Non-Linear Trellis Codes [ISIT'06].
- The NLTC consists of:
  - A 2^{ν}-state trellis structure (block *S*).
  - A look-up table (LUT) stores an output per branch.
  - The outputs satisfy the required ones density *p* (non-systematic)
- PC-NLTC: Two constituent (n_0, k_0) non-linear trellis codes (NLTC) linked by an interleaver (Π) of length *K*.



$$\text{rate} = \frac{k_0}{2n_0}$$

Distance on the Z-Channel

- The proper definition of distance between two codewords on the Z-Channel is the *directional distance.*
- *Definition:* Directional distance between two codewords X and $\tilde{X}$ (denoted $d_D(X,\tilde{X})$) is the number of positions at which X has a 0 and $\tilde{X}$ has a 1.
- Then, for an (n,k) code over the Z-Channel:

$$BER \le \frac{1}{2k}\frac{1}{2^k}\sum_{(U,\tilde{U})} d_H(U,\tilde{U})\cdot\left(\alpha^{d_D\left(X,\tilde{X}\right)}+\alpha^{d_D\left(\tilde{X},X\right)}\right)$$

- Where $U,\tilde{U}$ are any possible data words, $d_H\left(U,\tilde{U}\right)$ their Hamming distance, and $X,\tilde{X}$ their respective codewords.

Design Criteria

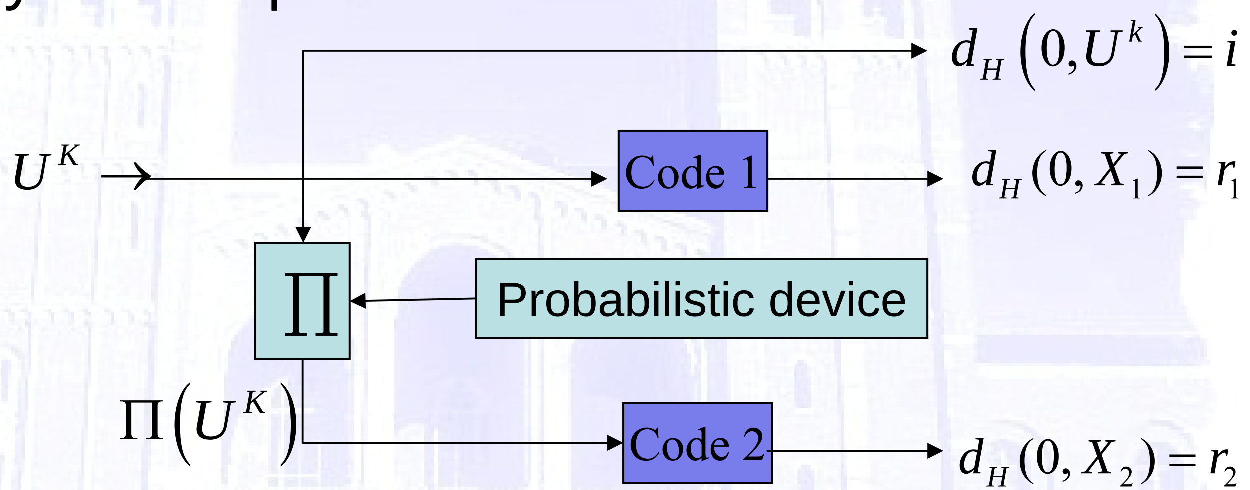
- Choose a target sum-rate *(SR).* In our case, *SR=0.6* (capacity ~ 0.7).
- Choose the number of states of the trellis code (more on this later).
- Choose k_0. For a 6-user OR-MAC, using 8-state trellis codes, and a SR=0.6, then the average Hamming weight of the output per branch is $\sim 0.54k_0$. We chose $k_0 = 2$.
- The length in bits of the output per trellis branch is then $n_0(N) \approx N \cdot k_0 / SR$.
- Assign a Hamming weight to each branch of the NLTC in order to maintain the optimal ones density.
- Total freedom on where to place those ones: branch and position.

Analytical BER Bound

- We provide a method to predict the BER of parallel concatenated non-linear codes over asymmetric channels, in particular the Z-Channel, under Maximum Likelihood decoding.
- We extend the uniform interleaver analysis proposed in [Benedetto '96].
- Uniform interleaver: given the two constituent codes, average over all possible interleavers of a certain length *K*.
- Key difference: **non-linearity** of the constituent codes.
  - We cannot assume that the all-zero codeword is transmitted. We need to average over all possible codewords.
- Constituent codes are **non-systematic**.

Uniform interleaver analysis

- **Linear case**: the all-zero codeword is assumed to be transmitted.
- Consider any other input data:



$$d_H\left(0, U^k\right) = d_H\left(0, \Pi\left(U^k\right)\right) = H_w\left(U^k\right) = i$$

$$d = i + r_1 + r_2$$

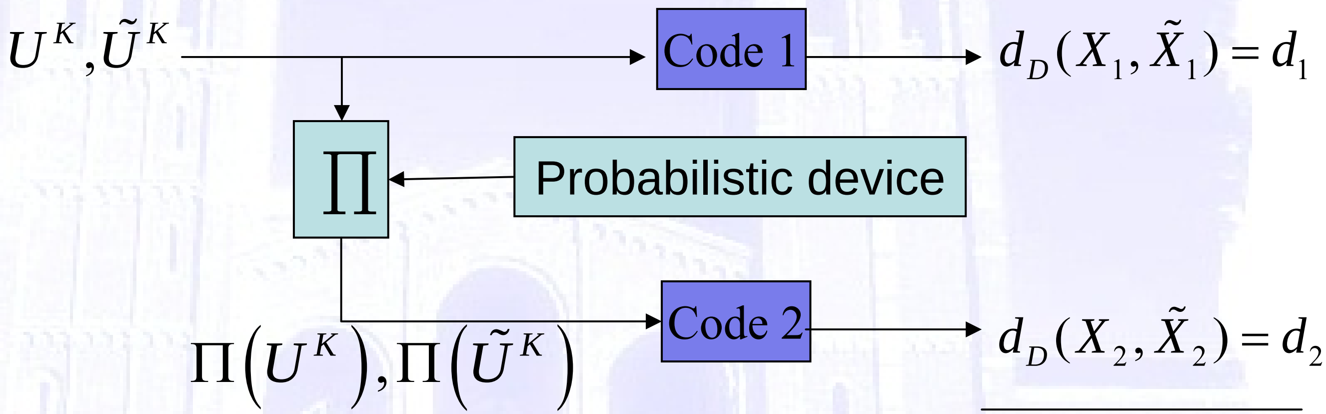
$$\text{Probability}\left(U^K \rightarrow \Pi\left(U^K\right)\right) = \binom{K}{i}^{-1}$$

Count the number of possible inputs with input Hamming weight *i*, and redundancy *r* :

$$A_{i,r_1}^{C_1}, A_{i,r_2}^{C_2} \rightarrow A_{i,r_1+r_2}^{C_p}$$

Uniform interleaver analysis (3)

- **Non-linear case**: we need to average over all possible transmitted codewords.



$$d = d_1 + d_2$$

$$d_H\left(U^K, \tilde{U}^K\right) = d_H\left(\Pi\left(U^K\right), \Pi\left(\tilde{U}^K\right)\right) = i,$$

$$\text{Probability}\left(U^K, \tilde{U}^K \rightarrow \Pi\left(U^K\right), \Pi\left(\tilde{U}^K\right)\right)$$

$$= \left[\binom{K}{w} \cdot \binom{K-w}{(i+w-\tilde{w})/2} \cdot \binom{w}{(i-w+\tilde{w})/2}\right]^{-1}$$

Count the number of possible input pairs with $(w, \tilde{w}, i, d_j)$

$$A^{C_1}_{(w,\tilde{w},i,d_1)}, A^{C_2}_{(w,\tilde{w},i,d_2)}$$

$$\rightarrow A^{C_P}_{(w,\tilde{w},i,d_1+d_2)}$$

Uniform interleaver (3)

- Hence:

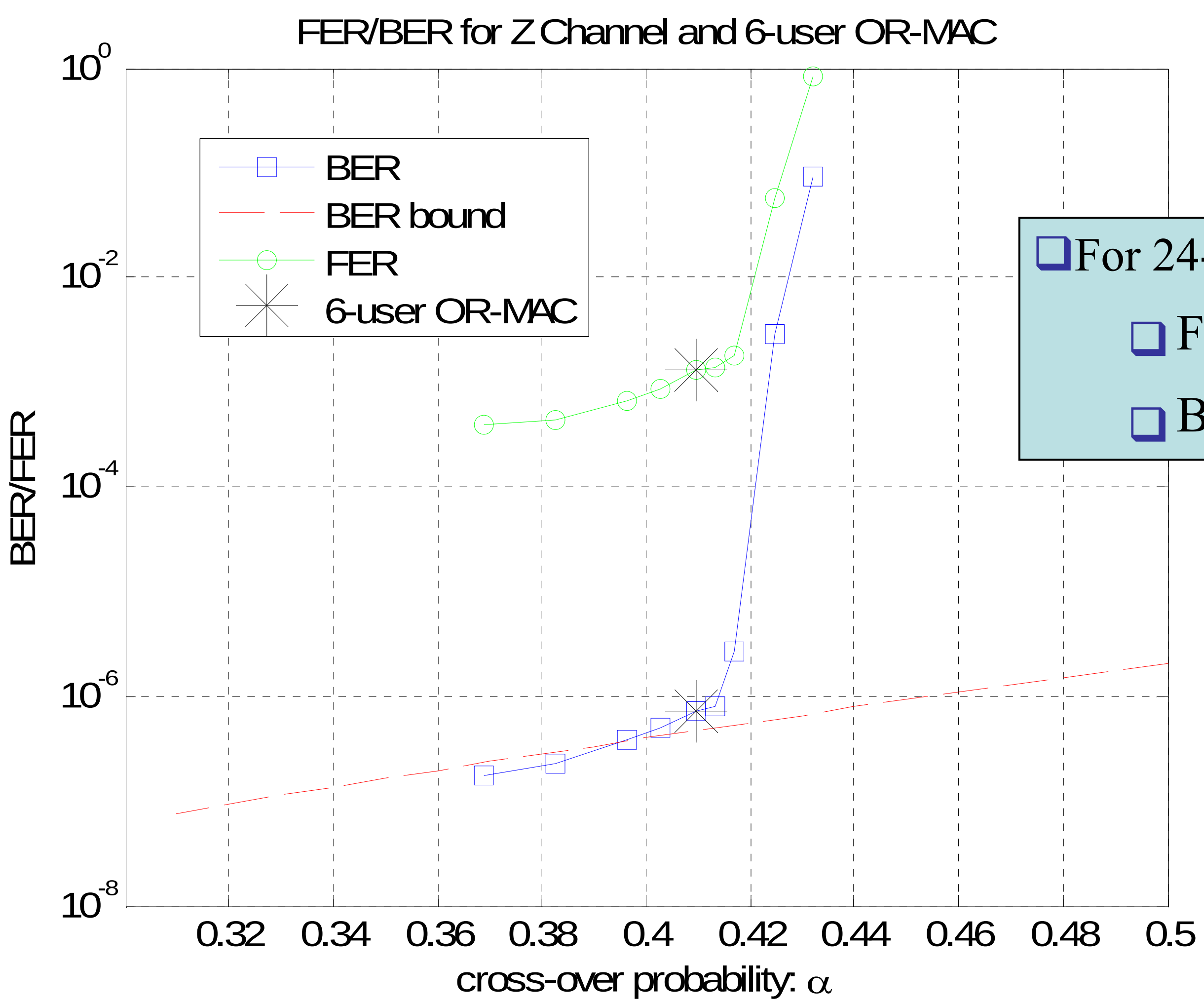
$$A^{C_P}(W,H,I,D)=\sum_w\sum_h\sum_i A^{C_P}_{(w,h,i)}(D)\,I^i H^h W^w$$

$$\text{where}\quad A^{C_P}_{(w,h,i)}(D)=\frac{A^{C_1}_{(w,h,i)}(D)\cdot A^{C_2}_{(w,h,i)}(D)}{\binom{k}{w}\cdot\binom{k-w}{(i+h)/2}\cdot\binom{w}{(i-h)/2}}$$

$$\Rightarrow BER\le\frac{1}{2K}\frac{1}{2^K}\left.\frac{\partial A^{C_P}(W,H,I,D)}{\partial I}\right|_{I=1,H=1,W=1,D=\alpha}$$

Results for 6-user OR-MAC

- Parallel concatenation of 8-state NLTCs.
- Sum-rate = 0.6, block-length = 8192, 12 iterations.



- For 24-user OR-MAC:
  - FER = 6.34×10^{-4}
  - BER = 4.37×10^{-7}

Limitation on the number of users

- There may be a limitation in the number of users for a certain number of states of the trellis.
- Notation:
  - 2^v-state encoder.
  - k_0 input bits per trellis section.
  - SR : Target sum-rate.
  - $n_0(N)$: # output bits per branch.
  - $M(N)$ total number of ones in all branches.

Limitation on the number of users

$$(1)\, n_0(N) = N \cdot k_0 / SR.$$

$$(2)\, M(N) = p(N) \cdot n_0(N) \cdot (\#\text{ branches})$$

$$\Rightarrow M(N) = \left(\frac{k_0 \cdot 2^{k_0+v}}{SR}\right) \cdot \underbrace{\left[N\left(1-(1/2)^{1/N}\right)\right]}_{\xrightarrow[N\to\infty]{} \ln(2)}$$

$$\Rightarrow \lim_{N\to\infty} M(N) = \ln(2) \cdot \frac{k_0 \cdot 2^{k_0+v}}{SR}$$

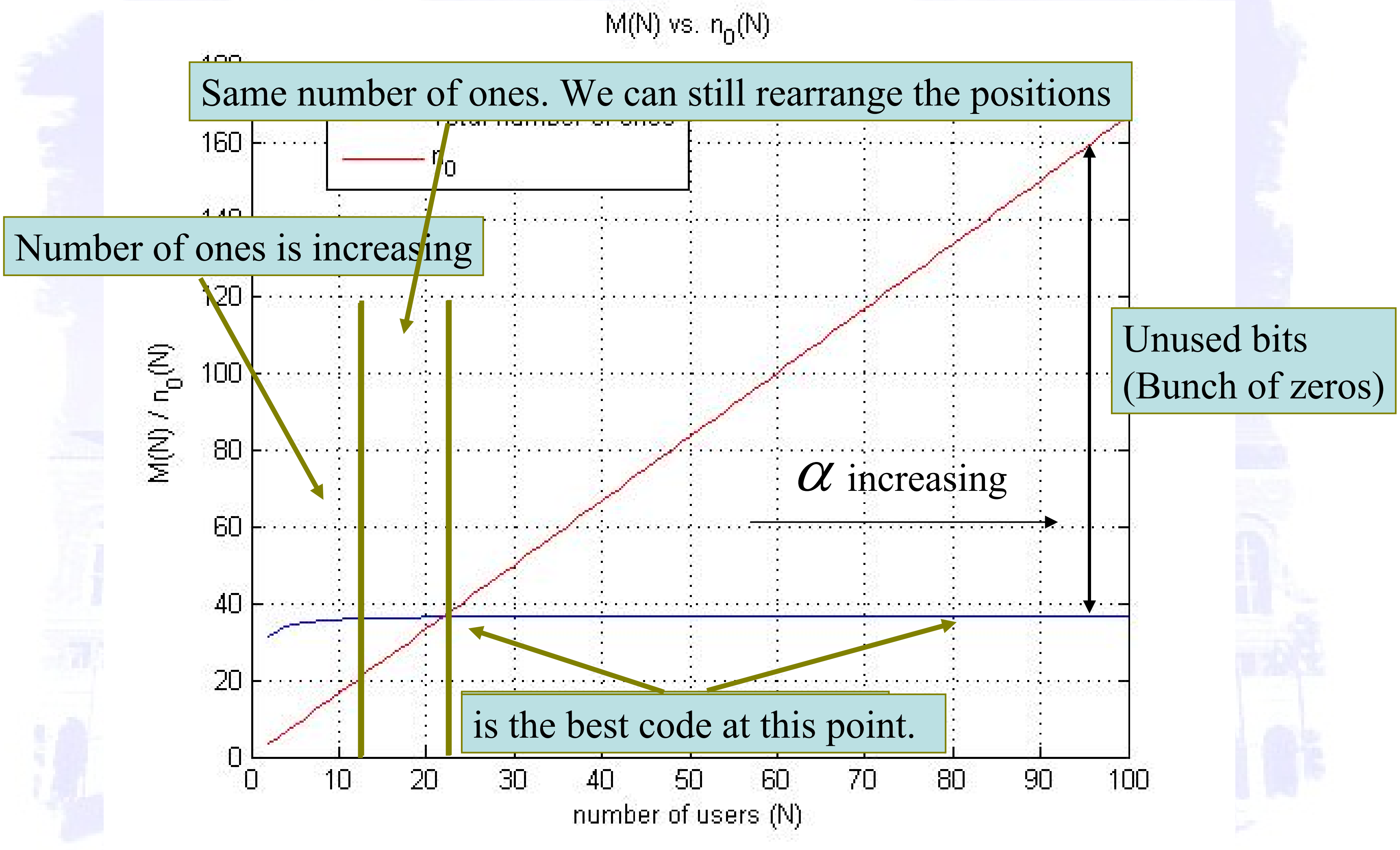
$$\to v = 3, k_0 = 2, SR = 0.6$$

- In our design:

$$\to M(N) = \frac{2^5}{0.6}\left[N\left(1-(1/2)^{1/N}\right)\right]$$

$$\to n_0(N) = \frac{N}{0.6}$$

Limitation after 23 users



For 8-state PC-NLTC

N	α	FER	BER
6	0.43877	9.45×10^{-4}	6.54×10^{-7}
24	0.48115	6.34×10^{-4}	4.37×10^{-7}
30	0.48312	1.01×10^{-3}	1.88×10^{-5}
48	0.48605	6.12×10^{-3}	2.58×10^{-4}
60	0.48702	1.50×10^{-2}	1.13×10^{-3}
72	0.48766	2.60×10^{-2}	2.98×10^{-3}

Conclusions

- A new family of codes has been presented: parallel concatenation of non-linear trellis codes.
- Application: uncoordinated access to the OR multiple access channel, using an IDMA-based system, with single-user decoding.
- They have proven to work well close to capacity (a sum-rate of 0.6 vs. ln(2)~0.7).
- A tight analytical prediction of the BER of a PC-NLTC over the Z-Channel has been derived. This technique can be generalized to non-linear codes in general, and other asymmetric channels.
- There is a limitation on the number of users for a fixed trellis structure. For an 8-state NLTC, the performance deteriorates for more than 24 users.
- We present a thorough analysis on this limitation.

Thank you!

Questions?