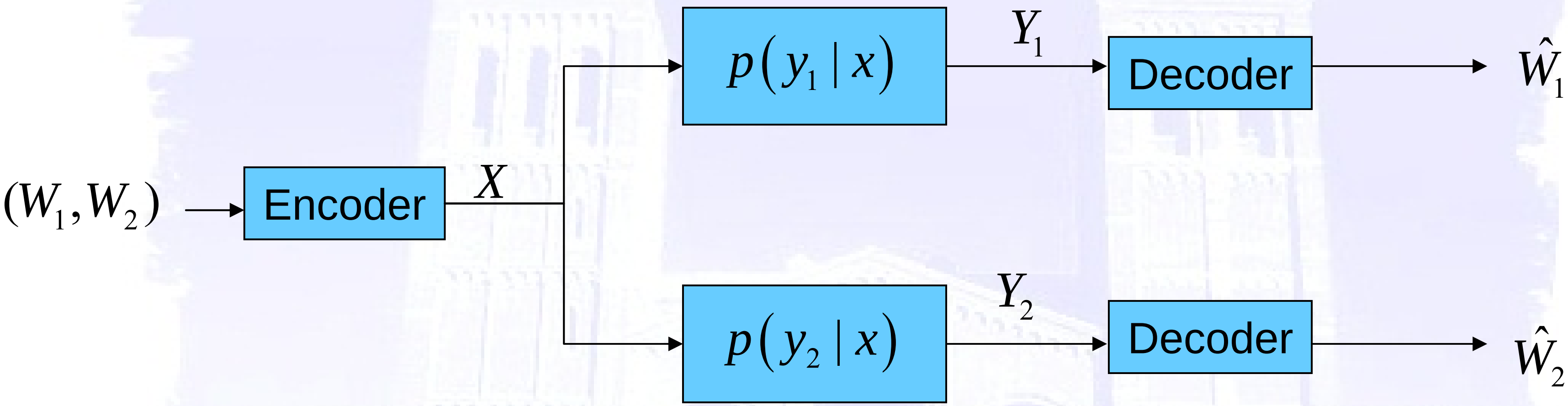
Nonlinear Turbo Codes for the broadcast Z Channel

Richard Wesel
Miguel Griot
Bike Xie
Andres Vila Casado

Outline

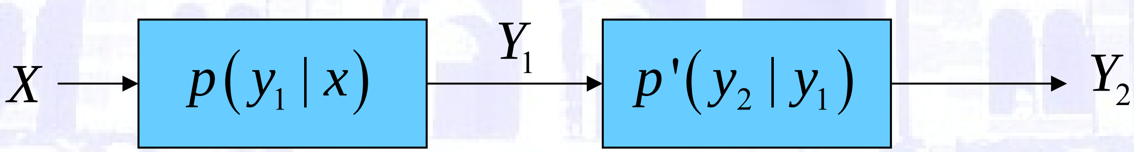
- The stochastically degraded Broadcast channel.
- The broadcast Z channel (B-Z channel)
  - Optimal transmission strategy.
  - Capacity region.
  - Channel coding design, nonlinear turbo codes:
    - Controlled ones density.
    - Designed for the Z channel and the Z channel with erasures.
  - Simulation results.
- Conjecture: optimal transmission strategy for a particular set of broadcast channels.
- Conclusions

The stochastically degraded broadcast channel



- *Stochastically degraded* if:

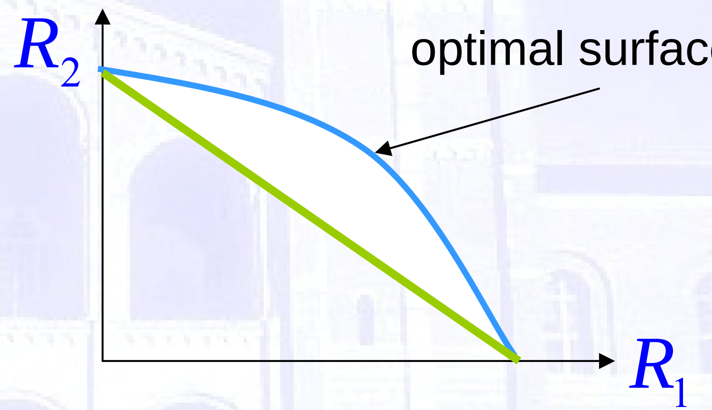
$$\exists\ p'(y_2 | y_1) \text{ such that } p(y_2 | x) = \sum_{y_1} p(y_1 | x)\, p'(y_2 | y_1)$$

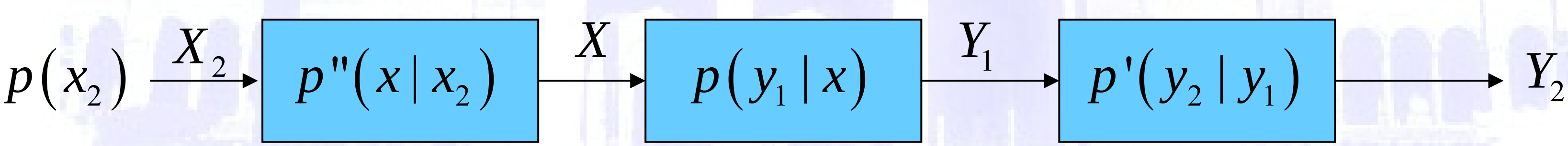


The stochastically degraded channel

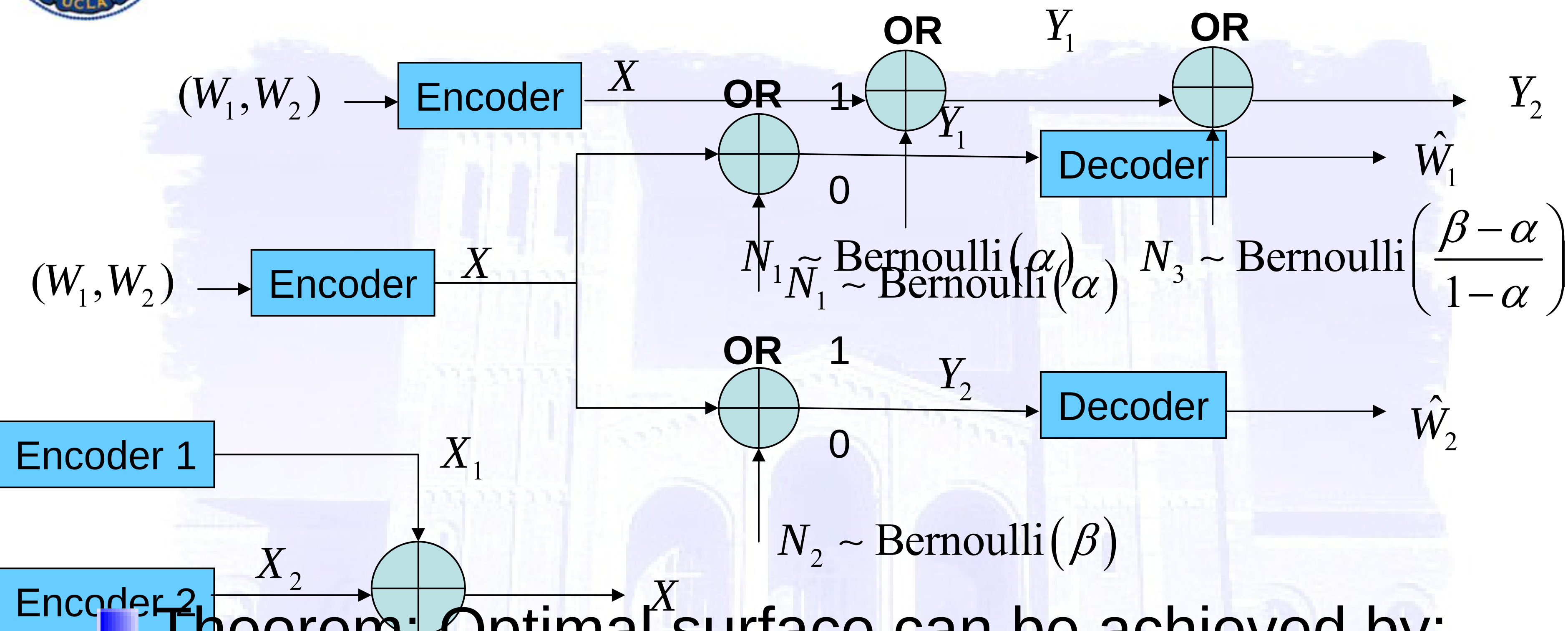
- Capacity region for sending independent information over the degraded channel $X \to Y_1 \to Y_2$ is the convex hull of the closure of the rate pairs

$$R_1 \le I(X;Y_1 \mid X_2)$$
$$R_2 \le I(X_2;Y_2)$$

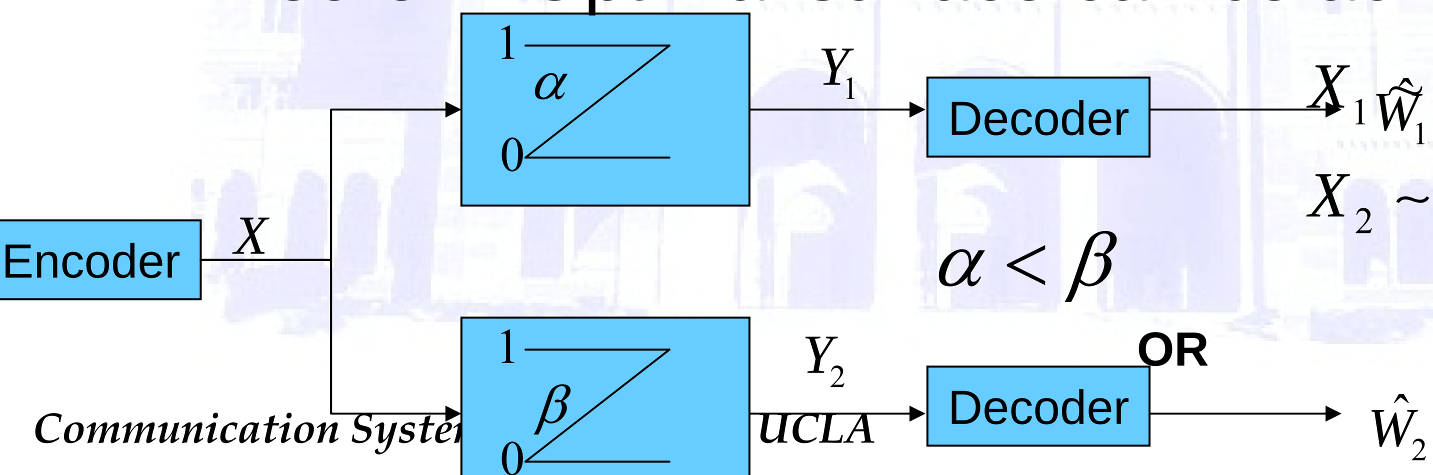




The broadcast Z channel



- Theorem: Optimal surface can be achieved by:



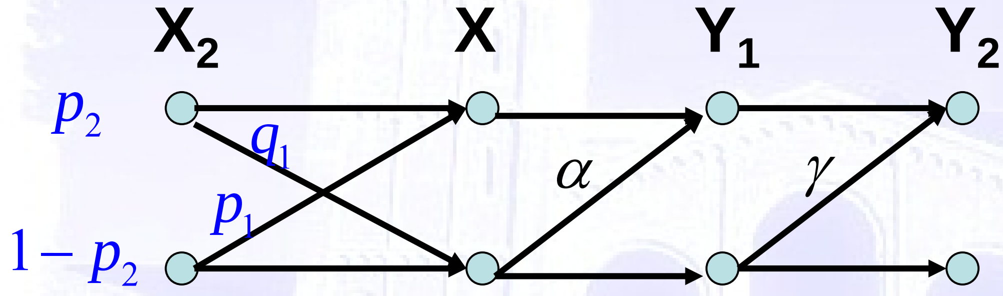
$X_1 \sim \text{Bernoulli}(p_1)$

$X_2 \sim \text{Bernoulli}(p_2)$

$\alpha < \beta$

Optimal transmission strategy

- Sketch of proof:
  - General case



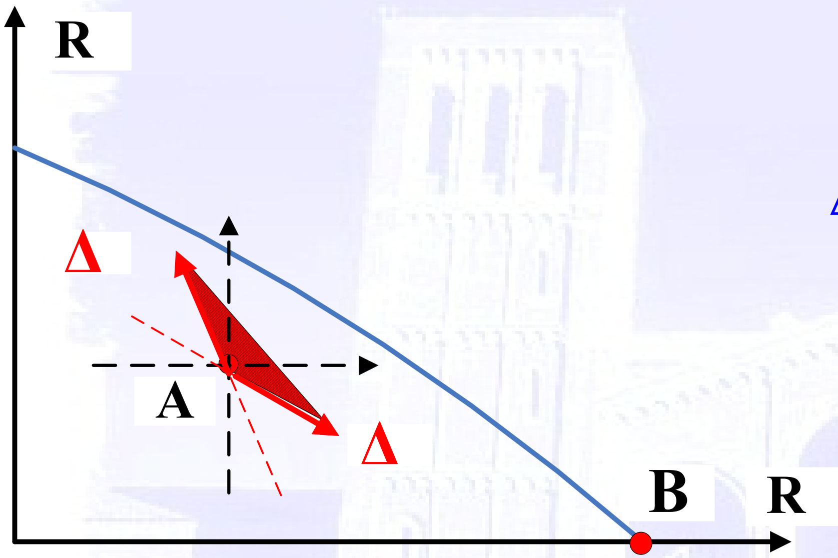
$$R_2 \le I(X_2;Y_2)$$

$$R_1 \le I(X;Y_1 \mid X_2)$$

- We need to prove that q_1=0 (or p_1=1)
- Without loss of generality $q_1 \le 1 - p_1$
- Consider any (R_1,R_2) point achieved with

$$p_2 \ne 0, p_2 \ne 1, q_1 \ne 0, p_1 \ne 1, q_1 \ne 1 - p_1$$

Proof for the B-Z channel



$$\Delta_1 : \begin{Bmatrix} p_2 \to p_2 \\ q_1 \to q_1 - (1-p_2)\tilde{\varepsilon} \\ p_1 \to p_1 - p_2\tilde{\varepsilon} \end{Bmatrix}, \tilde{\varepsilon} > 0$$

$$\Delta_2 : \begin{Bmatrix} p_2 \to p_2 + (1-p_2)\hat{\varepsilon} \\ q_1 \to q_1 \\ p_1 \to p_1 + (1-p_1-q_1)\hat{\varepsilon} \end{Bmatrix}, \hat{\varepsilon} > 0$$

$$\mathbf{A} : (R_1, R_2)|_{p_2, q_1, p_1}$$

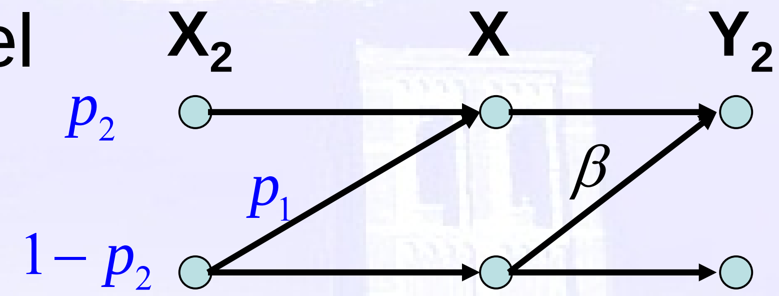
$$R_1 \leq I(X_2; Y_2)$$

$$R_2 \leq I(X; Y_1 | X_2)$$

$$p_2 \neq 0, p_2 \neq 1, q_1 \neq 0, p_1 \neq 1, q_1 \mathbf{2} 1 - p_1$$

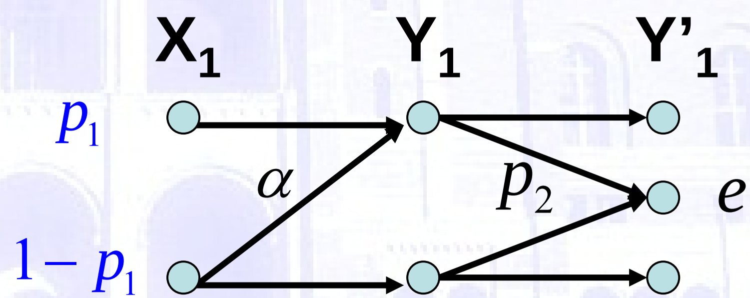
Perceived channels

- Receiver 2: Z channel



$$R_2 \le I(X_2;Y_2)$$
$$= H\left[(1-p_2)(1-p_1)(1-\beta)\right] - (1-p_2)H\left[(1-p_1)(1-\beta)\right]$$

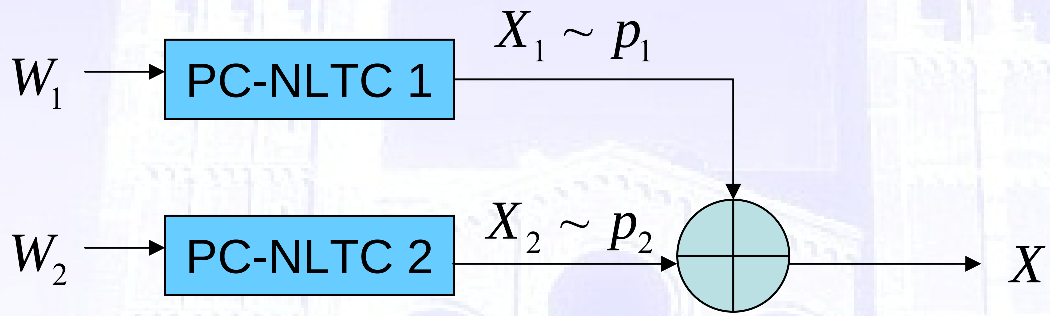
- Receiver 1: Z channel + erasure channel.



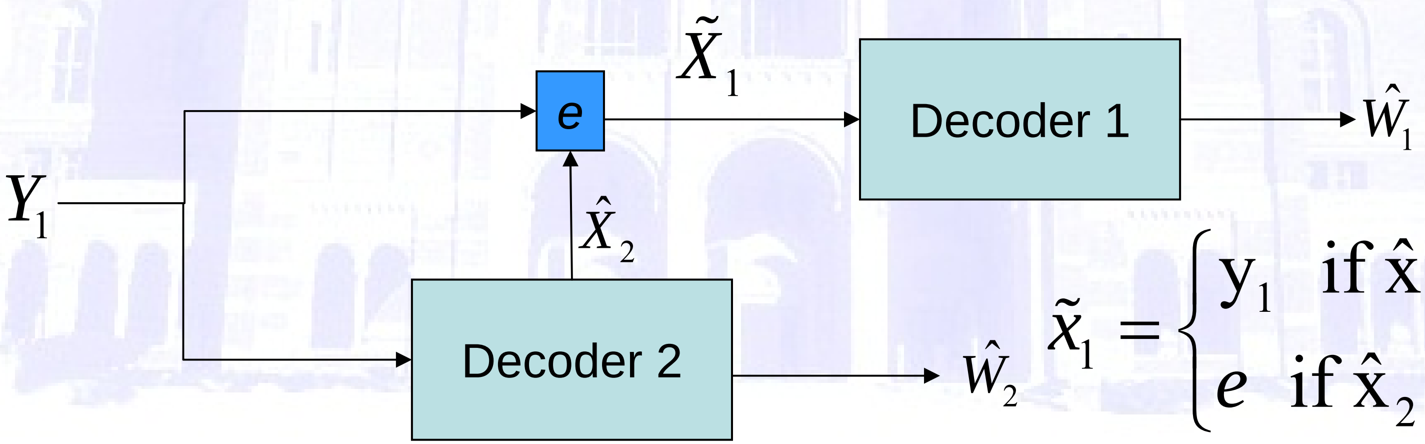
$$R_1 \le I(X;Y_1 \mid X_2)$$
$$= (1-p_2)\left\{H[(1-p_1)(1-\alpha)] - (1-p_1)H[1-\alpha]\right\}$$

Implementation

- Encoding: OR of two parallel concatenated nonlinear trellis codes [GlobeCom'06].



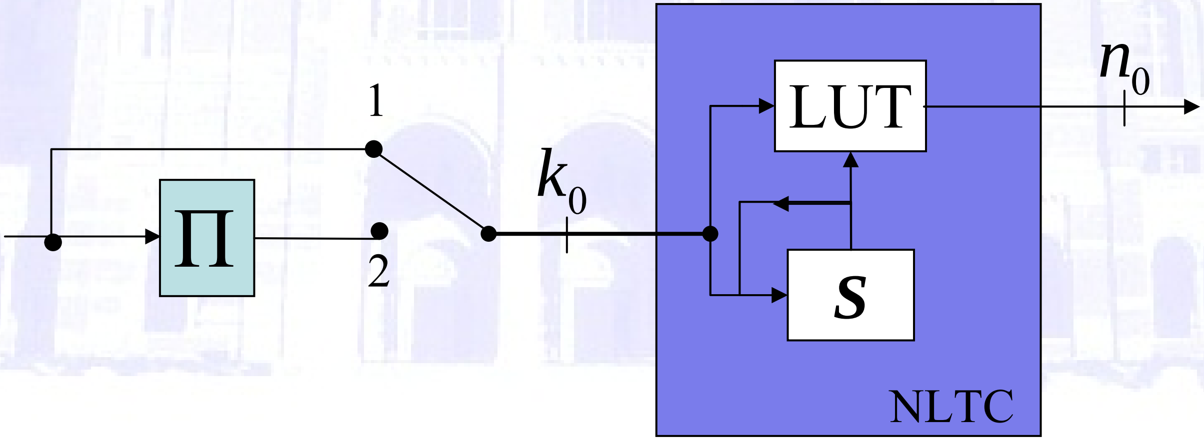
- Decoding receiver 1(hard):



$$\tilde{x}_1 = \begin{cases} y_1 & \text{if } \hat{x}_2 = 0, \\ e & \text{if } \hat{x}_2 = 1. \end{cases}$$

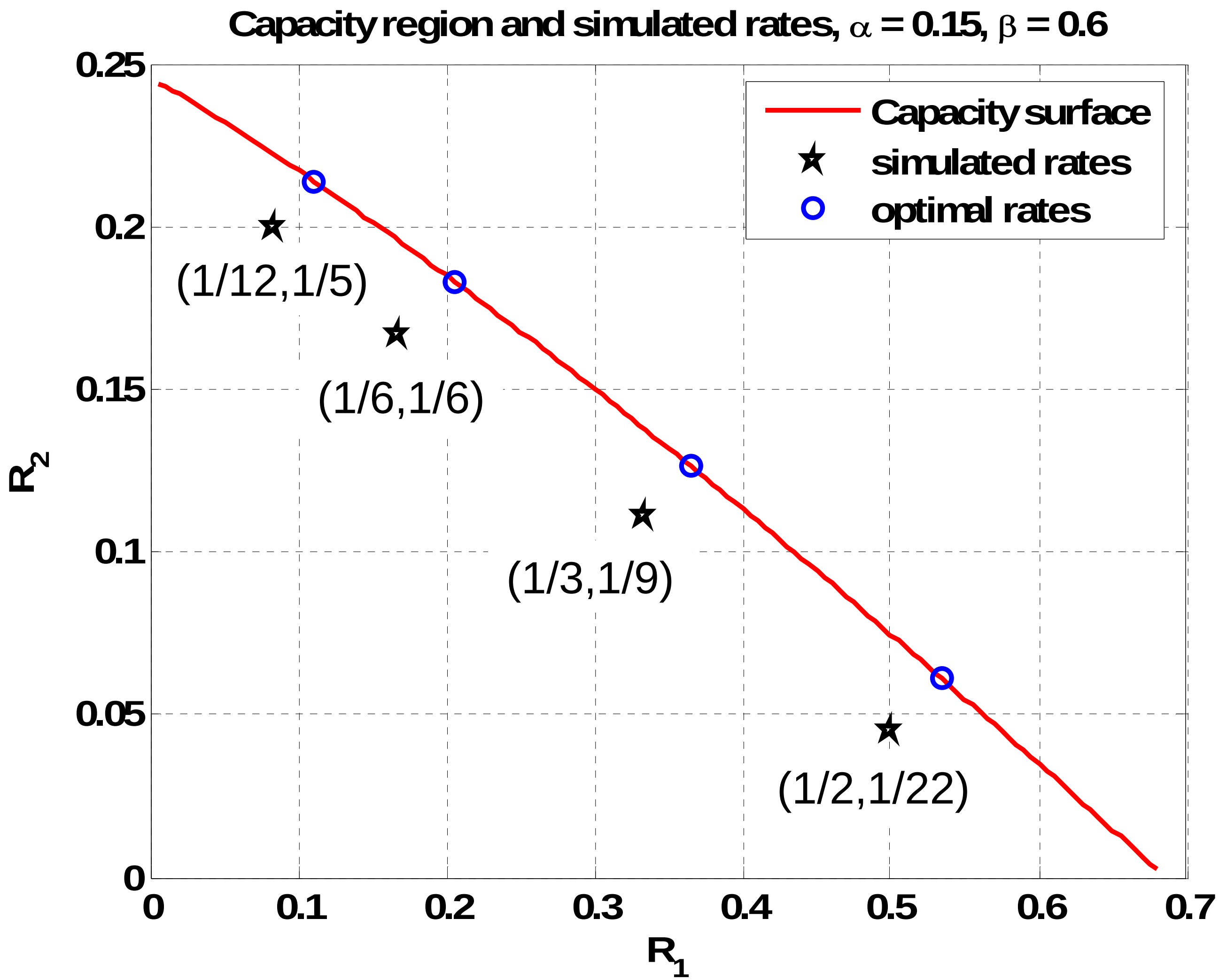
Parallel Concatenated Nonlinear Trellis Codes

- Presented in GlobeCom'06 (for Z channel).
- The NLTC consists of:
  - A 2^{ν}-state trellis structure (block *S*).
  - A look-up table (LUT) stores an output per branch.
  - The outputs satisfy the required ones density *p* (non-systematic)
- PC-NLTC: Two constituent (n_0, k_0) non-linear trellis codes (NLTC) linked by an interleaver (Π) of length *K*.



$$\text{rate} = \frac{k_0}{2n_0}$$

Example



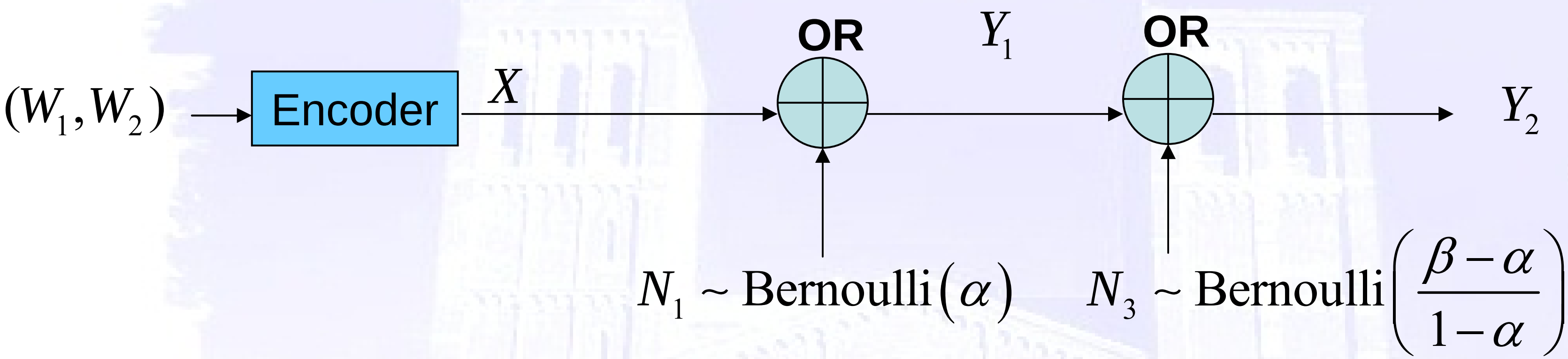
Capacity region and simulated rates, α = 0.15, β = 0.6
Capacity surface
simulated rates
optimal rates
(1/12,1/5)
(1/6,1/6)
(1/3,1/9)
(1/2,1/22)
0.25
0.2
0.15
0.1
0.05
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
R2
R1

Results

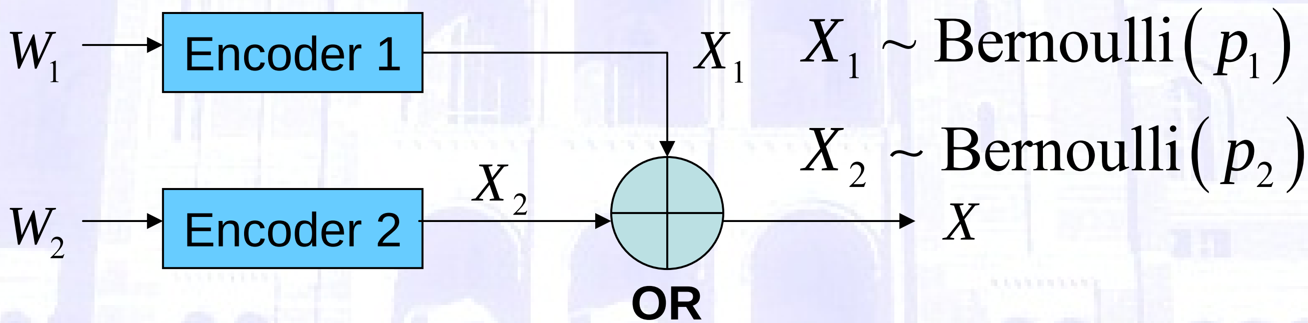
- 8-state nonlinear turbo codes.
- $k_0 = 2$

R_1	R_2	p_1	p_2	K_1	K_2	BER_1	BER_2
1/12	1/5	0.106	0.56	4800	1700	2.54×10^{-5}	1.24×10^{-5}
1/6	1/6	0.196	0.5	2048	2048	7.01×10^{-6}	5.33×10^{-6}
1/3	1/9	0.336	0.3739	1536	1536	7.13×10^{-6}	6.70×10^{-6}
1/2	1/22	0.463	0.1979	5632	1024	9.27×10^{-7}	3.27×10^{-6}

The broadcast Z channel

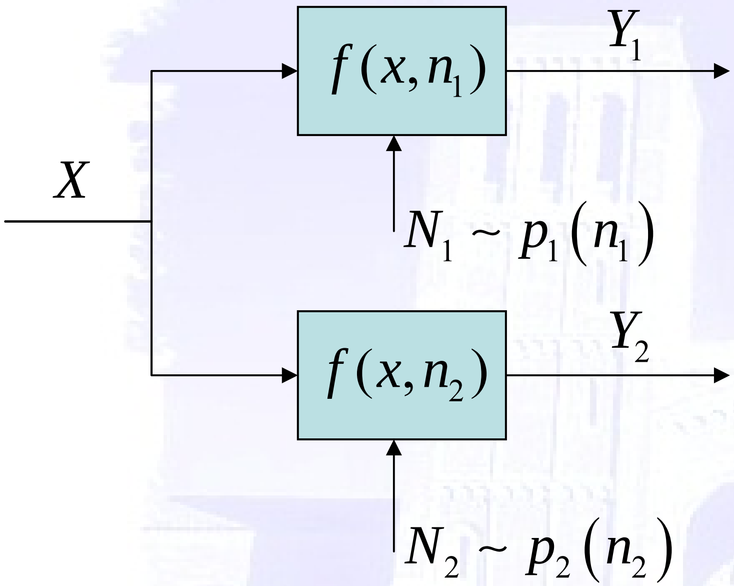


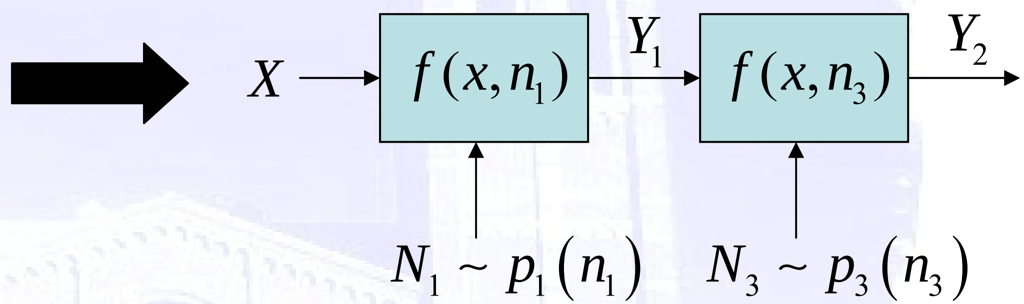
- Optimal surface can be achieved by:



- Also true for (AWGN, + operator), (BSC, XOR).

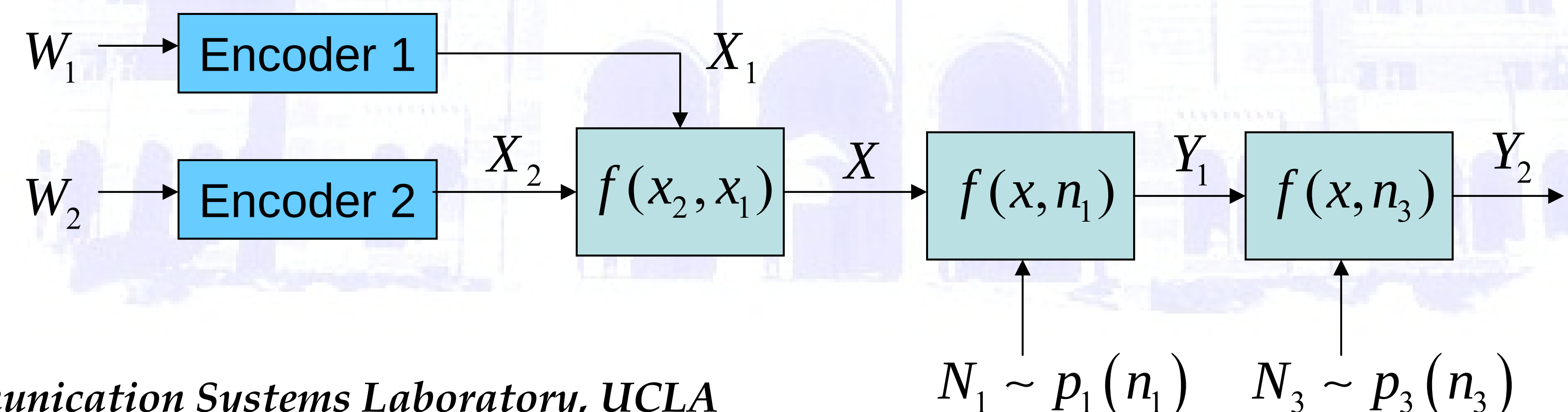
Conjecture





$$|X| = |Y_1| = |Y_2| = |N_1| = |N_2|$$

- Points on the optimal surface can be achieved by:



Conclusions

- We have presented an optimal transmission strategy for the Broadcast Z Channel.
- Simple encoding and decoding.
- A practical implementation that works close to capacity has been presented.
- Nonlinear turbo codes, specifically designed for the Z channel and the Z channel + erasures, have been designed.
- Conjecture: simple transmission strategy could be used to a set of stochastically degraded broadcast channels.