I have shown in [1] that

$$\frac{1}{2} \leq \liminf_{n\to\infty} \frac{N(\mathcal{L}_n)}{\sqrt{n}} \leq \limsup_{n\to\infty} \frac{N(\mathcal{L}_n)}{\sqrt{n}} \leq 15 \text{ a.s.},$$

where $N(\mathcal{L}_n)$ is the maximum number of non-crossing line segments in $\mathcal{L}_n$ (a set of $n$ random line segments in the unit square). Having established this result, we can say a lot more with relatively little effort. An application of Talagrand's Inequality [2, Section 7.7] reveals that $N(\mathcal{L}_n)$ is tightly concentrated about its median $m_n \in [\sqrt{n}/2, 15\sqrt{n}]$ in an interval of order no larger than $n^{1/4}$. Further, by applying the subadditive ergodic theorem (in a manner similar in spirit to [3, Example 7.5.2] albeit to a four-dimensional Poisson process), one can show that there exists some constant $c \in [1/2, 15]$ such that

$$\frac{N(\mathcal{L}_n)}{\sqrt{n}} \to c \text{ a.s.}$$

Hence, almost everything is known about asymptotic the behavior of $N(\mathcal{L}_n)$, except the exact value of $c$. Thus, I pose this as a challenge to whoever may stumble across this note. For the sake of making a guess, I'll conjecture that $c = 2$ (as in the case of the longest increasing subsequence of a random permutation). If you succeed in determining $c$, I would be very interested in hearing from you. A good starting place would be Hammersley (1970), Logan and Shepp (1977), and Vershik and Kerov (1977). In these works, the authors determine the constant $c$ for the problem of longest increasing subsequence of a random permutation.

Thomas Courtade
University of California, Los Angeles
November 1, 2011

## References

[1] T. Courtade, "On Random Line Segments in the Unit Square", unpublished but available on my website (2011).

[2] N. Alon and J. Spencer. The probabilistic method (2ed). New York: Wiley-Interscience (2000). ISBN 0-471-37046-0.

[3] R. Durrett, Probability: Theory and Examples, Fourth Edition (2010), Cambridge University Press.