Oscillator network based control of unsteady fluid flows

Aditya G. Nair* Kunihiko Taira* Steven L. Brunton**

* Department of Mechanical Engineering, Florida State University, Tallahassee, FL 32310, USA
** Department of Mechanical Engineering, University of Washington, Seattle, Washington 98195, USA

Abstract: A network-based analysis is performed to examine the transfer of kinetic energy amongst modal structures in unsteady fluid flows. Spatial modes are extracted from the flow fields with the corresponding amplitudes serving as a set of oscillators describing unsteady fluctuations. The ensuing dynamics are analyzed using a networked oscillator model describing the amplitude of energy transfers and phase dynamics among the modes. Leveraging the network interactions extracted, we demonstrate the effectiveness of model-based controller in DNS to control the energy transfer dynamics. The validity and accuracy of the approach is demonstrated for a canonical example of two-dimensional unsteady flow over a circular cylinder.

Keywords: Unsteady fluid flow, Active flow control, Oscillator network

1. INTRODUCTION

Modal decomposition techniques extract coherent structures embedded in fluid flows (Holmes et al., 2012; Taira et al., 2017). In particular, proper orthogonal decomposition (POD) (Sirovich, 1987; Berkooz et al., 1993) decomposes snapshot flow field data into discrete modes based on energy content. Using these modes, reduced-order models are developed which reduce the computational complexity of analyzing fluid flows dramatically (Aubry et al., 1988; Holmes et al., 2012; Noack et al., 2003). However, unsteady fluid flows are characterized by strong nonlinear interactions which pose a significant challenge to the effectiveness of flow control and limit applicability of traditional reduced-order models (Brunton and Noack, 2015). In the present effort to control nonlinear interactions in fluid flow, we leverage network analysis to examine how these modes interact to distribute energy in non-equilibrium conditions.

The application of network analysis (Newman, 2010) has recently been extended to represent vortical interactions in fluid flows (Nair and Taira, 2015) and uncovered the scale-free network characterization of decaying two-dimensional turbulence (Taira et al., 2016). In this work, we extend network analysis to describe and control modal interactions in fluid flows by casting the fluid flow problems in terms of a network of coupled oscillators. An oscillator is a set of self-sustaining dynamical features that exhibit periodic fluctuations. Coupling of multiple oscillators leads to collective rhythms with transfer of energy between them representing interaction physics. There have been many rich studies examining coupled oscillators, in particular by Kuramoto and Strogatz. The foundational work laid out by Kuramoto (1975, 1984) elegantly describes the interactive phase dynamics between oscillators. In the works of Aizawa (1976) and Matthews and Strogatz (1990), the oscillator phase model was generalized to incorporate amplitude variations. We extend these ideas to characterize and control interactions in unsteady fluid flows.

In this work, we utilize modal decomposition techniques in conjunction with coupled oscillator models to highlight the interactive physics involved in unsteady fluid flows. Our objective in this work is three-fold: (1) characterize the nonlinear energy transfer between modes in unsteady fluid flows, (2) describe interactive dynamics between modes from a network-theoretic perspective and (3) control the perturbations about the limit cycle state of periodic flows. We demonstrate the strength of our present approach on a canonical example of two-dimensional cylinder flow. In what follows, we first lay the theoretical foundation of this work in §2. We then demonstrate the oscillator network based representation of modal interaction for incompressible flow over a cylinder in §3. Concluding remarks are offered in §4.

2. FORMULATION

A network or graph $\mathcal{G} = \{V, E, w\}$ consists of a set of nodes $V$ connected by edges $E$ with associated edge weights $w$ (Chung, 1997; Newman, 2010). For a set of $N$ nodes, the network is concisely described by its adjacency matrix $A_{\mathcal{G}} \in \mathbb{R}^{N \times N}$ given as

$$[A_{\mathcal{G}}]_{mn} = \begin{cases} w_{mn} & \text{if there is an edge from } n \text{ to } m \\ 0 & \text{otherwise.} \end{cases} \quad (1)$$

Another matrix closely related to the adjacency matrix is the (in-degree) graph Laplacian, defined as

$$[L_{\mathcal{G}}]_{mn} = \sum_{n=1}^{N} [A_{\mathcal{G}}]_{mn} - [A_{\mathcal{G}}]_{mn}. \quad (2)$$
In the present work, we consider the modes (pairs) from POD to be the nodes (V) and the interaction between them as edges (E). The unsteady incompressible velocity field \( \mathbf{u} \) to be examined can be approximated by a finite expansion in terms of the mean velocity field \( \mathbf{u}_0 \) and N spatial POD modes \( \phi_n^u \) as
\[
\mathbf{u}(x,t) \approx \mathbf{u}_0(x) + \sum_{n=1}^{N} a_n(t)\phi_n^u(x),
\]
where \( a_n(t) = \langle \mathbf{u} - \mathbf{u}_0, \phi_n^u \rangle \) are the temporal coefficients and \( \langle \cdot, \cdot \rangle \) denotes the inner product over the computational domain. The kinetic energy of the modes are given by \( \bar{a}_n^2/2 \), where \( \bar{\cdot} \) represents the time average, and the total modal energy is provided by \( E = \sum_n^N \bar{a}_n^2/2 \).

The present formulation is concerned with time periodic flows for which conjugate mode pairs describing periodic coherent structures are obtained from POD. Conjugate mode pairs \( (\phi_{2n-1}^u, \phi_{2n}^u) \) with temporal coefficients \( (a_{2n-1}, a_{2n}) \) can be represented in the complex plane as,
\[
z_m = a_{2n-1} + i a_{2n} = r_m \exp(i\theta_m),
\]
where \( m = 1, 2, \ldots N/2 \), \( n = 1, 2, \ldots N/2 \), \( r_m = |z_m| \), and \( \theta_m = \angle z_m \). The temporal coefficient associated with the mean flow \( (\omega_0) \) is fixed to unity, \( z_0 = 1 \). Analogous to Eq. (3), the velocity field can be recovered in terms of the mean field and modes as
\[
\mathbf{u}(x,t) \approx \mathbf{u}_0(x) + \sum_{n=1}^{N/2} [\Re(z_m)\phi_{2n-1}^u + \Im(z_m)\phi_{2n}^u],
\]

The Stuart–Landau equation is related to the normal form of the Andronov–Hopf bifurcation and describes generalized dynamics of limit-cycle oscillators in the baseline case that is time-periodic without any forcing as
\[
z_m^b = z_m^b(\lambda_m - |z_m^b|^2 + i\Omega_m^b),
\]
with \( z_m^b = r_m^b \exp(i\theta_m^b) \), \( \lambda_m = (r_m^b)^2 \) and \( \Omega_m^b \) being the oscillator frequency. The superscript \( b \) denotes the baseline case. It is noteworthy that there is no coupling between oscillators in the baseline dynamics which leads us to a networked oscillator model. To characterize these interactions, we introduce impulsive perturbations to the baseline temporal coefficients of oscillators at \( t = t_0 \). These perturbations are introduced in direct numerical simulation (DNS) to the baseline shedding state, and they lead to the emergence of nonlinear interactions and energy exchange among the oscillators resulting in fluctuations described by
\[
z_m = r_m \exp(i\theta_m) = z_m^b + z_m'.
\]

The initial velocity field for DNS for perturbed cases is constructed by substituting Eq. (8) at \( t = 0 \) in Eq. (5).

Once the perturbations are introduced, the unsteady flow tries to return to the baseline flow (natural limit cycle) by distributing the effect of the perturbation among the POD modes. The resulting fluctuations in the temporal dynamics of the modes are attributed to the interactions between them. To capture the fluctuating amplitude and phase of the temporal coefficients, we track the normalized fluctuation \( \zeta_m = |z_m'/z_m^b| \approx \epsilon_m \exp(i\theta_m') \). To extract oscillator interactions, we consider a networked oscillator model of linearly coupled oscillators given by
\[
\zeta_m = \sum_{n=1}^{N/2} [\mathbf{A}_G]_{mn}(\zeta_n - \zeta_m) = -\sum_{n=1}^{N/2} [\mathbf{L}_G]_{mn}\zeta_n,
\]
where \( [\mathbf{A}_G]_{mn} \) and \( [\mathbf{L}_G]_{mn} \) are the adjacency and the in-degree Laplacian matrices, respectively. The rows of the adjacency matrix indicates the dependence of the oscillators \( n \) on the dynamics of oscillators in column \( m \), i.e., \( w_{mn} \). Using the knowledge of time evolution of \( \zeta \) from DNS, we use a linear regression procedure to compute the adjacency matrix weights.

As the linear regression is performed over temporal coefficients in the complex plane, the adjacency matrix weights are complex-valued. These weights \( w_{mn} \) can be broken down to a magnitude \( |w_{mn}| \) and phase \( \angle w_{mn} \). The magnitude of the weights signifies the influence of oscillator \( n \) on oscillator \( m \) as illustrated in Figure 1 (b). The phase of the edge weights emphasizes the individual modal contributions in oscillator interactions. The networked oscillator model describes both amplitude and phase dynamics of collective oscillation and captures the energy exchange and phase dynamics among the modes. Below, we apply our approach to analyze a canonical problem of two-dimensional incompressible flow over a cylinder.
3. APPLICATION TO CYLINDER FLOW

3.1 Baseline (unperturbed flow)

We first perform DNS of incompressible flow over a cylinder using the immersed boundary projection method (Taira and Colonius, 2007; Colonius and Taira, 2008; Munday and Taira, 2013) at a diameter-based Reynolds number of 100. We perform POD using method of snapshots (Sirovich, 1987) with velocity field data \( u \) in the baseline case. As 99.98% of the modal kinetic energy is captured by the first eight POD modes, we choose \( N = 8 \) for our analysis. The extracted spatial modes and oscillators corresponding to temporal coefficients of the individual mode pairs are shown in Figure 2. Each mode pair is ordered in terms of decreasing energy content. These modes were validated against those obtained from the work by Noack et al. (2003). The temporal dynamics of these oscillators (modes) are independent with no coupling between them.

3.2 Perturbed flow

To examine the interaction between the modes in cylinder flow, we introduce an amplitude perturbation at \( t = t_0 \) in DNS. Thus, the initial velocity field for DNS of the perturbed case is prescribed introducing additional energy into the simulation through the initial condition (impulse perturbation). As the introduced perturbation in oscillator II convects downstream, it interacts with other oscillators. The projected temporal coefficients from the perturbed case are then extracted from DNS and the time history of normalized fluctuation \( \zeta_m \) is tracked using the networked oscillator model. To construct the normalized fluctuation time history, \( \zeta_m(\theta_I) = [z_m(\theta_I) - z_m^b(\theta_I)]/z_m^b(\theta_I) \), we align the phase of oscillator I (containing a majority of the energy) for the perturbed case and the baseline case. Once the normalized fluctuation is constructed, we also compute its time derivative \( \dot{\zeta}_m \). We then construct the library of functions \( \zeta_n - \zeta_m \) for each oscillator \( m \) and perform a simple regression to obtain \( [A_G]_{mn} \).
Here, we consider full-state feedback and a column input transfer dynamics effectively for unsteady fluid flows. Thus, using a networked oscillator model in control in DNS is shown in Figure 3(d). We can drive the oscillators I, II and III. With the network interactions characterized, we can develop a control strategy to suppress the perturbations. The networked oscillator model framework helps us extract a linear differential equation given by Eq. (9) governing the dynamics of perturbations introduced. We demonstrate the use of the linear quadratic regulator (LQR) to control the amplitude perturbations imposed on oscillator I. An optimal control strategy using LQR minimizes the quadratic cost function of the form,

\[ J = \int_0^\infty x(t)^T Q x(t) + u(t)^T R u(t) \, dt, \]

where \( Q \) and \( R \) are the weights. Here, the state variable \( x \) is the normalized fluctuation in the temporal coefficient of the oscillators \( \zeta \). We use \( Q = I \) and \( R = 0.1 \) above with the cost function associated with the state weighted more than the input. We implement an optimal feedback control law \( u = -K x \) yielding a control system of the form,

\[ \dot{\zeta}_n = - \sum_{m=1}^{N/2} [L_n - BK]_{mn} \zeta_m. \]  

Here, we consider full-state feedback and a column input matrix \( B = [0 \ 1 \ 0 \ 0] \). The results with the application of control in DNS is shown in Figure 3(d). We can drive the oscillators to the natural limit cycle much faster than the baseline case. Thus, using a networked oscillator model in conjunction with optimal control, we can control energy transfer dynamics effectively for unsteady fluid flows.

4. CONCLUDING REMARKS

We constructed a networked oscillator model for describing modal interactions in unsteady fluid flows. Amplitude and phase perturbations introduced in the Navier–Stokes equations were tracked using this networked oscillator framework. Using a canonical example of flow over a cylinder, the energy transfer dynamics were highlighted and excellent agreement of the model with DNS were observed. With the knowledge of network interactions between oscillators, an optimal control strategy was designed for suppressing oscillator fluctuations to the natural limit cycle faster. The present approach leverages the knowledge of modal interactions providing insights beyond traditional approaches for flow control of unsteady fluid flows.

REFERENCES


