Distributed Frequency Control of Prosumer-Based Electric Energy Systems

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***Abstract*—In this paper, we propose a distributed frequency regulation framework for prosumer-based electric energy systems, where a prosumer (producer-consumer) is defined as an intelligent agent which can produce, consume, and/or store electricity. Despite the frequency regulators being distributed, stability can be ensured while avoiding inter-area oscillations using a limited control effort. To achieve this, a fully distributed one-step model-predictive control protocol is proposed and analyzed, whereby each prosumer communicates solely with its neighbors in the network. The efficacy of the proposed frequency regulation framework is shown through simulations on two real-world electric energy systems of different scale and complexity. We show that prosumers can indeed bring frequency and power deviations to their desired values after small perturbations.**

***Index Terms*—Distributed frequency regulation, distributed optimization, long-term freqeuncy stability, model-predictive control, prosumers.**

I. Introduction

TODAY'S electric power systems are operated and controlled in a hierarchical way under which monitoring and control tasks are handled at different hierarchical levels. However, emerging technologies such as renewable energy sources, distributed generators (DGs), plug-in hybrid electric vehicles (PHEVs), and demand response, pose new challenges to the operation and control of legacy electric power systems due to the increased scale and complexity. In fact, it will no longer be possible for control centers and SCADA (supervisory control and data acquisition) systems to fully monitor and control emerging electric energy systems.

To overcome these challenges, it is envisioned that future, smart energy grids will be populated with multiple decision makers. Each such decision maker, or agent, will be able to produce, consume, and/or store electricity, as well as make strategic decisions enabled by deployment of communication and information technologies. These economically motivated agents are known as prosumers (producers-consumers) [1]. Under the prosumer-based framework, electric energy systems are largely flat [2] in that small prosumers, such as individual buildings with energy management systems, share the same functionality as large prosumers, such as regional utilities, but at the smaller scale. The challenges are thus how to gracefully extend the current control and management paradigm to energy-systems comprised of prosumers, where the control objectives of the individual prosumers are similar, although more advanced, to those associated with traditional control centers.

In this paper, we address one particular, technical aspect of the prosmer-based energy systems, namely the frequency regulation problem. We propose a general framework for *distributed frequency regulation in prosumer-based electric energy systems* and address the problem of how thousands of spatially distributed and heterogeneous prosumers can regulate frequency in a distributed and robust manner. To this end, we first review today's practices for regulating frequency and balancing mid-term hard-to-predict fluctuations in load.

The rest of the paper is organized as follows: In Section II, today's industry practices for frequency regulation are revisited and the prosumer model is introduced, followed by a general framework for distributed frequency regulation in Section III. The simulation results for distributed frequency regulation on two real-world electric energy systems are presented in Section IV, and the paper concludes in Section V with discussions of the overall findings.

II. Today's Industry and Beyond

In today's electric power industry, hard-to-predict fluctuations in demand and renewable energy production are stabilized by means of governor control of fast-ramping power plants. These fluctuations cause steady state errors in frequency and tie-line flows. In order to bring steady state frequency to the nominal value[1] and tie-line flows to their scheduled values, it is essential to regulate frequency minute-by-minute. To achieve this, automatic generation control (AGC) systems are used to adjust set-points of the governor control of fast-ramping power plants at a regional level. The implementation of such AGC systems to regulate frequency requires centralized communication/control systems, while for smaller systems, such as micro-grids, a few fast-ramping generators can be equipped with integral control systems to regulate frequency directly [3].

Decentralized frequency regulation is already employed by utilities and area-controls in an ad-hoc fashion by reacting to local frequency measurements and then adopting *unilateral* corrective control actions, i.e., actions that neglect the effects of coupling between neighboring utilities/area-controls. It is well-

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[1]60 Hz in the U.S. and 50 Hz in Europe

known that the lack of coordination between the AGC systems of neighboring utilities/area-controls may cause inter-area oscillations and potentially lead to instability problems. In the worst-case scenario, inter-area oscillations can result in blackouts [4].

Much research has been done to mitigate this, and several methods have been proposed to overcome inter-area oscillations [5], [6]. Although the proposed methods could increase the damping of inter-area modes, they do not guarantee system-wide stability, which is an explicit aim in this paper.

Recently, there have been proposals to mitigate inter-area oscillations by means of wide-area monitoring and control (WAMC) systems based on synchronized phasor measurements [7].

The implementation of WAMC systems requires extensive hierarchical communication/control infrastructure. This makes centralized control systems more vulnerable to cyber-attacks, meaning that the power systems will have a single point of failure. To remedy this, we propose a new distributed frequency regulation framework for prosumer-based electric energy systems. In order to ground the technical discussion, we first describe what is meant by a prosumer-based electric energy system.

Traditionally, electric power systems are divided into generation, transmission, distribution, and consumption sections. Each section is strictly producing, transporting, or consuming electricity. As already discussed in the introduction, the boundary between producers and consumers will become increasingly blurry as emerging technologies allow consumers to produce and/or store electricity. Therefore, the conventional producers and consumers transform into hybrid agents that we denote "prosumers".

A prosumer can be an independent system operator (ISO), a utility, a microgrid, or even a building. All prosumers will have to have a physical layer, a control layer, and a communication layer, where the physical layer includes physical devices inside a prosumer, such as generators, loads, and wires [1]. The control layer consists of the computation center and control devices of the prosumer. The communication layer allows the prosumer to communicate with its neighbors and to share key local information.

Prosumers have multiple controllable components, such as deferrable loads, storages, PHEVs, distributed energy resources (DERs), and/or conventional power plants. For the purpose of this paper, prosumers can regulate frequency by adjusting set-points of the controllable components. Since all prosumers are coupled through the grid, the control strategy of a prosumer will inevitably have to depend on the behaviors of other prosumers. Therefore, a coordinated control framework needs to be implemented in order to achieve system-wide stability with minimal control effort. This is the topic of the next section.

III. Distributed Frequency Regulation

In this section, we investigate how prosumers can optimize the system-wide performance index by iteratively communicating key local information so that the entire system converges to the nominal values, in order to

1) achieve system-wide frequency stability;
2) avoid inter-area oscillations;
3) minimize system-wide control effort;
4) reduce the required amount of sensing and communication.

The proposed solution that satisfies all of these objectives will be based on a variation on model predictive control (MPC), where the look-ahead horizon is limited to a single step in order to limit the need for information sharing [8]. But, before we can arrive at this formulation, we first need to pose the frequency regulation problem.

A. Quasi-Steady State Dynamic Model

Recalling from previous sections, frequency regulation involves bringing the steady state frequencies at the individual prosumers to 60 Hz, or, more generally, to ω^{des}. We moreover need to achieve this in a distributed manner, and we start by observing that, once the so-called primary control dynamics have been stabilized, the quasi-static description of the frequency at prosumer i satisfies the three-way droop equation [9]

$$\Delta\omega_i(k) = \gamma_i u_i(k) - \sigma_i \Delta P_i(k) \tag{1}$$

where $\Delta\omega_i(k) = \omega_i(k) - \omega^{\mathrm{des}}$ is the deviation in frequency at prosumer i from the desired frequency at time step k, whose internal dynamics is characterized at steady state by the parameter γ_i. Moreover, u_i is the reference set point, which is going to act as our control input, ΔP_i is the deviation in output power from the scheduled interchange, and σ_i is the droop constant of prosumer i, which in essence acts as a proportional control gain in the droop controller. For a treatment of the three-way droop equation, see for example [9].

Gathering together the contributions from all n prosumers in the network as $\Delta\omega = [\Delta\omega_1, \ldots, \Delta\omega_n]^T$, $u = [u_1, \ldots, u_n]^T$, and $\Delta P = [\Delta P_1, \ldots, \Delta P_n]^T$, yields

$$\Delta\omega(k) = \Gamma u(k) - \mathcal{S} \Delta P(k) \tag{2}$$

where $\Gamma = \mathrm{diag}(\gamma_1, \ldots, \gamma_n)$ and $\mathcal{S} = \mathrm{diag}(\sigma_1, \ldots, \sigma_n)$.

As both Γ and $\mathcal{S}$ are diagonal, this formulation seems to indicate that there is no coupling whatsoever between prosumers. This is of course not the case, and the prosumers are indeed coupled through the power grid via the output powers. We can express this coupling by relating the active power injections to the bus angles across the grid

$$\Delta P(k) = J_{P\delta} \Delta\delta(k). \tag{3}$$

Here $J_{P\delta}$ is the Jacobian matrix for real powers and voltage angles of the grid, whose entries are functions of the susceptances and conductances of the grid and those of the voltage magnitudes and angles of the prosumers. This matrix contains the structural information about the grid, i.e., $J_{P\delta}$ is not diagonal, and $\Delta\delta = [\Delta\delta_1, \ldots, \Delta\delta_n]^T$ is a vector containing the deviations in voltage angles from the scheduled voltage angles of the individual prosumers.[2] Note that in this formulation, we as-

[2]In the terminology of graph-based, distributed control, the Jacobian is a weighted Laplacian. For the purpose of small changes in frequency, the Jacobian matrix $J_{P\delta}$ can be assumed to be constant.

sume that the sensitivity of real powers with respect to changes in voltage magnitudes is negligible. That is,

$$\|J_{P\delta}\|_\infty \gg \|J_{PV}\|_\infty \tag{4}$$

where $\|\cdot\|_\infty$ represents the infinity norm of the indicated matrix.

The model we will be considering is a discrete-time model, as is appropriate for the quasi-static models and is standard in the frequency regulation literature, e.g., [9]. The relationship between voltage angle and frequency becomes

$$\Delta\delta(k+1) = \Delta\delta(k) + T_s\Delta\omega(k) \tag{5}$$

given the sample time T_s. The time scales under consideration when doing frequency regulation is at the order of tens of seconds or a minute, i.e., a typical value for T_s would be 30 seconds [9].

Gathering all these terms together and setting $J = J_{P\delta}$ give

$$\begin{aligned}\Delta P(k+1) &= J\Delta\delta(k+1) = J\Delta\delta(k) + JT_s\Delta\omega(k)\\ &= \Delta P(k) + JT_s\Gamma u(k) - JT_s\mathcal{S}\Delta P(k)\\ &= (I - T_sJ\mathcal{S})\Delta P(k) + T_sJ\Gamma u(k)\end{aligned}$$

where I is the $n \times n$ identity matrix. As such, if we let the state of the system be the more conventional $x(k) = \Delta P(k)$, the discrete-time, quasi-static state-space model for the prosumer dynamics becomes

$$x(k+1) = Ax(k) + Bu(k) \tag{6}$$

where the system matrices (the system matrix and control matrix of the grid) are

$$A = I - T_sJ\mathcal{S} \tag{7}$$
$$B = T_sJ\Gamma. \tag{8}$$

As a final note, this model allows for the prosumers to be heterogeneous in the sense that they can support different technologies, such as renewable energy sources, DGs, conventional power plants, demand response, and/or PHEVs.

B. Model-Predictive Control

The aim of the prosumers from the frequency regulation point of view, is to drive the steady state frequencies and power levels to the desired values. Moreover, this must be achieved in a distributed manner, where information is confined to flow among prosumers that are adjacent in the information exchange network. We can view this as an optimal control problem, where the prosumers collaboratively try to minimize the global quadratic cost function

$$\begin{aligned}\min_u \quad & \sum_{k=t_c}^{t_c+K} \left(x(k+1)^T\mathbf{P}x(k+1) + u(k)^T\mathbf{R}u(k)\right)\\ \text{s.t.} \quad & x(k+1) = Ax(k) + Bu(k)\end{aligned} \tag{9}$$

where t_c is the current time at which the minimization takes place, K is the look-ahead horizon (finite or possibly infinite), and $\mathbf{P}$ and $\mathbf{R}$ are *diagonal*, positive definite weight matrices.[3]

If the distributed nature of the problem is not important, the solution, if it exists, to the discrete-time LQ problem is given by

$$u(k) = -\mathbf{K}(k)x(k) \tag{10}$$

where the gain matrix $\mathbf{K}(k)$ is obtained by solving an appropriate Riccati equation, and where $\mathbf{K}$ is constant in the infinite horizon case. Unfortunately, $\mathbf{K}$ does not in general have the same sparsity structure as A or B, i.e., $u = -\mathbf{K}x$ is *not* a distributed solution. But can we turn this somehow into a distributed solution?

As a first observation, since the information exchange network over which the prosumers interact is an undirected and connected network, its transitive closure [4] is a complete graph—or fully connected network. This implies that the conditions prescribed in [10] are met, i.e., the information exchange network is sufficiently rich relative to the system dynamics to be *quadratically invariant*, which in turn implies that a distributed version of $u = -\mathbf{K}x$ is obtainable in a computationally efficient way (the problem is convex). However, obtaining this distributed solution requires that the centralized $\mathbf{K}$ has been obtained, which implies complete knowledge of A and B, which is something that we cannot always ask of the prosumer network.

As such, solving the distributed MPC problem using quadratic invariance is not possible in this particular case. Instead, we have to go directly to the MPC formulation [11], [12]. However, what makes distributed *optimal control* harder than distributed *optimization*, is that it requires predictions into the future. In order for prosumer i to predict its state one step into the future, it needs to know its neighbors' states. To see this, we note that $x(k+1) = Ax(k) + Bu(k)$, where the network sparsity structure is encoded in the A and B matrices. To predict two steps into the future, it needs to know its neighbors' neighbors' states, since $x(k+2) = A^2x(k) + ABu(k) + Bu(k+1)$, where A^2 means that information from two hops away are needed, and so forth for larger prediction horizons. This means that as long as the time horizon K is long enough, knowledge is required of *all* prosumers' states, which is not a scalable proposition, as shown in [13].

One possible remedy to this problem is to turn the MPC problem into an optimization problem by simply letting the time horizon be *one*. The benefit from this is that individual prosumers only need to know their neighbors' states. However, the drawback is that the approach is not always guaranteed to achieve system stabilization. In the next subsection, we explore conditions under which the one-step MPC can stabilize the system. And, we note that this is indeed the key technical contribution of this paper—an investigation of whether or not distributed frequency regulation is possible when restricting the

[3]Note that in general these matrices do not need to be diagonal as long as they are positive definite, but in the prosumer network, we insist on this since we only care about individual deviations from the schedule rather than couplings through cost weights.

[4]The transitive closure of a graph is a graph which contains an edge between any two vertices which are connected in the original graph through a path.

information flow and, subsequently, the length of the prediction horizon.

C. One-Step MPC

Before we can attempt to solve the distributed MPC problem over a single time step, we need to fundamentally understand if it is doable to stabilize the system by one-step MPC. What we want to do is to minimize

$$\min_u \; x(t_c+1)^T \mathbf{P} x(t_c+1) + u(t_c)^T \mathbf{R} u(t_c). \tag{11}$$

Now, from the theory of MPC we know that as long as the system is completely controllable, there exists a positive definite $\mathbf{P}$ that renders the closed-loop system stable when driven by the minimizing solution [14]. In fact, assuming that $u(t_c) = -\mathbf{K}x(t_c)$ is a stabilizing, nominal solution, the optimal solution to the one-step MPC is stabilizing as long as $\mathbf{K}$ and $\mathbf{P}$ jointly satisfy the discrete-time Lyapunov equation

$$(A - B\mathbf{K})^T \mathbf{P}(A - B\mathbf{K}) - \mathbf{P} = -\mathbf{K}^T \mathbf{R} \mathbf{K}.$$

Once this P has been obtained, the optimizing solution can be computed. But, this computation requires—as was also the case with quadratic invariance—a centralized computation.

But, since $x(t_c+1) = Ax(t_c) + Bu(t_c)$, we can set $x(t_c) = x$ and $u(t_c) = u$ and reformulate (11) as

$$\mathcal{C}_x(u) = (Ax + Bu)^T \mathbf{P}(Ax + Bu) + u^T \mathbf{R} u. \tag{12}$$

Note that, at time t_c, x is given (the current state), which is why we use this as a subscript to the cost rather than as a decision variable.

The minimizing u is found by letting

$$0 = \frac{\partial \mathcal{C}_x}{\partial u} = 2B^T \mathbf{P} A x + 2(B^T \mathbf{P} B + \mathbf{R})u \tag{13}$$

i.e.,

$$u = -(B^T \mathbf{P} B + \mathbf{R})^{-1} B^T \mathbf{P} A x. \tag{14}$$

The closed-loop dynamics becomes

$$x(k+1) = (I - B(B^T \mathbf{P} B + \mathbf{R})^{-1} B^T \mathbf{P}) A x(k). \tag{15}$$

As a direct consequence of this, we have the following result:

Lemma 3.1: The minimizing controller to (11) asymptotically stabilizes the system if and only if $|\lambda| < 1$ for all eigenvalues λ to

$$(I - B(B^T \mathbf{P} B + \mathbf{R})^{-1} B^T \mathbf{P}) A.$$

But, this condition is not checkable by the individual prosumers. Instead, we can do better by first observing that by making $\mathbf{P}$ significantly "larger" than $\mathbf{R}$, the control cost goes down, allowing for potentially large control signals that will stabilize the state. An instantiation of this observation could be to let $\mathbf{R} = \epsilon I$ and $\mathbf{P} = I$, and then make ϵ small, in which case the closed-loop system matrix becomes

$$(I - B(B^T B + \epsilon I)^{-1} B^T) A. \tag{16}$$

Taking the limit as ϵ goes to zero from above yields the so-called Tikhonov Regularization [15] of the Moore-Penrose pseudo-inverse

$$\lim_{\epsilon \to 0^+} (B^T B + \epsilon I)^{-1} B^T = B^\dagger \tag{17}$$

which in turn yields

$$\lim_{\epsilon \to 0^+} (I - B(B^T B + \epsilon I)^{-1} B^T) A = (I - BB^\dagger) A. \tag{18}$$

As such, an existence condition that is slightly easier to check is

Lemma 3.2: There exist diagonal, positive weight matrices $\mathbf{P}$ and $\mathbf{R}$ such that the minimizing controller to (11) asymptotically stabilizes the system if $|\lambda| < 1$ for all eigenvalues λ to

$$(I - BB^\dagger) A. \tag{19}$$

As a final observation, we note that $BB^\dagger$ is the orthogonal projector onto the range of B, i.e., $I - BB^\dagger$ is the orthogonal projector onto the kernel of B^T. In other words, if v is an eigenvector to A such that $v \in \mathrm{Range}(B)$ then

$$(I - BB^\dagger) A v = 0 \tag{20}$$

and a direct consequence of this is

Corollary 3.1: There exist diagonal, positive weight matrices $\mathbf{P}$ and $\mathbf{R}$ such that the minimizing controller to (11) asymptotically stabilizes the system if B has full rank.

Now, this corollary is not, as a first attempt, particularly useful. It is well-known that the Jacobian matrix for real powers and voltage angles of electric power girds is singular [16] and that B has rank $n-1$, i.e., the corollary does not apply. But even worse than that, the centralized solution is also not achievable since the system is not even controllable, i.e., the controllability matrix is rank deficient:

$$\mathrm{rank}\,(B \quad AB \quad \cdots \quad A^{n-1}B) = n - 1.$$

As a result, the system model has a one-dimensional, subspace where the behavior is not controllable. The way around this conundrum is through the introduction of the reference bus (slack bus). In the next section we do this through a singular perturbation technique that mitigates the controllability problem.

D. Singular Perturbations for Controllability

As observed, the frequency regulation problem is not fully controllable. This is known as the structural singularity in the literature [16], and one way in which this is remedied is by introducing a so-called slack bus that absorbs any excess power or meets any excess demands. In practice, what this corresponds to is a generator with very large inertia. Mathematically, this means that one bus is removed from the system dynamics, i.e., if bus i is the slack bus, then column i and row i in the Jacobian (J) is removed, resulting in a full rank Jacobian and, subsequently, a completely controllable system as well as a full rank B matrix.

We do not necessarily want to go this route since it is essentially a centralized way of solving the frequency regulation problem. Instead, we introduce a singular perturbation approach for achieving the same results. We start with the assumption that each prosumer with large inertia has the ability to balance power and self-select as a slack bus. Moreover, we assume that there is at least one such prosumer present in the network.

As such, let $\rho \subset \{1,\ldots,n\}$ be the set of prosumers that act as slack buses, and $P_{S,i}$ be the excess power absorbed/generated by the prosumer $i \in \rho$. The sensitivity of $P_{S,i}, i \in \rho$, with respect to changes in the voltage angle of prosumers is very small and it is related to the inverse of the inertia

$$\frac{\partial P_{S,i}}{\partial \Delta\delta_j} = \begin{cases} \mathcal{O}\left(\frac{1}{H_i}\right) = \xi_i & i = j \in \rho \\ 0 & i \neq j \end{cases} \tag{21}$$

where H_i is the inertia of prosumer i, and ξ_i is a small positive number. What (21) says is that the sensitivity of slack power at prosumer i with respect to changes in the voltage angle of prosumer i is on the order of $\mathcal{O}((1)/(H_i))$, and zero with respect to changes in the voltage angle of other prosumers.

Combining (3) and (21) leads to a new formulation for the Jacobian

$$J(\xi) = J + J_\xi \tag{22}$$

where J is the old Jacobian, and J_ξ is a diagonal matrix whose ith entry is zero if prosumer i is not a slack bus, and non-zero (ξ_i) if prosumer i is a slack bus. And, as long as there is at least one slack bus, the Jacobian is non-singular. However, the new Jacobian matrix has the same sparsity structure as the initial Jacobian matrix. Note moreover that this formulation is entirely consistent with the single slack-bus scenario, in which case J_ξ has a single non-zero entry associated with the slack bus.

Adding the new Jacobian matrix to (7) and (8) leads to a new system matrix and control matrix for the grid $A(\xi)$ and $B(\xi)$. Both of these matrices have full rank and, as such, the condition in Corollary 3.1 is satisfied. This has two direct implications, namely 1) there are diagonal weight matrices such that the one-step MPC has a stabilizing solution, and 2) each prosumer can pick their state weight "large" and their control weight "small" and the resulting solution is stabilizing. But, just because there exists a stabilizing solution it does not follow that the prosumers can obtain it in a distributed manner, which is the topic of the next subsection.

E. Distributed One-Step MPC

As the one step MPC problem has a stabilizing, minimizing solution, we can now move on to actually solving this problem in a distributed manner. As the cost is convex, we have access to a rich literature on distributed optimization for convex costs (see for example [17] for a representative sample). Note, however, that a shorter horizon in MPC problems runs the risk of introducing oscillations in the trajectories, so just because we have a stabilizing solution it does not follow that we have a good solution, which is something we have to keep in mind when applying the controllers to the frequency regulation problem.

The problem that we have to solve in a distributed manner is the following:

$$\min_{u_1,\ldots,u_n} \sum_{i\in N} [p_i x_i(t_c+1)^2 + r_i u_i(t_c)^2] \tag{23}$$

where $N = \{1,\ldots,n\}$ is the set of prosumers, subject to coupling constraints

$$x_i(t_c+1) = \sum_{j\in\mathcal{N}_i\cup\{i\}} [a_{ij}x_j(t_c) + b_{ij}u_j(t_c)],\ \forall i \in N \tag{24}$$

here $\mathcal{N}_i$ is the set of prosumer i's neighbors, and the system matrices are $A(\xi) = [a_{ij}], B(\xi) = [b_{ij}]$.[5] For the sake of simplicity, we drop (t_c) in our notations onward and indicate $u(t_c)$ and $x(t_c)$ by u and x, respectively.

To solve this optimization problem in a distributed way, we use Alternating Direction Method of Multipliers (ADMM) [17]. ADMM is known to converge for convex optimization problems with non-empty closed feasible sets and finite optimal solutions [17], which is the case for our problem. Because we desire a distributed formulation, we assume that each prosumer does not have direct access to the decision variables of other prosumers. Therefore, as described in [18], the coupling constraint at prosumer i can be reformulated by adding a new term, which represents the perception of prosumer i from the control action of prosumer j (neighboring prosumer). We call this perception U_{ij} and set the matrix of decision variables as $U = [U_{ij}]$, $\forall i \in N$ and $\forall j \in \mathcal{N}_i \cup \{i\}$. Therefore, the one-step MPC is recast as follows:

$$\begin{aligned} \min_{U_{1\cdot},\cdots U_{n\cdot}} \quad & \sum_{i=1}^{n} \left(p_i \left[A_{i\cdot}^T x^i + B_{i\cdot}^T U_{i\cdot}\right]^2 + r_i U_{ii}^2\right) \\ \text{s.t.} \quad & U_{ij} = U_{jj},\ \forall i \in N, j \in \mathcal{N}_i \end{aligned} \tag{25}$$

where $A_{i\cdot}$, $B_{i\cdot}$, and $U_{i\cdot}$ are the ith rows of A, B and U matrices, respectively. In addition, x^i is a column vector, which includes $x_j, j \in \mathcal{N}_i \cup \{i\}$. Constraints in (25), guarantee that prosumer i has a correct perception from the control strategy of its neighbors.

By relaxing the constraints, augmenting them in the objective function, and using ADMM [18], we can iteratively solve (25). Now, let the augmented Lagrangian function of prosumer i at iteration $h+1$ be

$$\begin{aligned} \mathcal{L}_{\rho,i}\left(U_{i\cdot}, \bar{U}_{i\cdot}^h, \lambda_{i\cdot}^h\right) = {} & p_i \left[A_{i\cdot}^T x^i + B_{i\cdot}^T U_{i\cdot}\right]^2 + r_i U_{ii}^2 \\ & + \lambda_{i\cdot}^{h^T} \left(U_{i\cdot} - \bar{U}_{i\cdot}^h\right) + \frac{\rho}{2}\left\|U_{i\cdot} - \bar{U}_{i\cdot}^h\right\|_2^2 \end{aligned} \tag{26}$$

where $\rho > 0$ is a given penalty factor, and $\bar{U}_{i\cdot}^h$ is a column vector, which includes the average control strategy of prosumer i and that of its neighbors

$$\bar{U}_{ij}^h := \frac{\sum_{l\in\mathcal{N}_j\cup\{j\}} U_{lj}^k}{|\mathcal{N}_j| + 1},\ \forall j \in \mathcal{N}_i \cup \{i\}. \tag{27}$$

Then, the primal and dual residuals $\alpha_{i\cdot}^h$ and $\beta_{i\cdot}^h$ at iteration h shall be updated as

$$\alpha_{i\cdot}^h = U_{i\cdot}^h - \bar{U}_{i\cdot}^h, \text{ and } \beta_{i\cdot}^h = \rho\left(\bar{U}_{i\cdot}^h - \bar{U}_{i\cdot}^{h-1}\right), \forall i \in N. \tag{28}$$

[5]Note that the diagonal entries in the weight matrix R can be chosen sufficiently small in order to ensure that $\|x\|^2$ is a Lyapunov function (since this is a Lyapunov function in the limit as R goes to zero). As such, as long as the states decrease along each dimension (each prosumer state), stability is ensured, which constitutes a constructive, as well as distributed, way of selecting the weight matrix R.

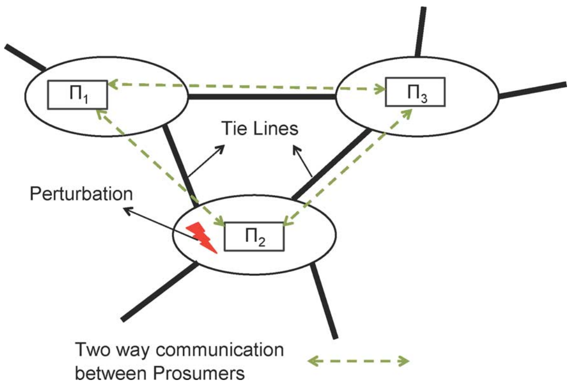

Fig. 1. Schematic of distributed frequency regulation architecture, which demonstrates the communication layer on top of the physical layer.

The dual variables are also updated as

$$\lambda_{i\cdot}^{h} = \lambda_{i\cdot}^{h-1} + \rho\alpha_{i\cdot}^{h}, \ \forall i \in N. \tag{29}$$

Fig. 1 shows the schematic of the proposed, distributed one-step MPC frequency regulation architecture for prosumer-based electric energy systems. As seen in this Figure, after a perturbation in the prosumer 2, the prosumer communicates with its neighbors. Then, all prosumers coordinate to stabilize deviations in the grid frequency and those in tie-line flows iteratively.

We propose the distrusted one-step MPC algorithm as follows:

Algorithm: Distributed One-Step MPC

Step 0. *Initialize control variables* $U_{i\cdot}^{0}$ *and dual variables* $\lambda_{i\cdot}^{-1}$, *for each prosumer* $i \in N$; *set* $h = 0$.

Step 2. *Each prosumer* i *sends* U_{ij}^{h} *to prosumer* j, $j \in \mathcal{N}_i$.

Step 3. *Each prosumer* j *computes* $\bar{U}_{ij}^{h}$ *using* (27), and sends it back to $i \in \mathcal{N}_j$.

Step 4. *Each prosumer* i *updates its primal and dual residuals from* (28), and dual variable from (29).

Step 5. *For given primal and dual tolerances* $\epsilon^{\mathrm{Pri}} > 0$ *and* $\epsilon^{Dual} > 0$, *if* $h > 0$, $\sum_{i\in N} \|\alpha_{i\cdot}^{h}\|_2^2 \le \epsilon^{Pri}$ and $\sum_{i\in N} \|\beta_{i\cdot}^{h}\|_2^2 \le \epsilon^{\mathrm{Dual}}$ STOP and output $\bar{U}_{ii}^{h}$ as optimal control strategy for each prosumer i; otherwise go to Step 6.

Step 6. *Each prosumer* i *updates* $U_{i\cdot}^{h+1}$ *by solving a self-contained problem of the following form*:

$$U_{i}^{h+1} = \underset{U_{i\cdot}}{\operatorname{argmin}} \quad \mathcal{L}_{\rho,i}\left(U_{i\cdot}, \bar{U}_{i\cdot}^{h}, \lambda_{i\cdot}^{h}\right). \tag{30}$$

Set $h \leftarrow h+1$ *and go to step 1.*

Note that local constraints such as maximum and minimum power output, and ramp rates can be included in (30). Without such constraints, quadratic optimization problem (30) has a closed form solution as follows:

$$U_{i}^{h+1} = \left(2p_i B_{i\cdot} B_{i\cdot}^{T} + E^{i}\right)^{-1} \left(\rho\bar{U}_{i\cdot}^{h} - \lambda_{i\cdot}^{h} - 2p_i B_{i\cdot} A_{i\cdot}^{T} x^{i}\right) \tag{31}$$

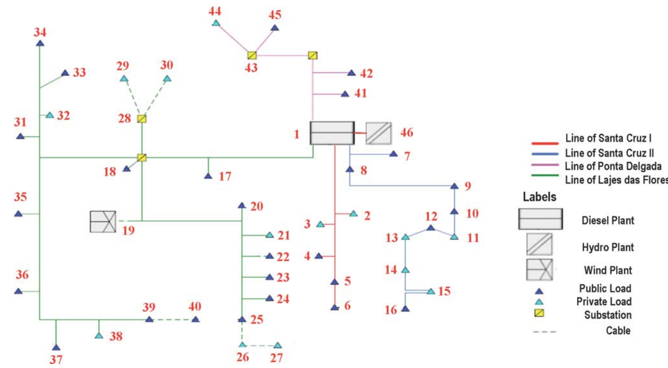

Fig. 2. Electrical network of Flores Island [19].

where E^i is a diagonal matrix of size $|\mathcal{N}_i| + 1$, $E_{ii}^{i} = 2r_i + \rho$ and $E_{jj}^{i} = \rho$, $\forall j \in \mathcal{N}_i$. It is worth mentioning that in (31), only $\bar{U}_{i\cdot}^{h}$ and $\lambda_{i\cdot}^{h}$ are changing in each iteration. This significantly decreases the amount of computation in each iteration.

Note that in designing the one-step MPC control the coupling between prosumers is already considered. Therefore, the minimizing control is naturally cancelling out interactions between prosumers and consequently inter-area oscillations are eliminated. This has been demonstrated on the electric power systems on the Azores Archipelago.

IV. Distributed Frequency Control Simulation

In this section, the distributed one-step MPC is simulated on two real-world electric power systems. The first system is the electric power grid on Flores Island, and the second one is the electric power system on Sao Miguel Island.

A. Frequency Regulation on Flores Island

Flores is one of the western group islands of the Azores Archipelago, islands of Portugal, which is located in the middle of North Atlantic Ocean. The island has small population (approximately 4000 inhabitants) and its average electricity demand is about 2 MW [19].

The electrical grid of Flores consists of three main load centers and three mid-size power plants. The load centers are located in Santa Cruz, the capital of Flores, in Lajes das Flores, the second largest community and the harbor of Flores, and in Ponta Delgada, the third-largest community located along the northern coast. A diesel power plant whose total capacity is 2.5 MW, and a hydro power plant with an overall capacity of 1.65 MW are located in Santa Cruz. They are producing about 85% of the electricity demand. The rest of the demand is provided by a 0.65 MW wind power plant located in the center of the island. Fig. 2 illustrates the schematic of the electric network on Flores.

Based on the geographical division of the electric power grid, we split the grid into three prosumers. Each prosumer includes a power plant. Also, the diesel prosumer is the slack. Fig. 3 illustrates the schematic of the prosumer-based electric power system on Flores. The data for the electric power system and that for the quasi-steady state characteristics of the prosumers are available in [19] and [20].

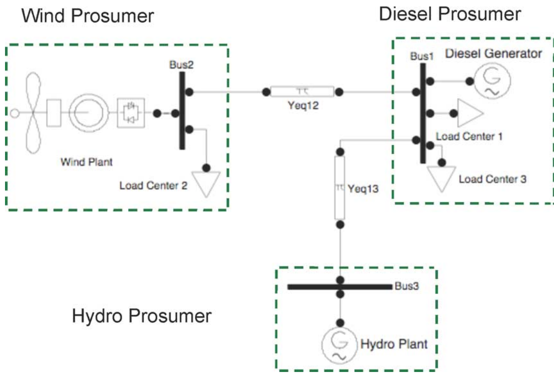

Fig. 3. Prosumer-based electric power system on Flores.

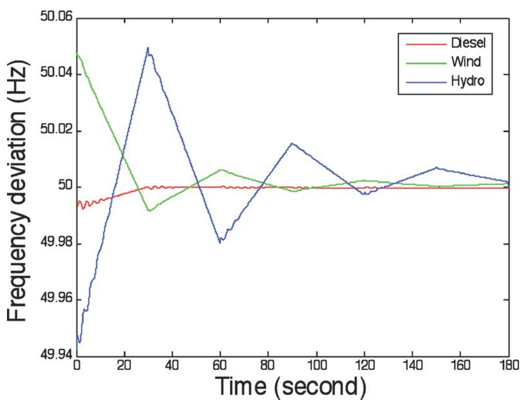

Fig. 4. Frequency deviations of prosumers after a perturbation.

We explore the frequency stability of the island in two scenarios. In the first scenario, we assume that frequency regulation is done in a conventional way. That is, prosumers do not communicate with each other. The control center measures frequency deviations locally (at the diesel prosumer) and every 30 s set points of the prosumers are adjusted in response to deviations in local frequency. This approach can create long-term stability problems, particularly if more than 50% of the electricity is produced by renewable energy sources.[6] For instance, Fig. 4 shows a scenario under which prosumers start with imbalance in quasi-steady state frequency at time zero. The control actions are calculated using the conventional approach and set-points of prosumers are adjusted accordingly. Since coupling between prosumers are neglected, after 30 s frequency imbalance is not eliminated and therefore prosumers need to re-adjust their set-points. This process continues until frequency converges to 50 Hz.

In the second scenario, we propose that the frequency stability of Flores would be guaranteed by implementing the distributed one-step MPC. Under the proposed framework, prosumers communicates with their neighbors to calculate the optimal control strategy every 30 s. As shown in Fig. 5, at time zero prosumers start with frequency imbalance. Then, they calculate their control action using consensus-based ADMM and implement it almost instantaneously. Fig. 5 shows that frequency deviations converge to 50 Hz after 30 s.

[6]http://apps1.eere.energy.gov/tribalenergy/pdfs/wind_aktech01_murdock.pdf

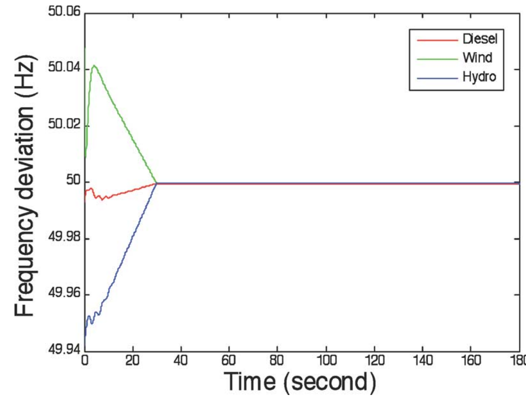

Fig. 5. Frequency deviations of prosumers after implementing the one-step MPC control.

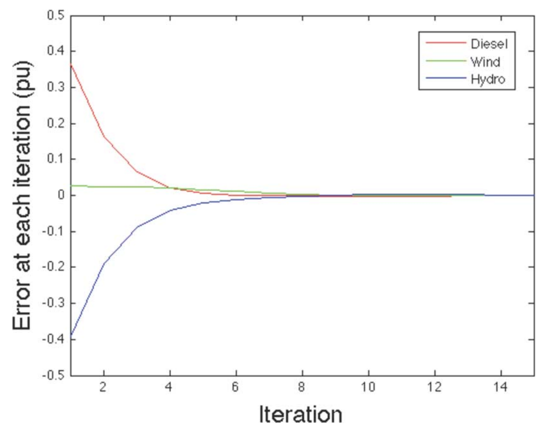

Fig. 6. Convergence rate of the optimization process for each prosumer.

Fig. 6 also illustrates the convergence rate of the optimization process. At each iteration, prosumers communicate twice with their neighbors. First prosumers send their control strategy and their perception of their neighbors' control strategy. Then, prosumers calculate their average control strategy and broadcast the result.

B. Frequency Regulation on Sao Miguel Island

Sao Miguel is the capital and the largest island of the Azores Archipelago. It has a population of over 140 000 inhabitants and its average electricity demand is about 65 MW. The island has a loop electric power system with over 1950 lines and 1900 nodes. Three large-scale diesel power plants are producing over 60% of the electricity demand. Also, two large-scale geothermal power plants are producing about 35% of the demand. The rest of the demand is provided by ten small-scale hydro power plants. The data for the electric power system on Sao Miguel is available in [19] and [20]. We divide the electric power grid of the island into fifteen prosumers. Each prosumer includes a generator and many loads located electrically close to the generator. Fig. 7 illustrates a schematic of the prosumer-based electric power grid on Sao Miguel.

We assume that each prosumer is capable of regulating frequency by adjusting the set point of its generator and/or by adjusting its deferrable loads. In addition, each prosumer can communicate with its neighboring prosumers. Similar to the pre-

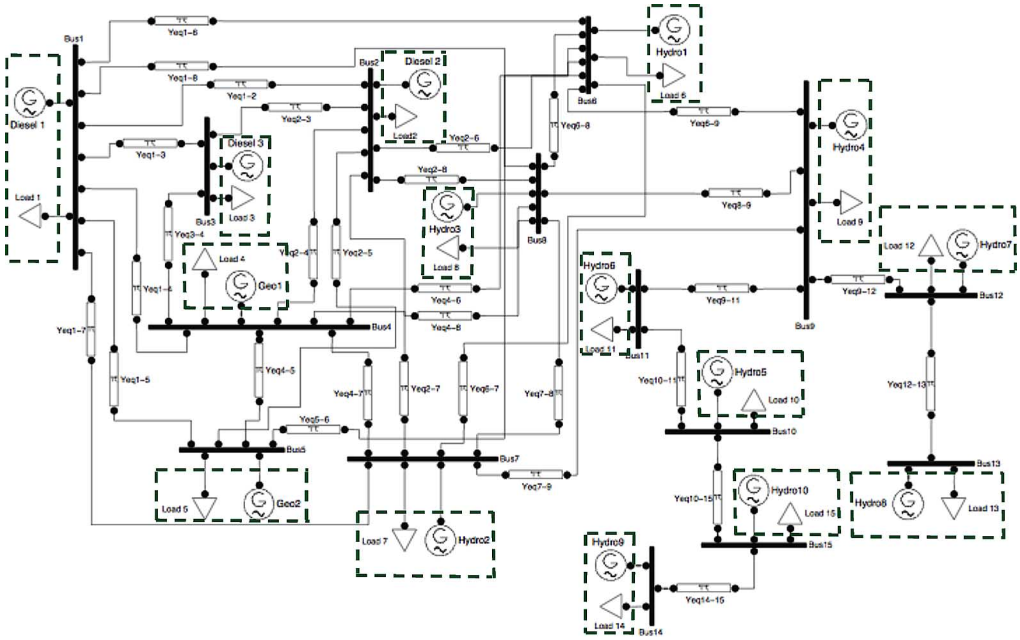

Fig. 7. Prosumer-based electric power system on Sao Miguel.

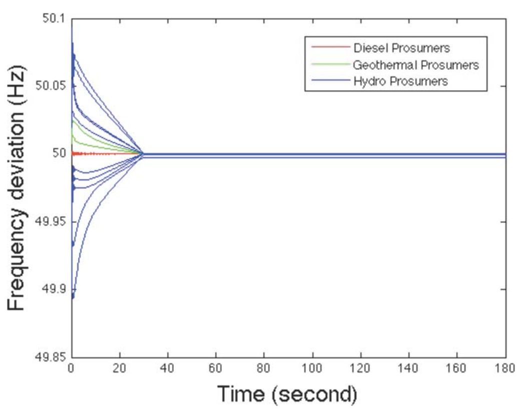

Fig. 8. Frequency deviations of prosumers on Sao Miguel after implementing the one-step MPC control.

vious case, we assume that prosumers start with frequency imbalance at time zero. Then, prosumers calculate their optimal control actions using consensus-based ADMM and implement them almost instantaneously. Fig. 8 shows how frequency deviations converge to 50 Hz after 30 s.

V. Conclusion

In this paper, we proposed a new architecture for frequency control in prosumer-based electric energy systems. The proposed architecture guarantees long-term frequency stability of electric energy systems; solves inter-area oscillations problems; minimizes system-wide control effort, and; needs less sensing and communication compared with today's centralized communication/control systems.

We first introduced a new quasi-steady state dynamic model for heterogeneous prosumer-based power grids and formulated the frequency regulation problem as a MPC problem. Through the introduction of slack buses and one-step MPC algorithms, the MPC problem is indeed solvable in a distributed manner, and the distributed one-step MPC was simulated on two real-world electric power systems on the Azores Archipelago. We showed that the proposed frequency control architecture could guarantee long-term system-wide frequency stability of the islands and minimize global control efforts.

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