JOURNAL OF HEAT TRANSFER
P OLICY

ASME International

Policy on Reporting Uncertainties in Experimental Measurements and Results

The purpose of this statement is to offer guidelines on estimating experimental uncertainty to: (1) ensure uniformity of presenting experimental data; and (2) to raise the author's awareness regarding the importance of giving a more precise statement about their measurement uncertainties.

GUIDELINES

An uncertainty analysis of experimental measurements is necessary for the results to be used to their fullest value. Authors submitting papers for publication to the JOURNAL OF HEAT TRANSFER are required to describe the uncertainties in their experimental measurements and in the results calculated from those measurements. The Journal suggests that all uncertainty evaluation be performed in accordance with a 95 percent confidence interval. If estimates are made at a confidence level other than 95 percent, adequate explanation of the techniques and rationalization for the choice of confidence interval should be provided.

For each result presented, the presentation of the experimental data should include the following information:

1. **The precision limit, P.** The $\pm P$ interval about a nominal result (single or averaged) is the experimenter's 95 percent confidence estimate of the band within which the mean of many such results would fall, if the experiment were repeated many times under the same conditions using the same equipment. Thus, the precision limit is an estimate of the lack of repeatability caused by random errors and unsteadiness.

2. **The bias limit, B.** The bias limit is an estimate of the magnitude of the fixed, constant error. It is assigned with the understanding that the experimenter is 95 percent confident that the true value of the bias error, if known, would be less than $|B|$.

3. **The uncertainty, U.** The $\pm U$ interval about the nominal result is the band within which the experimenter is 95 percent confident that the true value of the result lies. The 95 percent confidence uncertainty is calculated from

$$U = [B^2 + P^2]^{1/2} \tag{1}$$

4. A brief description of, or reference to, the methods used for the uncertainty analysis.

The estimates of precision limits and bias limits should be made over a representative time interval for the experiment. The following additional information should be presented preferably in tabular form.

(a) The precision and bias limits for each variable and parameter used.
(b) The equations by which each result was calculated.
(c) A statement comparing the observed scatter in results on repeated trials (if performed) with the expected scatter ($\pm P$) based on the uncertainty analysis.

A discussion of sources of experimental error in the body of the text without the above does not satisfy our requirement. All reported data must show uncertainty estimates. All figures reporting new data should show uncertainty estimates of those data either on the figure itself or in the caption. A list of references on the topic is provided below.

EXAMPLE

Consider an experiment in which an air-cooled device is being tested and it is desired to determine the rate of heat transfer, q, to the cooling air. This might be accomplished by; measuring the mass flow rate, m, and the inlet and outlet air temperatures, T_i and T_o, and computing:

$$q = mc\,(T_o - T_i) \tag{2}$$

where c is the constant-pressure specific heat of air.

The 95 percent confidence uncertainty, U_q, in the experimental result, q, is given by the following combination of a precision (random) contribution to the uncertainty of q, P_q, and a bias contribution to the uncertainty of q, B_q:

$$U_q = \sqrt{P_q^2 + B_q^2} \tag{3}$$

These two contributions can be evaluated separately in terms of the sensitivity coefficients of the result, q, to the measured quantities (e.g. $\partial q/\partial m$) following the propagation equation of Kline and McClintock (1955),

$$P_q^2 = \left(\frac{\partial q}{\partial m}\right)^2 P_m^2 + \left(\frac{\partial q}{\partial c}\right)^2 P_c^2 + \left(\frac{\partial q}{\partial T_o}\right)^2 P_{T_o}^2 + \left(\frac{\partial q}{\partial T_i}\right)^2 P_{T_1}^2 \tag{4}$$

and

$$B_q^2 = \left(\frac{\partial q}{\partial m}\right)^2 B_m^2 + \left(\frac{\partial q}{\partial c}\right)^2 B_c^2 + \left(\frac{\partial q}{\partial T_o}\right)^2 B_{T_o}^2 + \left(\frac{\partial q}{\partial T_i}\right)^2 B_{T_i}^2 + 2\left(\frac{\partial q}{\partial T_o}\right)\left(\frac{\partial q}{\partial T_i}\right) B'_{T_o} B'_{T_i} \tag{5}$$

where B'_{T_o} and B'_{T_i} are the portions of B_{T_o} and B_{T_i} that arise from identical error sources (such as the calibration error for thermocouples that were calibrated using the same standards, equipment, and procedures) and are therefore presumed to be perfectly correlated.

Using Eq. (2) to evaluate the derivatives, defining $\Delta T = T_o - T_i$ and rearranging, one obtains

$$\left(\frac{P_q}{q}\right)^2 = \left(\frac{P_m}{m}\right)^2 + \left(\frac{P_c}{c}\right)^2 + \left(\frac{P_{T_o}}{\Delta T}\right)^2 + \left(\frac{P_{T_i}}{\Delta T}\right)^2 \tag{6}$$

and

$$\left(\frac{B_q}{q}\right)^2 = \left(\frac{B_m}{m}\right)^2 + \left(\frac{B_c}{c}\right)^2 + \left(\frac{B_{T_o}}{\Delta T}\right)^2 + \left(\frac{B_{T_i}}{\Delta T}\right)^2 - 2\left(\frac{B'_{T_o}}{\Delta T}\right)\left(\frac{B'_{T_i}}{\Delta T}\right) \tag{7}$$

These derivatives could be evaluated numerically, using a data reduction program or analytically.

The precision limits, P_m, P_{T_o} and P_{T_i}, can each be calculated as 2 times the standard deviation of unsteadiness of a set of observations of m, T_o and T_i, respectively, measured with the apparatus in normal running condition. These terms must include the process unsteadiness: instrument unsteadiness (or imprecision) is not sufficient. A sufficiently large number of samples (>30) should be taken over a sufficiently long sampling period, relative to the longest period of the unsteadiness, in order for unsteadiness values to be representative of the process. The precision limit of the specific heat, P_c, would arise due to the variation in the average temperature used to enter the property table or curve-fit. Evaluation requires determining the relationship between c and T. In most practical cases, this term would be negligible, relative to the other precision limit terms.

The bias limits, B_m, B_{T_o}, and B_{T_i}, are each determined either by calibration tests conducted before and after the experiment or by combining, by the root-sum-square method, estimates of elemental bias errors that influence the measurement of the respective variables. The elemental bias errors include the estimated bias errors of the calibration standards and in the calibration procedure, and less-than -perfect curve fitting of the calibration data. One component of the bias limit, B_c, is the "fossilized" error, which represents the bias error inherent in the specific heat taken from a table of properties. It is due to errors that may have arisen in the measurements of those properties and in the tabulation of the results. Such contributions are usually at least 0.25-0.5 percent, and are often much larger than that (Coleman and Steele, 1989)

In estimating the precision limits and bias limits for a variable, the true definition of the variable much be acknowledged. For example in Eq. (2), the temperatures T_o and T_i , represent the bulk mean air temperatures at the outlet and inlet cross sections, respectively. If point measurements of temperature are used to represent T_o and T_i , in Eq.(2), then a bias error (which Moffat (1988) calls a "conceptual bias") occurs that is equal to the difference between the measured temperatures (T_o and T_i ,) and the bulk mean temperatures at the outlet and inlet cross sections, respectively. A correction must be made for that difference and the bias error evaluation would include an estimate of the residual uncertainty in that correction in addition to the bias errors from the probe calibration, etc.

Consider the situation in which the bias limits in the temperature measurements are uncorrelated and are estimated as 0.5^o C and the bias limit on the specific heat property value is 0.5 percent. The estimated bias error of the mass flow meter system is specified as "0.25 percent of reading from 10 to 90 percent of full scale." Discussion with the manufacturer reveals that this is a fixed error estimate (it cannot be reduced by taking the average of multiple readings and is, thus, truly a bias error), and B_m is taken as 0.0025 times the value of m. For $\Delta T = 20\,^o$C, Eq. (7) gives:

$$\frac{B_q}{q} = \sqrt{(0.0025)^2 + (0.005)^2 + \left(\frac{0.5^{\mathbf{o}}C}{20^{\mathbf{o}}C}\right)^2 + \left(\frac{0.5^{\mathbf{o}}C}{20^{\mathbf{o}}C}\right)^2} \qquad = 0.036 \text{ (= 3.6 percent)} \tag{8}$$

Obviously, the bias limits on the temperature measurements are dominant for the specific case considered. Notice that if the bias errors in the two temperature measurements were totally correlated, the last term on the right side of Eq. (7) would cancel the third and fourth terms and then Bq/q would be 0.0056 instead if 0.036.

If the random errors and process unsteadiness were such that the precision limit for q, P_q , calculated from Eq. (5), was 2.7 percent, the overall uncertainty in the determination of q, U_q , would be:

$$\frac{U_q}{q} = \sqrt{\left(\frac{B_q}{q}\right)^2 + \left(\frac{P_q}{q}\right)^2} = \sqrt{(0.036)^2 + (0.027)^2} \qquad = 0.045 \text{ (= 4.5 percent)} \tag{9}$$

References and Recommended Readings:

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