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Analysis of Singular Stress Fields in Duplex Fusion Components

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By

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Dedication

This dissertation is dedicated to my mother, Mrs. Hollie Fitz Blanchard, and to my father, the late William Page Blanchard.

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PUBLICATIONS

J. Blanchard, N. Ghoniem, and S. Chou
An Approximate Solution to the Scattering Integral for General Interatomic Potentials
J. Appl. Phys., 61 (1987):3120

R. Mattas, M. Baskes, J. Brooks, J. Blanchard, A. Hassanein, and A. Tobin
Divertor Studies for the International Tokamak Reactor
Fusion Tech., 10 (1986):298

J. Blanchard and R. Watson
Residual Stresses in Bonded Armor Tiles for In-Vessel Fusion Components
Nuc. Eng. Des./Fusion, 4 (1986):61

J. Blanchard and N. Ghoniem
The Influence of Uncertainties in Material Properties and the Effects of Dimensional Scaling on the Prediction of Fusion Structure Lifetimes
Nuc. Eng. Des./Fusion, 4 (1986):67

J. Blanchard and N. Ghoniem
Inelastic Structural Analysis of the MARS Tandem Mirror Reactor
Nuc. Eng. Des./Fusion, 2 (1985):19

J. Blanchard and N. Ghoniem
The Bowing of Solid Breeder Rods in a Pin-Type Fusion Reactor
Fusion Tech., 10 (1986):1623

J. Garner, J. Blanchard, G. Orient, and K. Taghavi
A Module for Testing a Lithium-Cooled Tokamak Blanket in a Tandem Mirror Test Reactor
Fusion Tech., 8 (1985):962

P. Gierszewski, et al.
Engineering Scaling and Quantification of the Test Requirements for Fusion Nuclear Nuclear Technology
Fusion Tech., 8 (1985):1100

J. Blanchard and N. Ghoniem
The Influence of Irradiation and Thermal Creep on Stress Redistribution in Fusion Blankets
J. Nuc. Mat., 122 (1984):101

N. Ghoniem, K. Taghavi, J. Blanchard, and S. Grotz
Limits on Transient Power Variation During Startup and Shutdown of Li-Pb Cooled TMR Blankets
Nuc. Tech./Fusion, 4 (1983):769

F. Najmabadi et al.
The TITAN Reversed-Field Pinch Fusion Reactor Study–Scoping Phase Report
UCLA-PPG-1100 (Jan. 1987)

R. Conn et al.
TOKOPS: The Influence of Reactor Operation on the Design and Performance of Tokamaks with Solid Breeder Blankets
UCLA-PPG-1009 (1986

R. Mattas et al.
U.S. Contribution: INTOR Critical Issue A, Impurity Control and First Wall Engineering
ETR-INTOR/ICE/8 (1984)

M. Abdou et al.
Finesse: A Study of the Issues, Experiments and Facilities for Fusion Nuclear Technology Research and Development
UCLA-PPG-821 (1984)

R. Conn et al.
START/TM: A Study of Start-Up and Fractional Power Operation of Tandem Mirror Fusion Reactors
UCLA-PPG-764 (1984)

ABSTRACT OF THE DISSERTATION

Analysis of Singular Stress Fields in Duplex Fusion Components

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Elastic stress singularities at the edge of the interface in duplex fusion components are studied using a series solution derived from the Airy stress function. For a crack-free interface, the order of the singularity is shown to depend on the material properties, the loadings, and the stress state (plane stress or plane strain). In all cases the singularity is weaker than the square root singularity seen in crack problems. The series solution, which features undetermined coefficients, satisfies the interface conditions and the traction conditions on free surfaces adjacent to the edge of the interface, but it does not satisfy the remote boundary conditions. A collocation technique is used to satisfy the traction conditions on these boundaries in the least-square sense. This solution technique is verified by comparing it to a semi-analytical solution for bonded quarter planes, which is obtained using the Mellin transform. This benchmark indicates that the collocation method provides

accurate information about the edge singularity, despite the existence of large errors in the bulk stresses far away from the edge of the interface.

The singularities in fusion components are studied by considering the thermal fields imparted by fabrication, start-up, and full-power operation. The influence of void swelling and irradiation creep on the singularities in bonded fusion components is considered by modeling the void swelling as an isotropic volume change, and by modeling irradiation creep as a linear viscoelastic process with a unit stress exponent. Two material combinations are chosen for the study of the effects of a typical fusion environment: graphite on copper and tungsten on vanadium. Because the graphite and copper are assumed to be creep-free, the densification of both materials causes boundary-layer stress intensities of similar magnitude to those caused by the fabrication process. The second duplex, though, is analysed using a relatively high irradiation creep rate and the effects of swelling are found to be quite small, leading to the conclusion that irradiation creep can be an advantage in the presence of significant void swelling.

Chapter 1

Introduction

Many technologies feature layered or bonded components, which must be designed with regard to interlaminar failure. Aerospace applications are constructed with increasing amounts of composite laminate materials, taking advantage of their high strength-to-weight ratios. Electronics applications feature bonded semiconductors, using various material combinations to produce different electrical properties. In magnetic fusion devices, bonded tiles are often used as a method for designing high heat flux components (such as limiters and divertors), maintaining reasonable temperatures and stresses by combining the high temperature and low sputtering capabilities of the tile materials with the strength and thermal properties of the substrate materials. Common to these applications of multi-layered structures is concern about the integrity of the bond. Generally this concern is rooted in the propensity of such components to fail at the interface, but loss of overall structural stiffness and thermal conductivity across the bond due to local delamination are also important.

The analysis of bonded structures has been accomplished in many guises, from

beam-type or variational approximations to semi-analytical two-dimensional elasticity solutions with varying degrees of success and/or usefulness. In this dissertation, the effects of tile size and material choice are explored using a series solution for the singular field at the edge of the interface to evaluate the stresses and displacements in an idealized model which represents a fusion component. This series solution only satisfies the traction-free boundary conditions on two surfaces, but it features undetermined coefficients which are used to satisfy the remaining boundary conditions in an approximate sense using boundary collocation techniques. An analog to the stress intensity factor is then defined to allow comparisons of the various loadings that a fusion component must endure.

This model for the analysis of interface stresses is applied to high heat flux components in fusion reactors. Most designs for these plasma-facing devices feature different materials for the surface and bulk, because of the conflicting material requirements dictated by surface erosion and bulk heat transfer, so the above model is applicable. The stress producing mechanisms to be studied include:

1. **fabrication**, in which residual stresses are induced by thermal expansion during cool- down from the bonding temperature

2. **thermal expansion and gradients**, which are produced during startup, steady operation, and shutdown

3. **swelling**, which is a permanent volume increase resulting from neutron-induced displacement damage

4. **creep relaxation** by irradiation induced creep mechanisms

Chapter 2

Previous Applications of Bonded Structures

2.1 Non-Fusion Applications

As the limits of technology are pushed, the demands imposed on materials become increasingly severe. In many cases, the structural requirements conflict with the required surface properties, so there is no single material which will function under the imposed conditions. An obvious design solution for these instances is to use layered components, which can provide better bulk and surface properties than any of the separate constituents. Some typical instances of this strategy are described below.

The use of composite structures appears to have begun over 2000 years ago with the use of laminated wood, but the concept has come of age only recently. The aerospace and automobile industries, among others, have increasingly taken advantage of the potential of hybrid structures for achieving strong, light-weight designs. The applications include:

- *Sandwich Constructions,* which feature two plates of a relatively strong material, with a weak core between them. This provides a high strength-to-weight

ratio and a durable, weather-tight finish[1]. Applications include helicopter rotor blades, aircraft wings, and fire walls in the aerospace industry and structural walls, roofs and furniture in the building industry.

- *Fiber Composites,* which provide high strength-to-weight ratios by setting fibers with high tensile strengths in a resin matrix, thus preventing catastrophic fracture. These materials typically consist of many layers of fibers of different orientations, thereby allowing the composite to have higher fracture toughness than any of its constituents. Applications include sporting goods, automobile parts (drive shafts, springs, wheels, bumpers, etc.), and aircraft structural components [2].

In addition to sufficient strength, these materials can provide many desirable surface properties, including heat-resistance, low moisture permeability, and high thermal emissivity. In all cases, the integrity of the interface is crucial to the design and reliability of the component.

The microelectronics industry also makes frequent use of bonded structures. In many cases, the silicon "chips" used in many electronic components are mounted to a ceramic or metal substrate to allow connections with other devices. The bond must conduct heat away from the chip, protect the silicon from shock, and meet reliability requirements [3], all of which are related to the bond integrity and the interfacial stresses. Other applications of bonded devices in the electronics industry include solar cells and ceramic or glass insulators.

Another interesting application of bonded structures has been implemented on the space shuttle, which is covered on much of its surface with low conductivity ceramic tiles to keep the underlying structure from overheating during re-entry [4].

Figure 1: Heat Resistant Tiles on the Space Shuttle[4].

(See figure 1.) In order to reduce the stresses associated with the differential thermal expansion of the tiles and substrate, a metal felt, called a "strain-isolation pad", is placed between them. This felt has very low shear and extensional moduli and is intended to protect the brittle ceramic tile from the deformations of the substrate. Unfortunately, the fibers in the pad caused stress concentrations where they were attached to the tile, so some unexpected failures occurred in the early flights. This problem was solved by local densification of the tile, and the overall design philosophy of the layered composition was retained.

2.2 High Heat Flux Components for Fusion Reactors

2.2.1 Desirable Surface Properties for Plasma-Facing Components

In a fusion reactor, the first wall, divertor, and/or limiter all face the plasma, so they generally experience the most severe loadings of any of the core components. The heat and particle fluxes that are expected in several near-term and commercial reactors are given in Table 1 (as indicated by McGrath [13], the heat fluxes in this table may be lower than the actual values, primarily because of off-normal events). In addition to the impurity control requirements imposed on these components, they must provide efficient heat removal and a sufficiently long lifetime. The lifetime is determined from two often conflicting mechanisms[1]: erosion caused by the particle flux from the plasma and mechanical failure, leading to the incentive for designing multi-layered fusion components.

Surface erosion in fusion plasma-facing components is caused by a flux of energetic neutral particles which scatter off the lattice atoms, thereby transferring their energy to the lattice. If the lattice atoms receive energy in excess of the surface binding energy, they will leave the component, thus eroding the already thin first wall or impurity control device. For a given particle flux, the erosion rate depends on the energy and mass of the incident particle, the angle of incidence, and the target material. The sputtering coefficient (sputtered atoms per incident particle) is shown

[1]The conflict arises from the fact that high erosion rates lead to a need for thick structures, while thermal stresses in plates subjected to surface heating increase with plate thickness. Hence, the design of such structures entails a compromise between the requirements for low stresses and a large allowable erosion.

Table 1: Typical Surface Heat Loads on High Heat Flux Components in a Variety of Fusion Reactors

	heat flux (MW/m^2)	**pulse length (s)**
Short Pulse		
TFTR	2-9	1.5 [5]
JET	5	20 [6]
JT-60	3-5	5 [7]
CIT	9	3.2 [8]
Long Pulse		
STARFIRE	4	steady state [9]
INTOR	4	200 [10]
NET	8	50-100 [11]
TITAN	6-8	steady state [12]

in figure 2 for deuterons incident on several different target materials [14]. This figure exhibits a threshold energy, below which sputtering does not occur. Since this threshold energy can be above 300 eV for high-Z materials, they provide good candidate materials for high heat flux components in machines that are able to achieve low plasma temperatures.

For some materials, particularly carbon, the sputtering rate can be enhanced by a chemical effect in which the incident particle and target material form a compound with a reduced binding energy. This is shown in figure 3, which shows the temperature dependence of this erosion enhancement mechanism. Because of this chemical sputtering, graphite must either be coated (usually with TiC or SiC), or its temperature must be controlled during operation.

As the eroded atoms leave the structure, they are ionized by the plasma and often return to the surface as energetic ions, leading to a situation in which the incident particle has the same mass as the target material. This is referred to as self-sputtering and can lead to large erosion rates if the sputtering coefficient is above

Figure 2: Sputtering rates for deuterons incident on several different metals. [14].

Figure 3: Temperature dependence of sputtering of graphite

Figure 4: Self-sputtering coefficient for deuterons incident on several different metals. [14].

one. Figure 4 shows the self-sputtering coefficient for several materials, indicating that many high-Z materials exhibit excessive sputtering rates at high energies.

Given these sputtering considerations, the likely surface material choices for plasma-facing components are:

- **Tungsten, Molybdenum, or Tantalum,** if the plasma edge temperature is low
- **Beryllium, Silicon-Carbide, or Titanium-Carbide,** for moderate edge temperatures (to avoid self-sputtering of high-Z materials)

- **graphite** if the tile temperature can be controlled to prevent chemical sputtering.

The erosion rate is not the only concern regarding material choice for plasma-facing components. As the first wall or impurity control device erodes, impurities are introduced into the plasma. These impurities interact with the plasma constituents and radiate power to the first wall, thus making ignition more difficult to achieve. The three primary mechanisms of radiation, Bremsstrahlung, line radiation, and recombination, are proportional to the impurity density and to Z^2, Z^4, and Z^6, respectively, where Z is the atomic number of the impurity. Hence, from the plasma impurity point of view, the plasma-facing components should be chosen such that the impurity density and atomic number are as low as possible. Therefore, both the erosion rate and the atomic number of the surface material are crucial to component design.

2.2.2 Bulk Property Requirements

While the surface material of fusion components must be chosen to reduce the impurities introduced into the plasma, their bulk properties must provide adequate heat removal and sufficient lifetime. For heat removal, the desirable substrate properties are high thermal conductivity and compatibility with efficient coolants. Another desirable feature is the ability to form strong bonds with the chosen surface material. The obvious metals satisfying these requirements are:

- **Copper alloys**, which have good thermal properties but are incompatible with liquid metal coolants

- **Stainless and ferritic steels**, which have adequate but unremarkable thermal properties and are susceptible to radiation damage
- **Vanadium**, which is incompatible with helium coolants due to the susceptibility to embrittlement by oxygen impurities in the helium coolant
- **refractory metals (Mo, Ta, W)**, which have good high-temperature strength and good thermal conductivities
- **Inconel**, a nickel-based alloy which also has good high-temperature strength

The reliability of duplex structures is dependent upon a number of factors that are related to design, fabrication and material properties. Reliable attachments can be achieved through the judicious choice of materials and bonding techniques. A typical structural material for use in a fusion reactor should demonstrate:

1. the ability to form strong bonds with the chosen surface material
2. good high temperature tensile strength
3. good high temperature fracture toughness
4. low thermal creep rates
5. fabricability
6. resistance to radiation damage
7. low activation

In addition to these properties, similarity of both the thermal expansion coefficients and elastic moduli of the coating and substrate is desirable to reduce thermal stresses

in a duplex structure. The impact of these properties on design will be discussed in later chapters of this dissertation.

2.2.3 Duplex Structures in Fusion Machines

Early Conceptual Designs

Given the material considerations described in the previous chapter, many different bulk and surface material combinations have been chosen for both experimental and conceptual reactors. The first attempt to use different surface and bulk materials in an impurity control device was the limiter design of the UWMAK-I [15] conceptual tokamak design in 1974. (A limiter is a device which defines the plasma edge and removes plasma exhaust impurities.) This design features a flowing lithium film on a stainless steel structure. The flowing lithium will not erode *per se*, because it is continually replenished, so the erosion rate is not a life-limiting factor. A similar concept is seen in the UWMAK- III [16] design, which features a very thin (0.1 mm) TZM sheet separated from a more substantial TZM backing plate by a falling lithium film, thus maintaining good thermal contact between the two. (TZM is a molybdenum- based alloy.) As in UWMAK-I, the design approach used here utilizes a replaceable surface material to extend component lifetimes.

STARFIRE [9] was by far the most prominent of the early conceptual tokamak designs. This design is less innovative than the UWMAK designs, but the need for duplex structures in impurity control devices was again acknowledged. The STARFIRE limiter, shown in figure 5, is a plate-like structure that extends toroidally around the reactor. The structural material is chosen from a group of four alloys

Figure 5: Schematic of the STARFIRE limiter[9].

(copper-, vanadium-, tantalum-, and, niobium-based), all of which are viable according to the study. The limiter and the first wall are coated with beryllium to eliminate sputtering of the underlying structural materials. The coating provides good thermal contact between the two layers, thus reducing the peak temperatures in the surface materials.

Existing Fusion Devices

Several existing experimental fusion devices use impurity control components or first walls that are composed of two materials. Doublet III [17], a tokamak physics testing device at GA technologies in San Diego, has used several limiter designs as the power and plasma heating has increased. A TiC coated graphite limiter was installed in

1980, but the graphite began to fail when the neutral beam power (used for plasma heating) was increased. The new design featured wider, thicker tiles that were contoured to reduce the peak heat flux. Also graphite tiles were mechanically bolted to parts of the first wall to provide neutral beam dumps. Later the graphite tiles on the limiter and first wall were coated (by chemical vapor deposition) with a mixture of SiC and pyrolitic carbon to suppress chemical sputtering of the graphite [18].

In contrast to other existing machines, ASDEX Upgrade [19], a tokamak in Garching, Germany, incorporates an actively-cooled duplex limiter. Presently, most devices use inertially cooled components, allowing the surface to heat up during the relatively short plasma burn and then radiate or conduct the heat away between burns. As fusion technology progresses, the burn times of tokamaks will increase, possibly leading to steady state machines, and active cooling will be required to maintain reasonable temperatures. The ASDEX design uses graphite as a surface material, brazed to molybdenum tubes. Molybdenum was chosen because its thermal expansion coefficient is very close to that of graphite. Brazing was shown to be a viable attachment scheme and testing with radiation heaters exhibited the ability to withstand a steady heat flux of at least 3.8 W/m^2. This is a promising result for long term designs.

TFTR [5], located at Princeton Plasma Physics Laboratory in New Jersey, is presently the largest tokamak in the United States. It was designed primarily to study the physics issues that are critical for the design of the next generation of fusion machines. For impurity control and definition of the plasma edge, the device uses a moveable (poloidal) bumper limiter, consisting of three Inconel blades covered with TiC- coated graphite tiles, as shown in figure 6. The tiles are mechanically attached to the Inconel plate, which is water cooled. Because of the short (1.5 s)

Figure 6: The TFTR bumper limiter, consisting of an Inconel backing plate covered with graphite tiles[5].

burn time for the device, mechanical attachment provides sufficient thermal contact with the substrate. During pulses, the graphite temperature rises rapidly, but the time between pulses allows conduction of the heat to the cooled backing plate and radiation to the vacuum vessel. Hence the graphite temperature does not exceed its temperature limit.

JT-60 [20] is another large tokamak, located at JAERI in Japan. Much like TFTR, it was designed to demonstrate reactor-relevant technologies, with somewhat more emphasis on aspects other than plasma physics. Duplex structures in JT-60 include:

- molybdenum and Inconel liner plates bolted to the Inconel vacuum vessel. (Mo

is used in the high heat flux regions of the first wall because it resists thermal shocks, and Inconel was used because it is less susceptible to electromagnetic forces caused by eddy currents.)

- TiC coatings on all plasma-facing surfaces to reduce the undesirable effects of impurities from the walls. (A process for *in-situ* TiC coating was developed to repair damage incurred during operation.)

The Joint European Torus (JET) [6], a tokamak located at Culham Laboratories in England, is slightly larger than both TFTR and JT-60. JET features an Inconel vacuum vessel, which was initially covered with additional Inconel heat shields. After 1984, damage of these heat shields was discovered, so inertially-cooled graphite tiles were attached to the first wall. The limiters have always been graphite. Beryllium was discarded as a surface material because it is toxic, but it has been retained as an option in case the graphite proves incapable of providing the desired performance.

Another existing fusion device which uses duplex structures is TEXTOR, a German experimental device which is equipped with a full toroidal belt limiter called ALT-II [21], as shown in figure 7. Originally, the design for ALT-II was a triplex structure, with a stainless steel backing plate, a copper cover plate, and a SiC-Al composite surface material, but the final design uses graphite tiles mechanically attached to an Inconel backing plate.

Near-Term Devices: Short Pulse

The next generation of fusion devices will be designed to demonstrate ignition in a D-T plasma and study burning plasma behavior, as well as further studying the

Figure 7: The ALT-II belt limiter, installed in TEXTOR[21].

Figure 8: First wall design for CIT[8].

engineering technologies associated with eventual commercial production of electricity. The Compact Ignition Tokamak (CIT) [22], which will be sited in the U.S., represents a compact version of such a device. The pulse length is to be 3-5 seconds, with a fusion power of 300-400 MW (corresponding to neutron wall loadings of 5-10 MW/m^2). It should be no surprise that the proposed vacuum vessel for CIT is made of Inconel, with mechanically attached graphite tiles on the first wall, as shown in figure 8. This attachment scheme is sufficient to maintain the graphite temperatures below the 2500 K limit for thermal loads of 8.5 MW/m^2, because the first wall backing plate is actively cooled and the pulse length is short. CIT will be capable of both limiter and divertor operation, and both components are similar to the first wall. (Like limiters, divertors are components designed to remove plasma exhaust, but they accomplish this by changing the magnetic field topology to cause the impurities in the edge to strike a collector plate.) The estimated time for a tile to return to its initial temperature is about one hour [8].

Near-Term Devices: Long Pulse and Steady State

The Fusion Experimental Reactor (FER) [23] is the Japanese version of a near-term tokamak. The burn mode for this device is essentially steady state, with burn times approaching 2,000 seconds, so the structural design is fundamentally different from previous studies. The proposed first wall is to be built from stainless steel, with no coating or tiles, so the erosion-limited lifetime is projected to be less than the reactor lifetime. For impurity control, a divertor was selected over a limiter because it is expected to allow denser, colder plasmas. The divertor is composed of tungsten armor brazed to a copper heat sink, and tests to verify the viability of such a design are in progress [24].

The International Tokamak Reactor (INTOR) [10] and the Next European Torus (NET) [11] represent the European designs for a near-term tokamak. (INTOR was precursor to NET.) The NET divertor is designed to withstand surface heat fluxes of 2-8 MW/m^2 for pulses of 50-1000 seconds. As seen previously, these conditions require actively-cooled, bonded structures if a duplex structure is to be used. NET is likely to use a tungsten-rhenium alloy as a surface material to reduce sputtering, brazing the tiles to a copper substrate for good heat transfer as shown in figure 9. Preliminary calculations indicate that this design is viable, but large-scale testing is required before any final judgements can be made [25].

Two other designs that have been proposed (but have been abandoned for various reasons) are Alcator-DCT [26] and DCT-8 [27]. Alcator DCT, a study at MIT, studies the issues associated with steady state operation. Both limiters and divertors are proposed for impurity control, and each is composed of graphite tiles attached to a metal substrate [28]. The tiles are attached either by direct brazing or with a soft intermediate layer, much like the shuttle tiles. DCT-8 is also proposed as a

Figure 9: Divertor design for NET[11].

steady state device, with beryllium tiles brazed to a copper substrate. Graphite has been rejected because of concerns about chemical sputtering.

Far-Term Reactors

Commercial fusion reactors will almost certainly be steady state, high power density machines, thus necessitating the use of actively cooled, bonded, plasma-facing components. This is demonstrated by the limiter design presented in the Blanket Comparison and Selection Study (BCSS) [29]. This limiter, shown in figure 10, is a triplex structure designed to withstand an average steady state heat flux of 3.5 MW/m^2. The surface materials used are beryllium on the top and tantalum on the leading edge, both of which are bonded to a vanadium base.

TITAN [12], a recent RFP design study, also resorted to layered designs for impurity control devices. The study developed two blanket concepts: a liquid

Figure 10: Limiter design for BCSS reactor design[29].

metal-cooled blanket and divertor with vanadium structure (TITAN-I) and a water-cooled design with a ferritic steel blanket (TITAN-II). TITAN-I coated the vanadium coolant tubes in the divertor with tungsten to keep the erosion rates low, while TITAN-II featured a one-material (tungsten) divertor. (The all-tungsten divertor was not used in TITAN-I because tungsten and liquid lithium are not compatible.) The TITAN study was the first fusion design study to address the design implications of stress concentrations that may occur at the free edge of bonded components, but the structural analysis did not include these concentrations (or singularities) self-consistently. Nevertheless, the study reinforced the need for further research into these phenomena.

It is evident from the above discussion that duplex structures are likely to be used for fusion impurity control devices in both near- and far-term fusion reactors. Unfortunately, the existing experimental and analytical studies are inconclusive in terms of the expected lifetime of such designs, so additional work is required in both areas.

Chapter 3

Previous Analyses of Bonded Structures

In the past, the analysis of duplex structures for fusion machines has focused on bulk behavior. The works of Mattas, *et al.* [10], Glasgow and Wolfer [30], Horie *et al.* [31], and Oomura *et al.* [32] all analyze duplex fusion structures using plate theory. None of these works consider end effects, so the emphasis is on the stresses in the bulk. Many of these works include irradiation effects, such as swelling, irradiation creep and loss of ductility, and some even use fracture mechanics to assess the impact of surface cracks on component lifetime, but the stress concentrations expected at the edge of the interfaces are ignored. Another approach, employed often by the NET team (as in the recent work by Renda, Federici, and Papa [33]), uses a finite element code (CASTEM) to analyse stresses in duplex structures. This technique provides a good picture of the stresses throughout the structure, even in complicated geometries with coolant channels, but the stresses near the edge of the interface are generally unreliable. Again, the treatment of the edge is inadequate. To date, no thorough study of the propensity of fusion components to fail at the edge of the interface has been carried out. This dissertation will address these edge effects, using techniques developed over several decades in the solid mechanics literature.

The analysis of bonded (or layered) structural components began in 1925 with Timoshenko's analysis of bi-metal thermostats[34]. This paper treats each layer of the strip as a beam and sets up an ordinary differential equation for the transverse deflection. End effects are ignored. Goland and Reissner[35] followed in 1944 with a more thorough analysis which included an adhesive layer and computed the shear and normal stresses to complement the axial stresses. Other beam-based models which solve ordinary differential equations are by Grimado[36], Chen and Nelson[37], and Ochoa and Marcano[38], all of which include thermal stresses, and find shear and normal stresses which are peaked (but finite) at the edge of the interface.

Another general technique for the analysis of bonded structures utilizes variational principles to minimize the complementary potential energy of the structure based on some assumed displacement profile. This work began with Weitsmann[39], who includes an adhesive layer between the two primary layers. The displacements are assumed to be quadratic and/or cubic polynomials and the shear and normal stresses are found to peak at the interface edge. Similar work has been conducted by Chen, Cheng, and Gerhardt[40], Chen and Cheng[41], and H. E. Williams[42, 43]. Williams considers thin layers by using matched asymptotic expansions for the displacement fields, giving a boundary layer with a width of order $\sqrt{t/l}$ (where t/l is the thickness-to-length ratio of the model) at the interface edge.

These variational methods require a great deal of algebra for even the simplest geometries and offer little advantage over numerical results from finite element codes, which are also based on potential minimization and can analyze many geometries and loadings. Unfortunately, neither of these two methods has treated the edge singularities predicted by infinitesimal elasticity. Many authors have studied stress singularities in bonded structures. M. L. Williams [44] studies stresses at the base

of a crack using an assumed solution for the stress function that is of the form (using the notation adopted for this dissertation): $\Phi(r, \theta; s) = r^{-s} F(\theta; s)$. This leads to a solution which consists of an expansion in terms of the eigenfunctions of the problem. Square root singularities are found in the stress fields at the tip of the crack. Applying a related technique to duplex models, Bogy[45, 46, 47] uses the Mellin transformation to calculate the asymptotic solutions for stresses near the edge of the interface. He considers a model consisting of two perfectly bonded quarter-planes and the singularity is found to be material dependent (as opposed to the simple geometry dependence associated with typical crack singularities), and, in most cases, of order $r^{-\delta}$, where δ lies between 0 and 0.40. For certain other material parameter combinations, though, the singularity is either logarithmic or nonexistent. Wang and Choi [48, 49] have made a similar study of crack-free singularities in anisotropic materials. Whereas Bogy only considers half-spaces, Wang and Choi study singularities in finite bodies using boundary collocation techniques. This work follows earlier work by Fadle [50], who considers double eigenfunction expansions for the analysis of rectangular bodies. One set of the coefficients in the expansion is found by minimizing the errors on the boundary in the least-square sense, as is done in this dissertation. Others, including Hein and Erdogan[51] and Dempsey and Sinclair[52, 53], consider arbitrary wedge angles and find complex singularities for some geometries, indicating oscillatory solutions. These oscillatory solutions occur, in particular, when the wedge angles are both 180^o, as in models for interface cracks. This leads to the unrealistic prediction of overlapping material on the crack faces near the crack tip. Some authors, including Erdogan [54], argue that the region over which the overlap occurs is small and can be ignored, but a more acceptable solution, by Comninou[55], assumes that the crack faces remain in contact over some

small region near the tip. In this case the asymptotic solutions for the near-tip fields are no longer oscillatory.

These analytical treatments of edge singularities are often overwhelmed by extensive algebra, so finite element approaches offer many potential advantages. Several authors, including Bartholomew[56], Lin and Mar[57], and Wang and Yuan [58] develop special elements for interfacial cracks. Also Raju and Crews[59], and Kokini and Perkins[60] use standard finite element methods and fit the near-edge fields to assumed singular fields to estimate the stress intensities associated with bonded structures. All these techniques are based on analytical solutions for the singular fields at the edge of the interface of some ideal model.

In addition to these hybrid element methods, a number of analyses have been conducted using standard finite elements. These include Trantina[61], Ishai and Gali[62], Gali and Ishai[63], Gulati and Hagy[64], Griffin and Roberts[65], Herakovich and Post[66], and Blanchard and Watson[67]. These studies consider various geometries and find finite stresses at the edge, leading to solutions much like the geometry-specific variational methods. In addition, Bauld and Goree[68] compare finite element and finite difference methods and conclude that the finite difference method is superior because it "characterizes the stress distributions near an interface corner in a more realistic manner." Whitcomb and Raju[69] review the use of finite element methods for analysis of bonded structures and find that standard finite elements are accurate to within two or three elements of the edge of the interface, regardless of the element size, indicating that these methods are useful for global solutions, but they do not accurately predict the near-edge fields. Hence, special elements must be used.

In terms of time-dependent solutions, only Delale and Erdogan[70, 71] deal with

the relaxation of thermal stresses in bonded structures. They consider viscoelastic materials and use variational methods to calculate the stresses, so there are no singularities. The relaxation of singular solutions has not been addressed.

Finally, there are numerous studies of stress singularities in composite structures/materials, typified by the work of Pagano and Soni[72]. These structures feature many very thin layers of orthotropic materials, generally built up from fibers embedded in a resin matrix. Techniques for analyzing such structures are not generally applicable in the case of relatively few thick layers, because of the approximations needed to handle the complex material constitutive equations, the numerous layers generally used in such designs, and the very small thickness of the layers. However, the local fields are quite similar.

Chapter 4

Problem Description

The problem to be studied in this dissertation consists of two equal-length, perfectly bonded strips (or plates) which have different material properties. If these types of structures are subjected to either applied surface loadings or thermal fields, classical elasticity theory predicts singular fields at the edge of the interface. The ultimate goal of this work is to determine the singular elastic stress fields in finite, layered fusion components under the many loading conditions expected in fusion devices. The task is composed of four parts: definition of the model, determination of the general stress fields associated with crack-free interface problems, tailoring of these solutions to finite bodies, and applications to fusion.

The interface is assumed to be perfect, with no slipping, delamination, or cracking. In this sense, the analysis presented here represents a study of the initiation of failure at the edge of the interface of a laminated structure. In bodies (either single- or multi-layered) containing cracks, linear elastic fracture mechanics predicts a stress field of the form

$$\sigma_{ij} \sim \frac{K}{\sqrt{r}} \quad r \to 0 \tag{1}$$

at the tip of the crack, where r is the distance from the crack tip. The stress intensity

factor K is considered to be a measure of the intensity of the stress singularity, and has been shown to be a useful predictor of crack growth. As will be shown in the following chapter, the stress field in a crack-free, bonded structure is generally of the form:

$$\sigma_{ij} \sim \frac{K}{r^{\delta}} \quad r \to 0 \tag{2}$$

where $0 \leq \delta \leq 0.41$. Therefore, crack-free, bonded components exhibit a relatively weak singularity analogous to that for the stress field near a crack tip. This dissertation assumes that the initiation of failure in bonded structures can be predicted by the "stress intensity" associated with the edge of a perfect interface, making knowledge of the characteristics of such stress fields vital to the design of bonded structures. This assumption, of course, requires experimental verification.

In studying the nature of stress singularities in layered fusion components, this work focuses on two areas:

- The treatment of radiation effects, such as void swelling, irradiation creep, and conductivity degradation in high heat flux components, and
- The comparison of the stress intensities in a particular component over its lifetime, to determine the critical points at which failure is most likely to occur.

The comparison of different material combinations, to determine the best choices for both near- and far-term fusion machines, cannot be made here because the critical stress intensities for the various material combinations have not been measured, nor has the importance of crack-free edge singularities been established.

The model to be studied herein features two rectangular, isotropic layers bonded along one surface, as shown in figure 11. The layers are assumed to be in perfect

Figure 11: Model used for analysis of singular stress fields in finite bodies

contact, with no defects or cracks anywhere in the structure. Slipping or debonding at the interface is not allowed. There are two singular fields in each of the small regions where the interface intersects a free surface and, if the layers are thin, the bulk of the material behaves like a beam or plate. This simple model is used in order to isolate the influence of the stress fields at the edge of the interface, thus discounting design dependent effects, such as external constraints. The strips are assumed be in a state of either plane strain or plane stress.

4.1 General Singular Stress Fields

The stresses at the edge of the interface between two materials can be sought by considering bonded semi-infinite quarter-planes. The model used for this purpose is shown in figure 12. Using the Airy stress function, one can determine a stress field that satisfies the field equations in the bulk, the interface conditions, and the traction

Figure 12: Model for semi-infinite quarter-planes to determine stresses at the edge of the interface

conditions on the free surfaces adjacent to the interface. This yields an infinite series with undetermined coefficients, which are be determined by considering the finite extent of the original model shown in figure 11.

4.2 Stresses in Finite Bodies

For the analysis of finite bodies, the general series solution mentioned in the previous section must be coerced to satisfy the traction conditions on the free surfaces which parallel the interface. This task is accomplished using two different techniques. The first, referred to here as point collocation, determines the coefficients in the stress

series so that the the traction conditions on the parallel surfaces are satisfied exactly at discrete (collocation) points. This method requires a correspondence between the number of terms retained in the stress series and the number of collocation points. The second method, referred to here as "least squares collocation", determines the coefficients in the series in order to minimize the integral of the squares of the residual surface tractions along the boundary.

4.3 Fusion Applications

The primary loading for this study consists of a thermal field which is either constant, to model a uniform temperature change, or varying linearly through the thickness, to model surface heating. In addition, neutron-induced volumetric swelling and stress relaxation due to creep will be considered to model the radiation damage caused by the fusion environment. Swelling can be regarded as a permanent, isotropic volume change of the material, so, until shutdown, it can be treated as a temperature change. Upon shutdown, though, the swelling strains do not reverse themselves, so the pre-irradiated state is not recovered. Also, as the swelling develops, irradiation creep is continuously relaxing the stresses. The interplay between the continuous swelling and the creep relaxation generally leads to some steady stress field that depends on the relative magnitudes of the creep and swelling. As with the swelling strains, creep strains are permanent and must be given special consideration during shutdown so that any residual stresses induced by the temperature reversal are accounted for. In this dissertation the irradiation creep is modeled assuming that the materials are viscoelastic. The details of this assumption will be discussed in a later chapter.

Chapter 5

Analysis of Singular Fields

5.1 Airy Stress Function

The analysis begins with the two-dimensional, steady-state, elastic field equations in polar coordinates (assuming no body forces are present), along with the traction-free boundary conditions and the assumed interface conditions:

strain-displacement:

$$\epsilon_{rr} = u_{r,r} \tag{3}$$

$$\epsilon_{\theta\theta} = \frac{u_r}{r} + \frac{u_{\theta,\theta}}{r} \tag{4}$$

$$\epsilon_{r\theta} = \frac{1}{2}\left(\frac{1}{r}u_{r,\theta} + u_{\theta,r} - \frac{u_\theta}{r}\right) \tag{5}$$

stress-strain:

$$\sigma_{rr} = 2\mu\epsilon_{rr} + \lambda\epsilon_{ll} - (2\mu + 3\lambda)\epsilon_{in} \tag{6}$$

$$\sigma_{\theta\theta} = 2\mu\epsilon_{\theta\theta} + \lambda\epsilon_{ll} - (2\mu + 3\lambda)\epsilon_{in} \tag{7}$$

$$\sigma_{r\theta} = 2\mu\epsilon_{r\theta} \tag{8}$$

equilibrium:

$$\sigma_{rr,r} + \frac{\sigma_{r\theta,\theta}}{r} + \frac{\sigma_{rr} - \sigma_{r\theta}}{r} = 0 \tag{9}$$

$$\frac{\sigma_{\theta\theta,\theta}}{r} + \sigma_{r\theta,r} + 2\frac{\sigma_{r\theta}}{r} = 0 \tag{10}$$

boundary conditions: $(\theta = \pm\pi/2)$

$$\sigma_{\theta\theta} = \sigma_{r\theta} = 0 \tag{11}$$

interface conditions: $(\theta = 0)$

$$\sigma'_{\theta\theta} = \sigma''_{\theta\theta} \tag{12}$$

$$\sigma'_{r\theta} = \sigma''_{r\theta} \tag{13}$$

$$u'_r = u''_r \tag{14}$$

$$u'_\theta = u''_\theta \tag{15}$$

In these equations, σ_{ij} represents a stress, ϵ_{ij} represents a strain, u_r and u_θ represent the radial and azimuthal deflections, respectively, and λ and μ are the Lamé material constants. The inelastic strain ϵ_{in} is the sum of the thermal and swelling strains, each representing an isotropic volume change, *i.e.*

$$\epsilon_{in} = \alpha T + \frac{\Delta V}{3V}, \tag{16}$$

where α is the thermal expansion coefficient, T is the difference between the temperature in the component and some zero-stress reference temperature, and $\frac{\Delta V}{3V}$ is the swelling strain. Also σ'_{ij} denotes a stress in the lower of the two strips, while σ''_{ij} denotes stress in the upper strip. A similar notation will be used for displacements and material properties as well.

In order to reduce the problem to determination of a scalar function, the Airy stress function Φ is introduced according to the standard definition:

$$\sigma_{rr} = \frac{1}{r}\Phi_{,r} + \frac{1}{r^2}\Phi_{,\theta\theta} \tag{17}$$

$$\sigma_{\theta\theta} = \Phi_{,rr} \tag{18}$$

$$\sigma_{r\theta} = \frac{1}{r^2}\Phi_{,\theta} - \frac{1}{r}\Phi_{,r\theta}. \tag{19}$$

Stress fields determined from a stress function of this type automatically satisfy the equilibrium equations.

If the displacements u_r and u_θ, determined from a known stress field are to be single-valued, then the stresses must satisfy the compatibility equation:

$$\nabla^2(\sigma_{rr} + \sigma_{\theta\theta}) + QE\nabla^2\epsilon_{in} = 0, \tag{20}$$

where Q is given by

$$Q = \begin{cases} 1 & for\ plane\ stress \\ 1/(1-\nu) & for\ plane\ strain \end{cases}. \tag{21}$$

Finally, in terms of the stress function, the displacements are given by:

$$u_{r,r} = \frac{1}{2\mu}\left[\frac{1}{r}\Phi_{,r} + \frac{1}{r^2}\Phi_{,\theta\theta} - \left(1 - \frac{m}{4}\right)\nabla^2\Phi\right] + n\epsilon_{in} \tag{22}$$

and

$$u_{\theta,r} - \frac{u_\theta}{r} + \frac{1}{r}u_{r,\theta} = \frac{1}{\mu}\left(\frac{1}{r^2}\Phi_{,\theta} - \frac{1}{r}\Phi_{,r\theta}\right), \tag{23}$$

where

$$m = \begin{cases} 4/(1+\nu) & for\ plane\ stress \\ 4(1-\nu) & for\ plane\ strain \end{cases}, \tag{24}$$

and

$$n = \begin{cases} 1 & for\ plane\ stress \\ (1+\nu) & for\ plane\ strain \end{cases} . \tag{25}$$

Inserting equations 17 and 18 into the compatibility equation yields the following fourth order partial differential equation for the stress function (again assuming no body forces):

$$\nabla^4 \Phi + QE\nabla^2 \epsilon_{in} = 0. \tag{26}$$

Solving for the stress function, subject to the appropriate surface traction and interface conditions, provides a means for computing the steady state thermal stresses, strains, and displacements in a planar medium.

In this study, the thermal fields are harmonic (satisfying Fourier's Law of conduction for a body in steady state) and the swelling strains are assumed to be uniform. The assumption of linear thermal fields is generally valid, but the swelling may not be uniform for three reasons:

1. Gradients in the damage rate, due to flux attenuation, can lead to swelling gradients. This is a small effect in the thin structures generally found in high-heat-flux fusion components.

2. The temperature dependence of the swelling rate can lead to swelling gradients in the presence of thermal gradients. No measurements of this effect have been conducted.

3. The stress dependence of swelling could lead to swelling gradients in the presence of stress gradients. Swelling is thought to be increased slightly by positive hydrostatic stresses.

These effects are all presumed to be small, but further testing is required to substantiate the uniform swelling assumption. Assuming that the inelastic strain fields are harmonic, the stress function is governed by:

$$\nabla^4 \Phi = 0. \tag{27}$$

5.2 An Assumed Solution

In order to reduce this partial differential equation for the Airy stress function to an ordinary differential equation, the solution is assumed to be of the form:

$$\Phi = r^{-s} F(\theta). \tag{28}$$

Under this transformation the equation for the stress function (equation 27) becomes

$$\left(\frac{d^2}{d\theta^2} + s^2\right)\left(\frac{d^2}{d\theta^2} + (s+2)^2\right) F = 0. \tag{29}$$

Also, the stresses are given by:

$$\sigma_{rr} = \left(\frac{d^2}{d\theta^2} + s^2\right) F r^{-(s+2)}, \tag{30}$$

$$\sigma_{\theta\theta} = s(s+1) F r^{-(s+2)}, \tag{31}$$

and

$$\sigma_{r\theta} = (s+1)\frac{dF}{d\theta} r^{-(s+2)}. \tag{32}$$

For $s \neq 0, -2$ [1] the general form of the stress function, from equation 29 is:

$$F = a \sin s\theta + b \cos s\theta + c \sin (s+2)\theta + d \cos (s+2)\theta \tag{33}$$

where a, b, c, and d are unknown constants.

[1] The case of $s = -2$ is handled separately in a later section.

Given a solution for the stress function in each quarter-plane of the model, the full solution is obtained by using the boundary and interface conditions to determine the four unknown constants in each strip. This process is begun by rewriting the stresses and displacements in terms of the unknown constants a, b, c, and d. Inserting equation 33 into equations 30-32, one obtains the following equations for the stresses:

$$\begin{aligned}
\sigma_{rr} &= [-as\sin s\theta - bs\cos s\theta - \\
&\quad c(s+4)\sin(s+2)\theta - d(s+4)\cos(s+2)\theta]\,(s+1)r^{-(s+2)} \\
\sigma_{\theta\theta} &= [a\sin s\theta + b\cos s\theta + \\
&\quad c\sin(s+2)\theta + d\cos(s+2)\theta]\,s(s+1)r^{-(s+2)} \\
\sigma_{r\theta} &= [as\cos s\theta - bs\sin s\theta + \\
&\quad c(s+2)\cos(s+2)\theta - d(s+2)\sin(s+2)\theta]\,(s+1)r^{-(s+2)}. \qquad (34)
\end{aligned}$$

The displacements can be found by inserting the solution for F into equations 22 and 23, and integrating them. First, though, one requires knowledge of the inelastic strain field, which is assumed to be of the form:

$$\epsilon_{in} = \epsilon_{in0} + \frac{r\sin\theta}{t}\alpha\Delta T, \qquad (35)$$

where ϵ_{in0} represents a uniform volume change (including both thermal expansion and swelling) from the stress-free state, and ΔT represents a linear temperature gradient, thus modeling a constant heat flux on the top surface. The swelling is assumed to be uniform, so it does not contribute to the gradient term. This strain field yields the following displacement fields:

$$\begin{aligned}
u_r &= u_0\cos\theta + v_0\sin\theta + n\epsilon_{in}r + \frac{n\alpha\Delta T r^2\sin\theta}{2t} + \\
&\quad \frac{r^{-(s+1)}}{2\mu}\,[sa\sin s\theta + sb\cos s\theta
\end{aligned}$$

$$
\begin{aligned}
& +(s+m)c\sin(s+2)\theta + (s+m)d\cos(s+2)\theta] \\
u_\theta \;=\; & -u_0\sin\theta + v_0\cos\theta + \omega_0 r - \frac{n\alpha\Delta T r^2\cos\theta}{2t} - \\
& \frac{r^{-(s+1)}}{2\mu}\left[sa\cos s\theta - sb\sin s\theta\right. \\
& \left.+(s+2-m)c\cos(s+2)\theta + (s+2-m)d\sin(s+2)\theta\right], \qquad (36)
\end{aligned}
$$

where u_0 and v_0 represent rigid body displacements in the x and y directions, respectively, and ω_0 represents a rigid body rotation.

Using these equations for the stresses and displacements, the boundary and interface conditions can be used to determine the eight unknown constants.

5.3 Boundary and Interface Conditions

The two traction-free surfaces adjacent to the interface (at $\theta = \pm\pi/2$) each provide two boundary conditions. Setting the normal stress $\sigma_{\theta\theta}$ to zero on the lower and upper free surfaces yields

$$-a'\sin\xi + b'\cos\xi + c'\sin\xi - d'\cos\xi = 0 \qquad (37)$$

and

$$a''\sin\xi + b''\cos\xi - c''\sin\xi - d''\cos\xi = 0, \qquad (38)$$

respectively, where $\xi = s\pi/2$. Similarly, setting the shear stresses on these surfaces to zero yields:

$$a's\cos\xi + b's\sin\xi - c'(s+2)\cos\xi - d'(s+2)\sin\xi = 0 \qquad (39)$$

and

$$a''s\cos\xi - b''s\sin\xi - c''(s+2)\cos\xi + d''(s+2)\sin\xi = 0. \qquad (40)$$

The assumed non-slip, crack-free interface conditions require that the radial and azimuthal displacements, as well as the shear and normal stresses of the two strips are matched, thus providing four additional equations. Matching the normal and shear stresses yields

$$b' + d' - b'' - d'' = 0 \tag{41}$$

and

$$sa' + (s+2)c' - sa'' - (s+2)c'' = 0, \tag{42}$$

respectively. The radial and azimuthal displacement continuity conditions lead to the final two equations for the unknown constants:

$$sb' + (s+m')d' - ksb'' - k(s+m'')d'' = -2\mu' \left(n'\epsilon'_{in0} - n''\epsilon''_{in0}\right) r^{(s+2)} \tag{43}$$

and

$$\begin{aligned} &-sa' - (s+2-m')c' + \\ &ksa'' + k(s+2-m'')c'' \;\; = \;\; 2\mu' \left(\frac{n'\alpha'}{t'}\Delta T' - \frac{n''\alpha''}{t''}\Delta T''\right) r^{(s+3)}, \end{aligned} \tag{44}$$

where k is the ratio of the shear moduli of the two materials ($k = \mu'/\mu''$). The resulting system of equations for the unknown constants can be written in the following form:

$$[\mathbf{X}]\,\{a\} = \{f\} \tag{45}$$

where

$$[\mathbf{X}] = \begin{bmatrix} -\sin\xi & \cos\xi & \sin\xi & -\cos\xi & 0 & 0 & 0 & 0 \\ s\cos\xi & s\sin\xi & -(s+2)\cos\xi & -(s+2)\sin\xi & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \sin\xi & \cos\xi & -\sin\xi & -\cos\xi \\ 0 & 0 & 0 & 0 & s\cos\xi & -s\sin\xi & -(s+2)\cos\xi & (s+2)\sin\xi \\ 0 & 1 & 0 & 1 & 0 & -1 & 0 & -1 \\ s & 0 & s+2 & 0 & -s & 0 & -(s+2) & 0 \\ 0 & s & 0 & s+m' & 0 & -ks & 0 & -k(s+m'') \\ -s & 0 & -(s+2-m') & 0 & ks & 0 & k(s+2-m'') & 0 \end{bmatrix}, \tag{46}$$

$$\{a\} = \begin{Bmatrix} a' \\ b' \\ c' \\ d' \\ a'' \\ b'' \\ c'' \\ d'' \end{Bmatrix} \text{ and } \{f\} = \begin{Bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ R_1 \\ R_2 \end{Bmatrix}, \tag{47}$$

$$R_1 = -2\mu' \left(n'\epsilon'_{in0} - n''\epsilon''_{in0}\right) r^{(s+2)} \tag{48}$$

and

$$R_2 = \mu' \left(\frac{n'\alpha'}{t'}\Delta T' - \frac{n''\alpha''}{t''}\Delta T''\right) r^{(s+3)}. \tag{49}$$

The solution of this system provides the stresses and displacements in bonded, semi-infinite quarter-planes.

5.4 Homogeneous Solution

The homogeneous solution of the problem is found by setting the forcing vector in equation 45 to zero, *i.e.*, by setting R_1 and R_2 to zero. A non-trivial solution to this equation only exists if the determinant of the matrix is zero. This leads to a characteristic equation of the form:

$$\|\mathbf{X}\| = 0, \tag{50}$$

where $\|\mathbf{X}\|$ is the determinant of the matrix in equation 45 and is given by:

$$\|\mathbf{X}\| = k_3^2\eta^2 + \frac{1}{4}k_1^2(\gamma\delta - \eta^2)^2 - k_2^2\gamma\delta + \frac{1}{2}k_2k_1(\delta - \gamma)(\gamma\delta - \eta^2), \tag{51}$$

where η, γ, and δ are given by:

$$\eta = -2\sin\xi\cos\xi \tag{52}$$

$$\gamma = 2(s+1) - 2\cos 2\xi \tag{53}$$

$$\delta = 2(s+1) + 2\cos 2\xi \tag{54}$$

and k_1, k_2, and k_3 are material constants determined by:

$$k_1 = 2(k-1), \tag{55}$$

$$k_2 = km'' - m' \tag{56}$$

and

$$k_3 = km'' + m'. \tag{57}$$

The values of s corresponding to the roots of the determinant are discussed in the next section.

5.4.1 Roots of the Determinant

Because the strain energy density is proportional to σ_{ij}^2, it will be proportional to $r^{-2(s+2)}$. Hence, for the total strain energy to be finite, s must be less than -1. Therefore, our interest lies in roots of the determinant that have real parts less than -1. In addition, roots that lie in the region $-2 < s < -1$ lead to singular stress fields, so they are of particular interest. For all k_1, k_2, and k_3 (i.e. for any combination of materials), s=0, -1, and -2 are always zeroes.

The root at s=-1

Because the stresses are all proportional to $(s+1)$ (as can be seen from equation 34), they all are identically zero at $s = -1$, and the displacements are given by:

$$\begin{aligned} u_r &= \frac{1}{2\mu}\left[a\sin\theta - b\cos\theta \right. \\ &\quad \left. +(m-1)c\sin\theta + (m-1)d\cos\theta\right] \\ u_\theta &= \frac{1}{2\mu}\left[-a\cos\theta - b\sin\theta \right. \\ &\quad \left. +(1-m)c\cos\theta + (1-m)d\sin\theta\right]. \end{aligned} \tag{58}$$

Solving the system represented by equation 45 for the eight unknown coefficients, these displacements are found to be:

$$u_r' = u_r'' = \frac{m'}{2\mu'}\left(c'\sin\theta + d'\cos\theta\right) \tag{59}$$

$$u_\theta' = u_\theta'' = \frac{m'}{2\mu'}\left(c'\cos\theta - d'\sin\theta\right), \tag{60}$$

which, if transformed to cartesian coordinates, yield constant displacements in the x and y directions. Hence, $s = -1$ leads to rigid body translations in the x and y directions.

The Root at s=-2

Although there is a root of the determinant at $s = -2$, this case must be handled separately because the general solution given in equation 33 is incomplete for $s = -2$. The previously assumed solution (equation 33) gives

$$\Phi = r^2\left[-a\sin 2\theta + b\cos 2\theta + d\right], \tag{61}$$

thus providing only three terms. The full solution for s=-2 is:

$$\Phi = r^2\left[-a\sin 2\theta + b\cos 2\theta + c\theta + d\right], \tag{62}$$

and the associated stresses and displacements are:

$$\begin{aligned}
\sigma_{rr} &= 2a\sin 2\theta - 2b\cos 2\theta + 2c\theta + 2d \\
\sigma_{\theta\theta} &= -2a\sin 2\theta + 2b\cos 2\theta + 2c\theta + 2d \\
\sigma_{r\theta} &= 2a\cos 2\theta + 2b\sin 2\theta - c \\
u_r &= \frac{r}{2\mu}\left[2a\sin 2\theta - 2b\cos 2\theta - (2-m)(c\theta + d)\right] \\
u_\theta &= \frac{r}{2\mu}\left[2a\cos 2\theta + 2b\sin 2\theta - mc\ln r\right] + \omega_0 r. \qquad (63)
\end{aligned}$$

When $k_2 \neq 0$, the last of these equations, combined with the zero-shear boundary conditions, requires that $c' = c'' = 0$ and $a' = a'' = 0$. Applying the zero-normal-stress boundary conditions and the axial displacement continuity conditions requires:

$$b' = b'' = d' = d'' \qquad (64)$$

and

$$[2k_1 - k_2]\, b' = 0. \qquad (65)$$

Therefore, the homogeneous solution for $s = -2$ and $k_2 \neq 0$ depends on the parameter $2k_1 - k_2$. (Hereafter, the quantity $2k_1 - k_2$ will be referred to as P.) When P is zero, there exists a solution for any b', representing a uniform tensile stress σ_{yy}. The physical explanation for this follows. When P is zero, the material properties of the two strips are such that:

$$\frac{\nu'}{E'} = \frac{\nu''}{E''} \qquad (66)$$

in the plane stress case, and

$$\frac{\nu'(1+\nu')}{E'} = \frac{\nu''(1+\nu'')}{E''} \qquad (67)$$

in the case of plane strain. In either case, these equations indicate that the lateral displacements in a material subject to a uniform tensile stress σ_{yy} are equal. Hence,

a uniform tension in the y-direction (perpendicular to the interface) is admissible if $P = 0$ because the displacements at the interface will still be continuous. This case ($P = 0$), leads to a solution of the form:

$$\begin{aligned}
\sigma_{rr} &= 2b\,[1 - \cos 2\theta] \\
\sigma_{\theta\theta} &= 2b\,[1 + \cos 2\theta] \\
\sigma_{r\theta} &= 2b\,[\sin 2\theta] \\
u_r &= \frac{-rb}{2\mu}\,[\cos 2\theta + (2 - m)] \\
u_\theta &= \frac{rb}{\mu}\sin 2\theta + \omega_0 r.
\end{aligned} \tag{68}$$

In the case where $P \neq 0$, one finds $a = b = d = 0$ in each strip, so the stresses are all zero, and the displacements are given by:

$$u_r = 0 \tag{69}$$

and

$$u_\theta = \frac{mc}{2\mu} r. \tag{70}$$

This last displacement is a rigid body rotation.

When $k_2 = 0$ there is an additional homogeneous solution. In this case, $c' = c'' \neq 0$ and the constants are given by:

$$\begin{aligned}
a' = a'' &= -\frac{1}{2}c' \\
b' = d'' &= \frac{-\pi}{2}c'\left(\frac{m' + k_1}{2k_1}\right) \\
b'' = d' &= \frac{-\pi}{2}c'\left(\frac{m' - k_1}{2k_1}\right).
\end{aligned} \tag{71}$$

These constants can be substituted into equation 63 to obtain the homogeneous solution for $k_2 = 0$.

To summarize, the root at $s = -1$ provides rigid body translations, u_0 and v_0. The root at $s = -2$ provides three possible solutions, depending on the material properties:

1. if $k_2(2k_1 - k_2) \neq 0$, then the only solution is a rigid body rotation
2. if $(2k_1 - k_2) = 0$, then a solution corresponding to uniform tension in the y-direction is also admissible
3. if $k_2 = 0$, another solution, represented by equation 71 is admissible

A discussion of the remaining roots follows.

Roots that cause singularities

Because the stresses are proportional to $r^{-(s+2)}$, they will be singular when there is a root on $-2 < s < -1$. Besides impacting the meaning of the root at $s = -2$, the parameter $P = 2k_1 - k_2$ also determines the existence of roots on the interval $-2 < s < -1$. As it turns out, the product k_2P is proportional to the derivative of the determinant of the matrix in equation 45, so it is an indicator of the slope of the determinant at $s = -2$. For $k_2P > 0$, there is exactly one zero on the interval $-2 < s < -1$, and it is a simple zero. For all admissible values of the material constants (*i.e.* $0 < \nu < \frac{1}{2}$), this zero occurs between -1.59 and -2.0. As k_2P approaches zero, the zero of the determinant moves closer to s=-2 until, when $k_2P = 0$, there are no zeroes on $-2 < s < -1$ and the zero at s=-2 becomes a double root. Finally, for $k_2P > 0$, there are no zeroes on $-2 < s < -1$ and the zero at s=-2 is simple; also, there is a simple zero between -2.4 and -2.0. These three basic behaviors of the determinant are shown schematically in figure 13.

Figure 13: Schematic of determinants for the three possible types of material combinations

Figure 14: Order of singularity for various material combinations

As one might expect, singularities tend to occur when the elastic properties of the two materials differ significantly. This is shown in figure 14, which shows the order of singularity as a function of the ratio of the shear moduli of the two layers for various values of m' and m''. The curves all peak at k=0, which models a situation in which one material has zero shear stiffness. In this case, the singularity is of order 0.41, which is still smaller than the 0.5 associated with linear elastic fracture problems. As k increases, the order decreases until it reaches zero somewhere between k=0.5 and k=0.9. Above these transition values, the shear moduli of the two materials are closely matched ($k \rightarrow 1$) and the algebraic singularity disappears. The stresses then exhibit either a logarithmic singularity or the stresses are of order one.

Table 2: Order of Singularity at Edge of Interface

	P		Order	
	plane stress	plane strain	plane stress	plane strain
graphite/ tungsten	-1.9	1.4	0	0.12
SiC/ tungsten	-0.1	-0.2	0	0
copper/ tungsten	-6.8	1.8	0	0.10
graphite/ copper	-113.0	101.0	0	0.07
SiC/ copper	-7.8	1.7	0	0.08

The order of singularity is presented for a variety of material combinations in Table 2. For the materials shown, all of which have been considered for fusion reactors, $P = 2k_1 - k_2$ is negative for plane stress, indicating no singularity. Unfortunately, bonded structures are rarely thin in the plane of the interface, so plane strain conditions are unavoidable. For plane strain, only the SiC/tungsten combination was not singular at the interface. This may be an indication that this combination is superior to the others, but at least two other factors must be considered. First, the order of singularity does not depend on the thermal expansion coefficients, so the concept of good matching of this important property does not influence the values given in the table. This will be pursued further when the particular solutions are sought. The second factor not considered here is fabricability. Some material combinations form better bonds with each other, and this higher strength can lead to larger allowable loads. This will not be considered further in this study.

Dundurs' Parameters

Although figure 14 is instructive, in that it vividly shows the effect of changing the ratio of the shear moduli of two materials in a bonded component, many such figures would be required to cover the whole spectrum of possible material combinations. Fortunately, Dundurs [73] has established that the order of singularity depends on only two material parameters, one set of which is known as Dundurs' parameters. These parameters, in terms of k_1, k_2, and k_3, are:

$$\alpha_D = \frac{k_2}{k_3} \quad \text{and} \quad \beta_D = \frac{k_2 - k_1}{k_3}. \tag{72}$$

Because Poisson's ratio is bounded by $0 \leq \nu \leq 1/2$, the material parameter m, defined in equation 24, is bounded by $2 \leq m \leq 4$ for plane strain and by $8/3 \leq m \leq 4$ for plane stress, thus restricting the admissible values of Dundurs' parameters. Rewriting Dundurs' parameters in terms of k, m', and m'', one finds:

$$\alpha_D = \frac{km'' - m'}{km'' + m'} \tag{73}$$

and

$$\beta_D = \frac{k(m'' - 2) - (m' - 2)}{km'' + m'}. \tag{74}$$

From these equations one can see that as the ratio of shear moduli (k) goes from 0 to ∞, α_D goes from -1 to 1, thus providing the bounds for the first Dundurs parameter. If k is at some intermediate value, the bounds on m provide upper and lower bounds for β_D as follows:

$$\frac{\alpha_D - 1}{4} \leq \beta_D \leq \frac{\alpha_D + 1}{4} \text{for plane strain} \tag{75}$$

$$\frac{3\alpha_D - 1}{8} \leq \beta_D \leq \frac{3\alpha_D + 1}{8} \text{for plane stress.} \tag{76}$$

Figure 15: Regions of relevant material parameters for state of plane strain

These bounds form parallelograms in $\alpha_D - \beta_D$ space, as shown in figures 15 and 16. Figure 15 represents the regions in $\alpha_D - \beta_D$ space for which there are relevant combinations of Dundurs' parameters for a body in a state of plane strain. The figure also shows the combinations of parameters which lead to singularities. As mentioned previously, singularities in bonded structures occur when $k_2(2k_1 - k_2)$ is positive, or, in terms of Dundurs' parameters, when $\alpha_D(\alpha_D - 2\beta_D)$ is positive. This allows us to determine the combinations of Dundurs' parameters which lead to singularities. This region, for a state of plane strain, is indicated by the hatched region in figure 15.

Figure 16 shows the regions of relevant parameters and singularities for a state of plane stress. The dotted line in this figure represents the parallelogram of relevant

Figure 16: Regions of relevant material parameters for state of plane stress

properties for plane stress, while the plane strain lines are also shown for comparison. As can be seen, the relevant material parameters for plane stress are a subset of those for plane strain. Also, the hatched region, indicating regions where singularities are expected, is smaller than that for plane strain because some of the values that caused singularities in plane strain cannot occur in plane stress.

Because the order of singularity depends on only two parameters, The whole spectrum of possible material parameters can be represented by a single plot. Figure 17 shows the order of singularity as a function of α_D for various values of β_D. As can be seen, the peak value is 0.41, which occurs when one material is incompressible ($\nu = 1/2$) and the other is infinitely stiff ($\mu \to \infty$). Figure 18 shows the analogous plot for plane stress. These latter curves are identical to those for plane strain, but they are cut off due to the smaller parallelogram of relevant values in the Dundurs' parameter space. The maximum possible order of singularity for plane stress is 0.31.

Other Integer Roots

In general, the only integer roots of the determinant are at s=-2, -1, and 0, but for certain material property combinations, there are other integer roots. If one assumes an integer solution, the determinant can be written:

$$\|\mathbf{X}\| = 4\left\{\left[k_1(s+1)^2 - (k_1 - k_2)\cos^2\frac{s\pi}{2}\right]^2 - k_2^2(s+1)^2\right\}. \tag{77}$$

If s is an odd integer, i.e. $s = -(2i-1)$, $i = 1, 2, 3\ldots$, then $\cos\frac{s\pi}{2} = 0$ and the determinant can be written:

$$\|\mathbf{X}\| = -16(i-1)^2 k_1^2\left[2(i-1) + \frac{k_2}{k_1}\right]\left[2(i-1) - \frac{k_2}{k_1}\right]. \tag{78}$$

Figure 17: Order of singularity in Dundurs' parameter space for state of plane strain

Figure 18: Order of singularity in Dundurs' parameter space for state of plane stress

From this equation, one can see that there are odd integer roots of the determinant if k_2 is an even multiple of k_1.

If s is an even integer, i.e. $s = -2j,\ j = 1, 2, 3 \ldots$, then $\cos\frac{s\pi}{2} = \pm 1$ and the determinant can be written:

$$\|\mathbf{X}\| = 16j(j-1)k_1^2 \left[2(j-1) + \frac{k_2}{k_1}\right] \left[2j - \frac{k_2}{k_1}\right]. \tag{79}$$

Again, there are additional integer roots when k_2 is an even multiple of k_1. As an example, consider the case where $k_2 = 8k_1$. Equation 78 indicates that there is a root for $i = 5$ ($s = -9$), while equation 79 indicates a root at $j = 4$ ($s = -8$). These eigenvalues must be accounted for when solutions for stresses in finite bodies are sought.

Complex Roots

Besides the real roots on the interval $-2 < s < 0$, there are an infinite number of complex roots. A typical example of the spectrum of complex roots, determined numerically using Muller's method [74], is shown in figure 19. Several observations can be made regarding these complex roots:

- they always appear as complex conjugates
- for large negative real part, the real part of one root is about one unit from its neighbors
- for increasing negative real part, the magnitude of the imaginary part increases much more slowly than the magnitude of the real part.

These complex roots, combined with the real root on the interval $-2 < s < -1$ (if one exists), lead to a solution for the stresses and displacements in the form of an

Figure 19: Roots of determinant for $-15 \leq s \leq 0$. The number of roots is doubly infinite and the roots are symmetric about $Re(s) = -1$

infinite series.

5.4.2 Series Solution for the Stresses and Displacements

Because the problem being considered in this section is homogeneous, a non-trivial solution exists only when the determinant of the matrix in equation 45 vanishes. Therefore, the solution for the eight unknown constants cannot be fully determined. Only seven of the constants can be determined in terms of the eighth (in this dissertation a' will be taken to be the undetermined constant), so the following ratios are defined:

$$A' = 1, \;\; B' = \frac{b'}{a'}, \;\; C' = \frac{c'}{sa'}, \;\; D' = \frac{d'}{sa'}, \tag{80}$$

and

$$A'' = \frac{a''}{a'},\ B'' = \frac{b''}{a'},\ C'' = \frac{c''}{sa'},\ D'' = \frac{d''}{sa'}. \tag{81}$$

Solving the system in equation 45 for these seven constants, one finds:

$$\begin{aligned}
B' &= \{k_1 s\eta[2s(s+1)+\gamma] + k_2 s\eta\delta - k_3\eta\gamma(s+2)\}/h \\
C' &= \left\{-k_1\gamma[2s(s+1)+\gamma] - k_2\gamma\delta - k_3\eta^2\right\}/h \\
D' &= \{-k_1\eta[2s(s+1)+\gamma] - k_2\eta\delta - k_3\eta\delta\}/h \\
A'' &= \left\{-k_1 s\delta[2s(s+1)+\gamma] - k_2 s\delta^2 - k_3(s+2)\eta^2\right\}/h \\
B'' &= \{-k_1 s\eta[2s(s+1)+\gamma] - k_2 s\eta\delta - k_3\eta\gamma(s+2)\}/h \\
C'' &= \left\{-k_1\gamma[2s(s+1)+\gamma] - k_2\gamma\delta + k_3\eta^2\right\}/h \\
D'' &= \{k_1\eta[2s(s+1)+\gamma] + k_2\eta\delta - k_3\eta\delta\}/h
\end{aligned} \tag{82}$$

where

$$h = -k_1 s\delta[2s(s+1)+\gamma] - k_2 s\delta^2 + k_3(s+2)\eta^2. \tag{83}$$

This last quantity, h, is proportional to the second derivative of the determinant of the matrix in equation 45, so it is only zero when the determinant has a double root. This case will be handled separately, so there is no concern with dividing by h in the above equations.

Now that seven of the constants are known in terms of the eighth, the stresses and displacements can be expressed in terms of the single unknown coefficients a':

$$\sigma_{ij} = a' f_{ij} r^{-(s+2)} \tag{84}$$

and

$$u_r = \frac{1}{2\mu} a' f_{u_r} r^{-(s+1)} \tag{85}$$

$$u_\theta = \frac{1}{2\mu} a' f_{u_\theta} r^{-(s+1)}, \tag{86}$$

where

$$
\begin{aligned}
f_{rr} &= [-A\sin s\theta - B\cos(s+2)\theta - \\
&\quad (s+4)C\sin(s+2)\theta - (s+4)D\cos(s+2)\theta]\, s(s+1), \\
f_{r\theta} &= [A\cos(s+2)\theta - B\sin(s+2)\theta + \\
&\quad (s+2)C\cos(s+2)\theta - (s+2)D\sin(s+2)\theta]\, s(s+1), \\
f_{\theta\theta} &= [A\sin(s+2)\theta + B\cos(s+2)\theta + \\
&\quad sC\sin(s+2)\theta + sD\cos(s+2)\theta]\, s(s+1), \\
f_u &= [A\sin(s+2)\theta + B\cos(s+2)\theta \\
&\quad +(s+m)C\sin(s+2)\theta + (s+m)D\cos(s+2)\theta]\, s, \\
f_v &= [A\cos(s+2)\theta - B\sin(s+2)\theta \\
&\quad +(s+2-m)C\cos(s+2)\theta + (s+2-m)D\sin(s+2)\theta]\, s. \qquad (87)
\end{aligned}
$$

Because there are an infinite number of roots of the determinant discussed in the previous section, the homogeneous solution for the stresses and displacements can be expressed in terms of an infinite series:

$$\sigma_{ij} = \sum_{k=1}^{\infty} a_k f_{ij(k)} r^{-(s+2)} \qquad (88)$$

$$u_r = \frac{1}{2\mu}\sum_{k=1}^{\infty} a_k f_{u_r(k)} r^{-(s+2)} \qquad (89)$$

$$u_\theta = \frac{1}{2\mu}\sum_{k=1}^{\infty} a_k f_{u_\theta(k)} r^{-(s+2)}, \qquad (90)$$

where

$$
\begin{aligned}
f_{rr(k)} &= [-A\sin s_k\theta - B\cos(s_k+2)\theta - \\
&\quad (s_k+4)C\sin(s_k+2)\theta - (s_k+4)D\cos(s_k+2)\theta]\, s_k(s_k+1),
\end{aligned}
$$

$$
\begin{aligned}
f_{r\theta(k)} &= [A\cos(s_k+2)\theta - B\sin(s_k+2)\theta + \\
&\quad (s_k+2)C\cos(s_k+2)\theta - (s_k+2)D\sin(s_k+2)\theta]\, s_k(s_k+1), \\
f_{\theta\theta(k)} &= [A\sin(s_k+2)\theta + B\cos(s_k+2)\theta + \\
&\quad s_kC\sin(s_k+2)\theta + s_kD\cos(s_k+2)\theta]\, s_k(s_k+1), \\
f_{u_r(k)} &= [A\sin(s_k+2)\theta + B\cos(s_k+2)\theta \\
&\quad +(s_k+m)C\sin(s_k+2)\theta + (s_k+m)D\cos(s_k+2)\theta]\, s_k, \\
f_{u_\theta(k)} &= [A\cos(s_k+2)\theta - B\sin(s_k+2)\theta \\
&\quad +(s_k+2-m)C\cos(s_k+2)\theta + (s_k+2-m)D\sin(s_k+2)\theta]\, s_k. \quad (91)
\end{aligned}
$$

Because the complex roots s_k appear as complex conjugates the stresses and displacements in this series are real. It remains to determine the unknown series coefficients for a particular thermal field.

5.5 Particular Solutions

5.5.1 Solvability and Logarithmic Singularities

As shown in the previous chapter, the solution for the near-edge stress fields in bonded structures can be described by the following equation:

$$[\mathbf{X}]\{a\} = \{f\}, \quad (92)$$

where the matrix and vectors are defined in equation 45. A solution to this equation exists if and only if the vector $\{f\}$ is orthogonal to all vector solutions of the equation [75]:

$$[\mathbf{X}]^t\{a\} = 0. \quad (93)$$

If no solution exists, then the following solution for the Airy stress function is adopted:

$$\Phi = \frac{\partial}{\partial s}\left\{r^{-s}\left[a\sin s\theta + b\cos s\theta + c\sin(s+2)\theta + d\cos(s+2)\theta\right]\right\}, \tag{94}$$

which can also be written as

$$\Phi = r^{-s}\left(\frac{\partial}{\partial s} - \ln r\right)\left[a\sin s\theta + b\cos s\theta + c\sin(s+2)\theta + d\cos(s+2)\theta\right]. \tag{95}$$

This leads to a matrix equation of the form:

$$r^{-s}\left(\frac{\partial}{\partial s} - \ln r\right)[\mathbf{X}]\{a\} = \{f\}, \tag{96}$$

which has a solution of the form:

$$\begin{aligned} [\mathbf{X}]\{a\} &= 0 \\ \frac{\partial}{\partial s}([\mathbf{X}]\{a\}) &= \{f\}. \end{aligned} \tag{97}$$

When the material properties and loadings are such that this type of solution holds, then a logarithmic stress singularity exists at the edge of the interface. The conditions for which such singularities are expected for thermal stresses in bonded structures will be mentioned in this chapter, but the analysis will not be carried further. Detailed analyses of logarithmic singularities in composites experiencing externally applied loads are discussed by Dempsey and Sinclair [52] and Zwiers, *et. al.* [76].

5.5.2 Particular Solution for Uniform Temperature Change

If the temperature field is uniform, then the last term (R_2) in the forcing vector is zero, so only R_1 remains. Because this vector must be independent of s, and R_1 is proportional to $r^{(s+2)}$, the particular solution must have $s = -2$. As with the homogeneous solution, the behavior of this particular solution depends on the material properties.

Particular solution for $k_2 = 0$

When k_2 is zero, the constants c' and c'' are non-zero and the system is non-linear, so the theory discussed in the beginning of this chapter for solvability of linear systems is not applicable. The solvability, though, can be shown by finding a solution. Because c does not appear in the interface condition for the continuity of the radial displacements u_r, a solution can be found by setting this constant to zero. This leads to a solution which can be used for all k_2, and is given in the following section.

Particular Solution of $k_2 \neq 0$ and $2k_1 - k_2 = 0$

As discussed in a previous section, if k_2 is not zero, then the constants c' and c'' must be zero, so the particular solution is found by solving the system represented by equation 45 with the forcing function retained. In this case, the matrix in equation 45 becomes:

$$[\mathbf{X}] = \begin{bmatrix} 0 & -1 & 0 & 1 & 0 & 0 & 0 & 0 \\ 2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 2 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & -1 & 0 & -1 \\ -2 & 0 & 0 & 0 & 2 & 0 & 0 & 0 \\ 0 & -2 & 0 & m'-2 & 0 & 2k & 0 & -k(m''-2) \\ 2 & 0 & m' & 0 & -2k & 0 & -km'' & 0 \end{bmatrix}, \tag{98}$$

and the only nonzero term in the function is R_1, which is given by:

$$R_1 = -2\mu' \left(n'\epsilon'_{in0} - n''\epsilon''_{in0} \right). \tag{99}$$

The existence of a solution here depends on the parameter $2k_1 - k_2$. If this parameter is zero, there is a solution vector to the associated homogeneous equation

(equation 93), given by:

$$\{a\} = \left\{ \begin{array}{cccccccc} m' & 0 & m' & 0 & 4-m' & 0 & 2 & 0 \end{array} \right\}. \tag{100}$$

This is not orthogonal to the forcing vector, so a logarithmic singularity is expected.

Solution for $k_2(2k_1 - k_2) \neq 0$

For most materials, the only solution to equation 93 is:

$$\{a\} = \left\{ \begin{array}{cccccccc} 0 & 1 & 0 & -1 & 0 & 1 & 0 & 0 \end{array} \right\}, \tag{101}$$

which is orthogonal to $\{f\}$. Hence, there is no logarithmic singularity and the particular solution to equation 45 is:

$$a' = a'' = c' = c'' = 0 \tag{102}$$

and

$$b' = b'' = d' = d'' = \frac{2\mu'(n''\epsilon''_{in0} - n'\epsilon'_{in0})}{4(k-1) - k_2}. \tag{103}$$

This particular solution provides a mechanism for reducing thermal stresses in bonded structures. For components in a state of plane stress, the stresses can be reduced by matching the the thermal expansion coefficients of the two materials, but for plane strain, the quantity:

$$(1+\nu'')\alpha'' - (1+\nu')\alpha' \tag{104}$$

should be minimized.

Using these constants, the stresses and displacements are found to be:

$$\sigma_{xx} = 0$$

$$
\begin{aligned}
\sigma_{yy} &= \frac{8\mu'(n''\epsilon''_{in0} - n'\epsilon'_{in0})}{2k_1 - k_2} \\
\sigma_{xy} &= 0 \\
u_x &= \left[\frac{n''\epsilon''_{in0} - n'\epsilon'_{in0}}{2k_1 - k_2}(m' - 4) + n'\alpha'\right] x \\
u_y &= \left[\frac{n''\epsilon''_{in0} - n'\epsilon'_{in0}}{2k_1 - k_2}m' + n'\alpha'\right] y
\end{aligned} \tag{105}
$$

This particular solution consists of a uniform tension or compression (on the surfaces parallel to the interface) that is of sufficient magnitude to cause matching displacements in two strips that expand at different rates. As mentioned previously, this mechanism relies on the fact that two strips will generally experience different transverse displacements when they are loaded by equal tension or compression. Hence, the solution breaks down when the lateral displacements of the two strips are equal, i.e., when $2k_1 - k_2 = 0$.

This analysis leaves us with a solution of the form:

$$
\begin{aligned}
\sigma_{rr} &= \sum_{k=1}^{\infty} a_k f_{rr(k)} r^{-(s+2)} + 2b(1 - \cos 2\theta) \\
\sigma_{\theta\theta} &= \sum_{k=1}^{\infty} a_k f_{\theta\theta(k)} r^{-(s+2)} + 2b(1 + \cos 2\theta) \\
\sigma_{r\theta} &= \sum_{k=1}^{\infty} a_k f_{r\theta(k)} r^{-(s+2)} + 2b \sin 2\theta \\
u_r &= \frac{1}{2\mu}\sum_{k=1}^{\infty} a_k f_{u_r(k)} r^{-(s+2)} + \frac{br}{2\mu}(m - 4\cos^2\theta) + n\epsilon_{in0} r \\
u_\theta &= \frac{1}{2\mu}\sum_{k=1}^{\infty} a_k f_{u_\theta(k)} r^{-(s+2)} + \frac{2br}{2\mu}\sin 2\theta
\end{aligned} \tag{106}
$$

where

$$
b = \frac{2\mu'}{2k_1 - k_2}(n''\epsilon''_{in0} - n'\epsilon'_{in0}). \tag{107}
$$

For any value of the series coefficients a_k, this general solution satisfies the equilibrium and compatibility equations in the interior of each of the two strips, the

traction conditions on the free surfaces adjacent to the interface, and the interface conditions.

5.5.3 Particular Solution for Linear Temperature Field

Components which experience uniform surface heating on a surface parallel to the interface will exhibit temperature fields that change linearly with the distance from the interface. To model this, the temperature gradient in each layer is represented by:

$$T = \Delta T \frac{r \sin\theta}{t}, \tag{108}$$

where ΔT, the temperature difference from the interface to the surface of the layer, depends on the surface heat flux and the thermal conductivity of that layer. The particular solution is found by solving the system represented by equation 45 with the forcing function retained. If the temperature field is linear, then the first non-zero term in the forcing function R_1 is zero, so only R_2 remains. Because the function must be independent of s, and R_2 is proportional to $r^{(s+3)}$, the particular solution must have $s = -3$. In this case, the matrix in equation 45 becomes:

$$[\mathbf{X}] = \begin{bmatrix} -1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & -3 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 3 & 0 & -1 \\ 0 & 1 & 0 & 1 & 0 & -1 & 0 & -1 \\ -3 & 0 & -1 & 0 & 3 & 0 & 1 & 0 \\ 0 & -3 & 0 & m'-3 & 0 & 3k & 0 & -k(m''-3) \\ 3 & 0 & m'+1 & 0 & -3k & 0 & -k(m''+1) & 0 \end{bmatrix}, \tag{109}$$

and the only nonzero term in the forcing function is R_2, which is given by:

$$R_2 = \mu' \left(\frac{n'\alpha'}{t'} \Delta T' - \frac{n''\alpha''}{t''} \Delta T'' \right). \tag{110}$$

This forcing vector again suggests ways in which the thermal stresses in bonded structures can be reduced by prudent design. For strips experiencing a constant heat flux, the stresses can by reduced by choosing the material properties of the strip such that R_2 is zero. For a constant heat flux, the temperature gradients in the strips are given by

$$\Delta T = \frac{qt}{k_T}, \tag{111}$$

where q is the heat flux and k_T is the thermal conductivity. Setting R_2 to zero requires:

$$\frac{n'\alpha'}{k'_T} = \frac{n''\alpha''}{k''_T}, \tag{112}$$

so matching the ratio of the expansion coefficient to the thermal conductivity will eliminate the stresses in a body subjected to a uniform heat flux.

As discussed in the beginning of this chapter, a solution to equation 109 always exists if the matrix is non-singular. A root of the determinant of this matrix exists at $s = -3$ if and only if the material properties are such that:

$$2k_1 \pm k_2 = 0. \tag{113}$$

When $2k_1 - k_2 = 0$, the eigenvector of the transpose of the matrix is given by:

$$\{a\} = \left\{ \begin{array}{cccccccc} 0 & m' & 0 & 2k_1 + m' & 3(m'-4) & 0 & -4 & 0 \end{array} \right\}. \tag{114}$$

This vector is orthogonal to the forcing vector $\{f\}$, so a solution exists. Hence, there are no logarithmic singularities for constant thermal gradients of the type described by equation 35. On the other hand, when $2k_1 + k_2 = 0$ the eigenvector of the transpose of the matrix is

$$\{a\} = \left\{ \begin{array}{cccccccc} -3m' & 0 & -3km' & 0 & 0 & m'+4 & 0 & 4 \end{array} \right\}. \tag{115}$$

This vector is not orthogonal to the forcing vector $\{f\}$, so no solution exists and there is a logarithmic singularity.

In situations where there is no logarithmic singularity (*i.e.* when $2k_1 + k_2 \neq 0$), the particular solution is given by:

$$b' = b'' = d' = d'' = 0 \tag{116}$$

and

$$a' = a'' = c' = c'' = \frac{\mu'}{4(k-1)+k2}\left(\frac{n''\alpha''\Delta T''}{t''} - \frac{n'\alpha'\Delta T'}{t'}\right). \tag{117}$$

Using these constants, the stresses and displacements are found to be:

$$\begin{aligned}
\sigma_{xx} &= 0 \\
\sigma_{yy} &= -8ay \\
\sigma_{xy} &= 8ax \\
u_x &= \frac{axy}{\mu}(4-m) + \frac{n\alpha\Delta T xy}{t} \\
u_y &= \frac{a}{2\mu}\left[(4+m)x^2 - my^2\right] + \frac{n\alpha\Delta T}{2t}(y^2 - x^2)
\end{aligned} \tag{118}$$

This particular solution can be added to the homogeneous solution and to the solution for constant temperature change to yield a full solution for stresses in a semi-infinite bonded structure. Collocation techniques can then be used to apply these solutions to finite bodies.

Chapter 6

Solutions for Finite Bodies

The series solution for the stresses and displacements near the edge of the interface between bonded structures satisfies the equilibrium and compatibility conditions in the interior of two bonded structures, and it satisfies the boundary conditions on the free surfaces adjacent to the interface, but it does not satisfy the other boundary conditions. Solutions for finite bodies must account for this deficiency. In this dissertation the model shown in figure 11 is used to explore the thermal stresses in bi-layered structures. Because the thermal field and model geometry are symmetric about $x = l$, only half of the model must be considered, leaving only one singularity to accommodate. Since there is only one singularity, a single series solution, originating where the interface intersects the free surface, can be used to model the stresses throughout the body. The previously unknown coefficients must be determined such that the remaining boundary conditions are satisfied in some approximate manner. As shown in figure 20, the additional stress boundary conditions are zero normal and shear stress on the surfaces parallel to the interface and zero shear stress along the symmetry line. The final boundary condition is that

the displacements in the axial (x) direction are uniform, which can be expressed by:

$$\frac{\partial u_x}{\partial y} = 0, \text{ at } x = l. \tag{119}$$

Putting this quantity into the series form used for the stresses and displacements, one finds:

$$\begin{aligned} \frac{\partial u_x}{\partial y} &= \frac{r^{-(s_k+2)}}{2\mu} \sum_{k=1}^{\infty} \Bigg\{ -(s_k+2) f_{u_r} \sin\theta\cos\theta + \frac{\partial f_{u_r}}{\partial\theta}\cos^2\theta + \\ &\quad \Big[(s_k+2)\sin^2\theta - 1\Big] f_{u_\theta} - \frac{\partial f_{u_\theta}}{\partial\theta}\sin\theta\cos\theta \Bigg\} + \omega_0 \end{aligned} \tag{120}$$

This chapter will focus on the various methods for determining the coefficients in the series to satisfy these additional boundary conditions.

6.1 Point Collocation

The first (and simplest) method for determining the unknown series coefficients is point collocation. This method determines the coefficients such that the boundary and symmetry conditions are satisfied exactly at discrete points, called collocation points. Because there are two boundary conditions at each point along the free surface or symmetry line, there must be two unknowns in the solution for each collocation point if the solution for the coefficients is to be uniquely determined. Hence, if m collocation points are used, the series solution must be truncated at $2m$ terms. This is a drawback of the method, because it forces the use of more terms in the series as more collocation points are used to improve the resolution of the solution. One might imagine that increasing the number of collocation points would always lead to a more accurate solution, but this is not necessarily the case. The magnitude of the n^{th} term in the series increases roughly as r^{n+1}, so it can be quite

Figure 20: Boundary and symmetry conditions in model of finite body. The axial displacement on edge BC is denoted by an undetermined constant (C).

large on the boundary. Hence, the precision required for accurate analysis increases rapidly as more terms are used and the accuracy of the solution will eventually decrease with increasing number of collocation points.

6.2 Least Squares Collocation

An alternative method for determining the coefficients in the series is called, for lack of a better term, least squares collocation, following Wang and Choi [49]. This technique minimizes, in the least squares sense, the integral of the error in the boundary and symmetry conditions along the outside of the symmetric model. This process is begun by defining the following integral:

$$I = \int_{AB} (w_{yy}\sigma_{yy}^2 + w_{xy}\sigma_{xy}^2)dx + \int_{BC} (w_{xy}\sigma_{xy}^2 + \frac{\partial u_x}{\partial y}^2)dy + \int_{CD} (w_{yy}\sigma_{yy}^2 + w_{xy}\sigma_{xy}^2)dx \tag{121}$$

which represents the integral of the errors in the series solution along the top, symmetry line and bottom, respectively. This residual integrand is given by the square of the difference between the prescribed boundary conditions and boundary values calculated from the approximate series solution. In most thermal stress problems the boundaries are traction-free, providing homogeneous boundary conditions. In this case the integrand is given by the stresses and displacements from the series solution alone. The normalization factors w_{ij} are used to non-dimensionalize the terms in this integral, but they also can be adjusted to emphasize a particular boundary condition on a particular side, in order to optimize the calculation of the unknown series coefficients. For this study, these normalization coefficients were taken to be the shear modulus of the associated strip. For a given number of terms in the

series solution, the minimization of the residual integral gives the best available solution for the unknown series coefficients. Inserting the series representations for the stresses and displacements into this integral, including the particular solution, and taking the partial derivative with respect to each coefficient yields the following system for the coefficients (in the case of a uniform temperature change):

$$[M]\{a\} = \{q\}, \tag{122}$$

where

$$\begin{aligned} M_{ij} &= \int_{AB} (f_{yy(i)} f_{yy(j)} + f_{xy(i)} f_{xy(j)}) dx \\ &\quad + \int_{BC} \left(\frac{\partial u_{x(i)}}{\partial y} \frac{\partial u_{x(j)}}{\partial y} + f_{xy(i)} f_{xy(j)} \right) dy \\ &\quad + \int_{CD} (f_{yy(i)} f_{yy(j)} + f_{xy(i)} f_{xy(j)}) dx \end{aligned}$$

and

$$q_i = \int_{AB} \sigma_{yy}^{part} f_{yy(i)} dx + \int_{CD} \sigma_{yy}^{part} f_{yy(i)} dx \tag{123}$$

where

$$f_{yy} = \left[\frac{1}{2}(f_{rr} + f_{r\theta}) - \frac{1}{2}(f_{rr} - f_{r\theta}) \cos 2\theta + f_{r\theta} \sin 2\theta \right] r^{-(s+2)} \tag{124}$$

and

$$f_{xy} = \left[\frac{1}{2}(f_{rr} - f_{r\theta}) \sin 2\theta + f_{r\theta} \cos 2\theta \right] r^{-(s+2)}. \tag{125}$$

For other loading conditions, only the definition of the forcing vector $\{q\}$ changes. Because of the increase of the magnitude of each term in the series with increasing n, the matrix in the above equation is ill-conditioned. Hence, as before, higher precision is required for cases where extra terms are used in the series.

Table 3: Material Properties Used for Comparison of Solution Methods for Finite Bodies

	Top Layer	Bottom Layer
elastic modulus (MPa)	20.69	6.89
expansion coefficient (K^{-1})	13×10^{-6}	6.5×10^{-6}
Poisson's ratio	0.25	0.33
thickness (cm)	5	5

6.3 Comparison of Point and Least Squares Collocation

For a given number of terms in the series solution for the stresses and displacements, the point collocation technique is much faster than the least squares method because it doesn't require integration. The usefulness of the two techniques, though, is another story. The comparison between the two will be made using the material properties and dimensions used by Chen, Cheng, and Gerhardt [40], as given in Table 3. All comparisons will be made assuming plane stress conditions. For these properties, the first ten roots of the determinant are given in Table 4. The first root indicates a stress singularity of the order of 0.071, i.e.

$$\sigma_{ij} \sim r^{-0.071} \quad r \to 0. \tag{126}$$

The point collocation analysis was conducted using an equal number of collocation points on the four boundary segments for which the boundary conditions are not yet satisfied. Therefore, multiples of four collocation points were chosen and multiples of eight terms in the series were necessary to satisfy the two conditions at each point. The least squares collocation approach is studied using Gaussian quadrature (48 Gauss points) to perform the integrals numerically.

Table 4: Roots of Determinant for Properties used in Comparison of Different Solution Methods

Real Part	Imaginary Part
-1.9289152	0
-2.8319658	0.3605005
-3.8066561	0.8169434
-4.8424640	0.9095635
-5.8452727	1.1700548
-6.8696655	1.1849641
-7.8711809	1.3908065
-8.8885541	1.3735112
-9.8890007	1.5530388
-10.902214	1.5177934

The accuracy of these methods is checked in two ways:

1. By comparing the calculated stress fields along the boundary to the required boundary conditions. This is especially important for the point collocation method, because satisfying the boundary conditions exactly at discrete points allows unlimited errors elsewhere.

2. By checking the convergence of the coefficients in the series with increasing number of terms in the series. Because the coefficient of the singular term (if one exists) may be useful in design, its convergence is necessary if a particular technique is to be useful.

Unfortunately, the point collocation method does poorly on both tests. Figure 21 shows the error in the normal stresses on the top surface, as calculated from both the point and least squares collocation methods. The errors shown in this figure are the relative difference between the stress computed by the approximation technique and the boundary condition, which is zero in this case. This error is normalized to the normal stress at the surface, from the particular solution. The top surface

Figure 21: Relative errors in normal stresses along top surface

exhibits, in both cases, the largest discrepancy between the calculated tractions and the traction-free conditions. As shown in the figure, the point collocation method yields surface stresses with enormous oscillations, giving errors which are several orders of magnitude greater than the particular solution. The extent of these errors are not shown on the figure, because anything greater than 100-150% errors are clearly unacceptable. The least squares collocation method fares much better. As shown in figure 21, the error is about 5-7% over most of the surface, and it peaks at about 27%.

The poor performance of the point collocation method is also shown by figure 22, which shows the convergence (or lack thereof) of the first series coefficient with increasing number of terms. While the least squares method converges quite

Figure 22: Convergence of the first term in the series solution for increasing number of retained terms

well, for two different model lengths, the point collocation method shows almost no convergence. This latter point is somewhat disturbing, and it precludes further consideration of this technique. The point collocation method could, conceivably be improved by fortuitous selection of the collocation points, but this would detract from the versatility of the method. Hereafter, only least squares collocation will be considered.

6.4 Results for Least Squares Collocation

6.4.1 Errors on the Boundary

As mentioned in the previous section, the least squares collocation method shows promise as a useful technique because it exhibits good convergence of the stress series coefficients and it provides a reasonable characterization of the boundary conditions. This latter point, is further exhibited in figure 23, which shows the relative error in both the shear and normal stresses along the top surface. This is essentially a magnified plot of the least squares collocation result for the normal stresses from figure 21, with the shear stresses shown as well. The error in the normal stress is on the order of 8% over most of the surface, with a peak of about 27% at $x = 0$, which represents the intersection of the top surface with the free edge. This has been shown previously to be the point of maximum error in a similar study of composites [49]. The peak error in the shear stress is less than 10%. The relative errors on the other surfaces are lower than those on the top surface.

One difficulty with these collocation methods is the added precision required from the computation as more terms are used in the series solution for the stresses and displacements. Because the size of the terms increase roughly as r^{n+1}, for n terms, the matrix is ill-conditioned and high precision is needed for accurate solution of the system. This is shown in figures 24 and 25. The first figure shows the peak error in the normal stress on the top surface as a function of the number of terms in the series solution. The solid line represents the results of double precision calculations on a Cray-II computer (32 bit real number representations), while the circles show the results of double precision on a VAX 8200 (8 bit reals). The higher precision calculations show rapid decrease in the error until about 16 terms are used, then

Figure 23: Relative errors in shear and normal stresses along top surface for least squares collocation method

Figure 24: Dependence of the peak error in the boundary conditions on the number of retained terms in the series for different amounts of precision in the numerical analysis

small improvement out to 64 terms. The lower precision calculations, on the other hand, show identical errors through about 27 terms, then a large error caused by the addition of just one term. A similar result is shown in figure 25, which plots the integral of the squares of the boundary conditions (see equation 121), normalized by the integral of the particular solution. This figure also shows a marked increase in the error of the lower precision calculations past 27 terms. Clearly high precision is required for the calculations, but it is not clear whether a lack of precision can be blamed for the peak error of 27% in the normal stresses at the boundary.

Figure 25: Dependence of the error in the integral of the boundary and symmetry conditions on the number of retained terms in the series for different amounts of precision in the numerical analysis

6.4.2 A Benchmark Problem

As discussed above, the least squares collocation method shows promise as a tool for studying singularities in bonded structures, but its usefulness is suspect considering the errors on the boundary; the method is certainly not useful for calculating stresses in the bulk. The errors on the boundary come from a number of sources, including:

- errors in the determination of the characteristic roots of the determinant, which becomes very important when large numbers of terms are used
- errors in solving the ill-conditioned linear system produced by the minimization of the integral around the boundary
- errors in the quadrature

Wang and Choi [49] used the same collocation technique to study crack-free singularities in anisotropic materials and obtained peak errors on the order of 1% on the boundary, as compared to the 30% in the calculations presented here. The difference, though, is that the anisotropic materials in Wang and Choi's study were identical, with the only difference between the two layers lying in the different fiber orientations. This case admits an infinite number of integer roots, in addition to the complex roots much like those found in the case of dissimilar isotropic materials, so there is less error in determination of the eigenvalues (the integer eigenvalues are exact) and there are twice as many eigenvalues below a certain magnitude. This last point is significant because the precision of a particular computer limits the allowable magnitude of the highest eigenvalue because the stresses are proportional to $r^{-(s+2)}$. Hence, the composite problem provides twice as many terms for a given precision. The reduction of the integral of the tractions along the boundary with an

increasing number of terms in the stress series, and the convergence of the first series coefficient, indicate that the calculations may be reliable. This must be verified with a known solution.

Since the author knows of no analyses of singularities in crack-free, finite, bonded structures composed of isotropic materials, an analytical solution must be used. The benchmark problem chosen to show the usefulness of the least squares collocation method is one of thermal stresses in bonded quarter planes, as studied by Bogy [47] using the Mellin transformation, which is defined for the various field quantities as:

$$\overline{\Phi}(s,\theta) = \int_0^\infty \Phi(r,\theta) r^{s-1} dr, \tag{127}$$

$$\overline{\sigma}(s,\theta) = \int_0^\infty \sigma(r,\theta) r^{s+1} dr, \tag{128}$$

$$\overline{T}(\theta;s) = \int_0^\infty E\alpha T(r,\theta) r^{s+1} dr, \tag{129}$$

and

$$\overline{u}(s,\theta) = \int_0^\infty u(r,\theta) r^{s} dr \tag{130}$$

assuming plane stress. In order to benchmark a code for a rectangular, finite body, consider a half-space problem with a thermal field of the form:

$$T = \begin{cases} T_0 & r < R_0 \\ 0 & r > R_0 \end{cases} \tag{131}$$

which represents a uniform temperature over a semi-circle of radius R_0. The solution of the half-space problem gives the stresses and displacements throughout the bonded quarter-planes. From this half-space, one can extract a rectangle from within the semi-circle of uniform temperature for analysis as a finite body. This finite body will have a uniform temperature change T_0 and surface tractions determined from the half-space solution. The least squares collocation method can

then be used to solve for the boundary-layer stress intensity for this rectangle and compare it to the semi-analytical solution found using the Mellin transform.

Applying the Mellin transform to the field equations presented in the beginning of the previous chapter, and using the transformed boundary conditions, one arrives at the same matrix equation (equation 45) as was found for the solution derived for use in the collocation solution. In this case, though, the solution of the system provides transformed stresses, which are given by (using $\sigma_{\theta\theta}$ as an example):

$$\overline{\sigma}_{\theta\theta}(\theta = 0) = \frac{-(s+1)}{s\|\mathbf{X}\|}\left[Q_3(s)m'\,\overline{T}' + Q_4(s)km''\,\overline{T}''\right] \tag{132}$$

where

$$\begin{aligned} Q_3 &= (1-\cos y)\left[-(k-1)\delta(\alpha\delta-\beta^2)+k_3\beta^2+k_2\alpha\delta\right]+ \\ &\quad \beta\sin y\left[-(k-1)(\alpha\delta-\beta^2)+2km''\alpha\right] \end{aligned} \tag{133}$$

and

$$\begin{aligned} Q_4 &= (1-\cos y)\left[(k-1)\delta(\alpha\delta-\beta^2)+k_3\beta^2-k_2\alpha\delta\right]+ \\ &\quad \beta\sin y\left[(k-1)(\alpha\delta-\beta^2)+2m'\alpha\right]. \end{aligned} \tag{134}$$

Inverting this relation gives the stresses in bonded elastic quarter planes. The Mellin inversion integral is given by:

$$\sigma(r,\theta) = \int_{c-i\infty}^{c+i\infty} \overline{\sigma}(s,\theta) r^{-(s+2)} ds, \tag{135}$$

where c lies in a strip of regularity of the integrand. As mentioned previously, we are concerned only with $s < -1$, so c lies between Re(s)=-1 and the first pole of the integrand associated with the inversion of the transformed stresses.

The transformed thermal field gives:

$$\overline{T} = \frac{E\alpha T_0 R_0^{s+2}}{s+2}, \tag{136}$$

so, as seen from equation 132, there are poles in the transformed azimuthal stress at $s = -2$ and at the roots of the determinant. According to the residue theorem, these poles provide the solution for thermal stress in bonded quarter-planes. Because the determinant in equation 132 is identical to the one derived previously, its roots are identical to those discussed earlier in this chapter. For the case studied here all the poles are simple poles, but the one at $s = -2$ must be treated differently, using L'Hospital's rule.

The pole at s=-2

At $s = -2$ the determinant has a double root, the transformed temperature has a simple pole, and Q_3 and Q_4 have simple roots, so the transformed stress has a simple pole. Using the residue theorem and L'Hospital's theorem, one finds that the residue at $s = -2$ provides:

$$\sigma_{\theta\theta}(\theta = 0) = \frac{8\mu'(\alpha'' - \alpha')T_0}{2k_1 - k_2}, \tag{137}$$

which is identical to the particular solution found in the previous chapter.

The remaining poles

Other than at $s = -2$, the poles of the transformed stresses are simple and the inversion gives the azimuthal stress as:

$$\begin{aligned}\sigma_{\theta\theta}(\theta = 0) &= \sum_{k=1}^{\infty} \left(\frac{-(s_k + 1)}{s_k}\right) \left(\frac{\partial \|\mathbf{X}\|}{\partial s}\right)^{-1}\Bigg|_{s=s_k} \\ &\quad \left[Q_3(s_k) m' \ \overline{T}'\Big|_{s=s_k} + Q_4(s_k) k m'' \ \overline{T}''\Big|_{s=s_k}\right] r^{-(s_k+2)}.\end{aligned} \tag{138}$$

This gives the solution for the azimuthal stresses in bonded quarter-planes as an infinite sum. The solution for the other stresses and displacements can be obtained

Figure 26: Convergence of stresses at a given distance along the interface, as obtained by varying the number of terms obtained from the Mellin inversion

in a similar manner.

Benchmark

To test the usefulness of the collocation technique, the material properties chosen for comparison of the two collocation methods earlier in this chapter were used to solve the half-space problem. The convergence of the series obtained from the Mellin transform inversion is demonstrated in figure 26, which plots the azimuthal interface stress on the symmetry line of the rectangle (at $r = l$) which will be considered for benchmarking the collocation solution. This figure shows that the boundary stresses require only about a dozen terms in the series for accurate representation.

The rectangle extricated from the half-space model was analysed by the collocation technique for comparison. The half-space solution is computed using 16 terms from the Mellin inversion and the number of terms in the collocation solution is varied to test its convergence. The rectangle to be analysed features a uniform temperature change and boundary conditions calculated from the half-space solution. The convergence of the first coefficient of the series with an increasing number of terms in the series is shown in figure 27, which shows the percent error in the collocation solution as compared to the known solution. This figure shows that the error is less than 1% when more than 12 terms are used. The peak errors in the surface stresses, calculated using 9 terms in the series, are roughly 25% of the particular solution, indicating that the first term in the series can be calculated with substantially greater accuracy than the surface stresses. Hence, it appears that the least squares collocation method yields accurate, reliable results for the asymptotic stresses near the edge of the interface.

6.4.3 Dimensional Effects

Given that the least squares collocation technique gives reliable information about the stress singularities in finite bonded structures, the technique can be used to investigate the effects of dimensional changes on the magnitude of such singularities. The effects of changing the model dimensions are shown in figures 28 and 29. The parameters plotted in these figures are (as defined by Wang and Choi [49]) "boundary-layer stress intensity factors", given by:

$$K_{ij} = \lim_{r \to 0} r^{s+2} \sigma_{ij}(r, 0; s). \tag{139}$$

Figure 27: Error in the boundary-layer stress intensity for collocation solution as compared to half-space solution

These boundary-layer stress intensity factors are analogous to the more common stress intensity factors used to represent crack tip stress fields, but it differs because the order of singularity is dependent on the material properties. For interface problems, there are six boundary-layer stress intensity factors, three in each layer, but those associated with the shear and normal stresses in the interface are identical, so there are only four distinct stress intensity factors. Figure 28 shows the dependence of the stress intensities on the ratios of the two strip thicknesses. These stress intensities are normalized to K_0, which is the value of $K_{\theta\theta}$ at $t'/t'' = 1$. For $T = 100°C$, $K_0 = -1.24\text{MPa (m)}^{0.071}$. As seen in this figure, changing the thickness of one layer relative to the other has little effect on the stress intensity until the ratio of the two thicknesses is less than about 0.2. Then the stress intensities drop rapidly to zero, because there are no stresses if one thickness is zero (*i.e.*, if the component is a single layer). Figure 29 shows a similar dependence of the stress intensities on the length-to-thickness ratio. As the length drops, relative to the thicknesses, there is little effect on the stress intensities until the length is less than about six times the thicknesses. As the length decreases further, the stress intensities decrease monotonically to zero.

Figure 28: Dependence of the boundary-layer stress intensity factors on the ratio of the strip thicknesses

Figure 29: Dependence of the boundary-layer stress intensity factors on the length-to-thickness ratio

Chapter 7

Viscoelastic Analysis

7.1 The Viscoelastic Analogy

Viscoelastic materials (including concrete and many polymers, as well as some metals) are materials that undergo time-dependent deformations under constant loading conditions. This is similar to creep, which occurs in metals, particularly at high temperatures. Both viscoelastic and creep models have been studied extensively, using surprisingly disparate methods, but the relationship between the two is not well established [77]. In general the constitutive equation of viscoelastic materials can be described by an equation of the form

$$\epsilon = \frac{1}{E}\left(\sigma + \int_{-\infty}^{t} K(t-\tau)\sigma(\tau)d\tau\right), \tag{140}$$

where $K(t-\tau)$ is a material-dependent kernel representing the viscoelastic constitutive behavior. This form implies that the instantaneous strain rate depends on the stress at time τ and on the time elapsed since the stress was applied, and it is the source of the differences between creep and linear viscoelastic behaviors. In many fusion applications (materials and temperatures), irradiation creep rates dominate

thermal creep rates, so the time dependent behavior is described by [78]

$$\frac{\partial \epsilon}{\partial t} = B\dot{\delta}\sigma^n, \tag{141}$$

where B and n are constants, and $\dot{\delta}$ is the dose rate (dpa/year). Because this type of creep rate equation makes the kernel in equation 140 independent of t and τ, the linear viscoelastic theory can be used to study the effects of irradiation creep in fusion machines. In addition, the creep rate B is, for most metals, independent of temperature, thus simplifying the analysis considerably.

In order to determine the stresses and strains in a viscoelastic body, the constitutive (stress-strain) relations must be modified to account for the time-dependent deformations. In general these relations are written:

$$P_1(D)s_{ij} = P_2(D)e_{ij} \tag{142}$$

and

$$P_3(D)\sigma_{ii} = P_4(D)(\epsilon_{ii} - 3\epsilon_{in}), \tag{143}$$

where

$$P_k(D) = \sum_{n=1}^{\infty} C_{kn}\frac{\partial^n}{\partial}. \tag{144}$$

A common solution method for these types of problems is to take the Laplace transform of the viscoelastic equations and compare the resulting set of equations to the steady-state formulation. Because the bulk behavior is often different from the shear behavior in viscoelastic materials, the stress-strain relations are usually written in terms of the stress and strain deviators s_{ij} and e_{ij}, defined as:

$$s_{ij} = \sigma_{ij} - \frac{\sigma_{ll}}{3} \tag{145}$$

and

$$e_{ij} = \epsilon_{ij} - \frac{\epsilon_{ll}}{3}. \tag{146}$$

In terms of these quantities, the stress-strain relations are

$$s_{ij} = 2\mu e_{ij} \tag{147}$$

and

$$\sigma_{ll} = 3\kappa\epsilon_{ll}, \tag{148}$$

where κ is the bulk modulus. Transforming equations 142 and 143 gives

$$\widetilde{s}_{ij} = \frac{P_2(p)}{P_1(p)}\widetilde{e}_{ij} \tag{149}$$

and

$$\widetilde{\sigma}_{ii} = \frac{P_4(p)}{P_3(p)}(\widetilde{\epsilon}_{ii} - 3\widetilde{\epsilon_{in}}), \tag{150}$$

where the Laplace transform of a function f is denoted by $\widetilde{f}$. By comparing the elastic and viscoelastic constitutive equations, it is apparent that the solution of the viscoelastic problem in the Laplace domain is equivalent to the solution of the steady-state problem, with the elastic properties 2μ and 3κ replaced by $P_2(p)/P_1(p)$ and $P_4(p)/P_3(p)$ respectively. The time-dependent behavior of the viscoelastic problem is thus recovered by substituting the equivalent transformed properties into the steady-state elastic solution and inverting the Laplace transform.

7.2 Laplace Transform Inversion

The analysis of bonded structures composed of viscoelastic materials is difficult, even with the use of the analogy described in the previous section. The substitution of functions of the Laplace parameter p for the material properties in equations such as equation 88 leads to very complicated expressions for the stresses in the Laplace domain and analytical inversion is not possible. The primary difficulty is the fact

that the eigenvalues s_k, which are functions of the Laplace parameter p, are not known explicitly because they are determined by solving a transcendental equation. Hence, numerical inversion is necessary. Unfortunately, numerical inversion of the Laplace transform is difficult because the operator's inherent unboundedness prevents explicit error control. Because a small change in the transformed function can lead to an arbitrarily large change in the real function, high precision is needed to obtain accurate results.

The method adopted for this work solves the Laplace transform integral definition as an integral equation, with f(t) as the unknown, using Gaussian quadrature. Other methods, such as those suggested by Miller and Guy [79] and Papoulis [80], may be used if the accuracy of the quadrature method is insufficient.

The quadrature method works for any function for which the transformed function $\tilde{f}(p)$ is known (and finite) on the real axis. The first step is to transform the integral into one with finite limits, then approximate the integral as a finite sum, using N^{th} order Gaussian quadrature. This can be evaluated for N arbitrary values of the Laplace parameter p, giving a linear system represented by

$$\widetilde{f}(p_k) = \frac{1}{2}\sum_{i=0}^{N} w_i \left(\frac{1+\tau_i}{2}\right)^{p_k-1} g(\tau_i). \tag{151}$$

This gives a system of N equations for the N unknowns $g(\tau_i)$, which represent values of the unknown function $g(\tau)$ at discrete locations τ_i. The solution from this point is trivial.

The accuracy of this method for numerical inversion depends on the order of the quadrature method and on where the p_k's are chosen. Figure 30 shows the comparison of analytical and numerical results of inversion of $\widetilde{f}(p) = 1/(p+1)$ using N=15 and $p_k = k$. The agreement is excellent, so these parameters should be useful

Figure 30: Analytical and numerical results for the Laplace Transform inversion of 1/(p+1)

for inverting functions similar to $1/(p+1)$, such as those that result from the analysis of certain viscoelastic materials, known as Maxwell materials.

7.3 Time Dependent Stresses in Maxwell Materials

In order to investigate the viscoelastic behavior of bonded viscoelastic quarter-planes, the constitutive equations for Maxwell materials will be considered. There

are many other, more sophisticated viscoelastic models available, but Maxwell materials capture the accepted behavior of irradiation creep in most materials (equation 141). For Maxwell materials, the stress-strain equations are:

$$\frac{\partial s_{ij}}{\partial t} + \frac{s_{ij}}{\tau_0} = 2\mu \frac{\partial \epsilon_{ij}}{\partial t} \tag{152}$$

and

$$\sigma_{ii} = 3\kappa(\epsilon_{ii} - 3\epsilon_{in}). \tag{153}$$

Hence, by comparison with equations 142 and 143,

$$\begin{array}{ll} P_1(D) = \frac{\partial}{\partial t} + \frac{1}{\tau_0} & P_2(D) = 2\mu\frac{\partial}{\partial t} \\ P_3(D) = 1 & P_4(D) = \kappa. \end{array} \tag{154}$$

By comparing equations 141 and 152, one can determine the relationship between the creep coefficient B and the decay constant τ_0, *i.e.*

$$\tau_0 = \frac{1}{2\mu B}, \tag{155}$$

where τ_0 is given in units of irradiation damage (dpa). Taking the Laplace transform of the operators in equation 154 and substituting into equation 149 yields the following equivalencies between the elastic material properties and the viscoelastic "properties" in the Laplace domain:

$$2\mu \rightarrow \frac{2\mu p}{p + \frac{1}{\tau_0}} \tag{156}$$

and

$$\kappa \rightarrow \kappa. \tag{157}$$

This represents a material for which the bulk behavior is elastic, and the stress decays exponentially (with a decay constant of τ_0) for a uniaxial fixed-grip test.

Substituting equations 156 and 157 into the elastic stress solutions represented by equation 88 yields expressions for the stresses in the Laplace domain. This is then inverted numerically to obtain the time-dependent behavior of duplex structures.

7.4 Viscoelastic Stresses in Bonded Materials

7.4.1 Solution Method

In order to solve for the time dependent stress intensity factors in bonded structures using the viscoelastic analogy, one begins by solving for their Laplace transforms. This is done by using the effective material properties,

$$\widetilde{\mu} = \frac{\mu p}{p + \frac{1}{\tau_0}} \tag{158}$$

and

$$\widetilde{\nu} = \frac{1}{2}\left[\frac{3\kappa(p + \frac{1}{\tau_0}) - 2p\mu}{3\kappa(p + \frac{1}{\tau_0}) + p\mu}\right]. \tag{159}$$

If the thermal and swelling strains are given by the following equation:

$$\epsilon_{in} = C_0 + C_1 t \tag{160}$$

C_0 and C_1 are constants, then the transform of these strains can be written:

$$\widetilde{\epsilon_{in}} = \frac{1}{p}\left(C_0 + \frac{C_1}{p}\right). \tag{161}$$

These transformed properties and loadings can be used in the elastic analyses provided in the previous chapter to determine the transform of the boundary-layer stress intensities. This can then be inverted to yield the time-dependent stresses.

If the characteristic values of the matrix in equation 45 are independent of time, then the time dependence of the solution depends only on the time dependence of

the particular solution, which, for plane stress and a uniform temperature change can be written (see equation 103):

$$\widetilde{\sigma_{yy}} = \frac{\widetilde{\epsilon''_{in}} - \widetilde{\epsilon'_{in}}}{\frac{\widetilde{\nu''}}{\widetilde{E''}} - \frac{\widetilde{\nu'}}{\widetilde{E'}}}, \tag{162}$$

or, in terms of the Laplace parameter (for $\tau'_0 = \tau''_0 = \tau_0$),

$$\widetilde{\sigma_{yy}} = 8\left(\frac{\mu'\mu''}{\mu' - \mu''}\right)\left[\frac{(C''_0 - C'_0)}{p + \frac{4}{3\tau_0}} + \frac{(C''_1 - C'_1)}{p\left(p + \frac{3}{4\tau_0}\right)}\right]. \tag{163}$$

Inversion of this equation gives the following time dependent behavior for the particular solution:

$$\sigma_{yy} = 8\left(\frac{\mu'\mu''}{\mu' - \mu''}\right)\left[(C''_0 - C'_0)e^{-\frac{4t}{3\tau_0}} + \frac{4}{3}\tau_0(C''_1 - C'_1)\left(1 - e^{-\frac{4t}{3\tau_0}}\right)\right]. \tag{164}$$

Hence, the initial stress, represented by C_0, decays exponentially at a rate governed by the creep constant τ_0, and the time dependent loading, which is typical of swelling phenomena and is represented by C_1, leads to a steady state stress proportional to $\tau_0 C_1$. Hence, if the characteristic values are independent of time, swelling that is linear in time leads to steady state stresses that are proportional to the swelling rate and inversely proportional to the creep rate $1/\tau_0$.

In reality, though, the characteristic values, and therefore the order of singularity for the stresses near the edge of the interface, are a function of time. This is dealt with in the following section.

7.4.2 The Order of the Singularity

One of the unique aspects of the analysis of perfectly bonded viscoelastic materials is the fact that the order of singularity is a function of time. This occurs because, contrary to crack problems, the order is material dependent and the viscoelastic

problem can be considered one of changing material properties. As an example, the transform of one of the material parameters, k, can be written:

$$\widetilde{k} = \frac{\mu'}{\mu''}\left(\frac{p + \frac{1}{\tau_0''}}{p + \frac{1}{\tau_0'}}\right). \tag{165}$$

As the Laplace parameter p ranges from 0 to ∞, k can take on virtually any value, depending on the creep constants of the two materials. This can be shown with the use of the limit theorems of the Laplace transform, giving the final value of k (at $t \to \infty$) as:

$$k_\infty = \frac{\mu'\tau_0'}{\mu''\tau_0''}. \tag{166}$$

Therefore, depending on the ratios of the creep constants of the two materials, k can take on any value during the life of the component. As shown in the previous chapter, this parameter can have a profound effect on the order of singularity. Figure 14 shows the dependence of the order of singularity on the ratio of shear moduli k for constant Poisson's ratios. Assuming that Poisson's ratio was constant, one could follow the curve in this figure as k changes with the Laplace parameter to determine the range of the the order of singularity over the life of the structure. Depending on the starting point ($p \to \infty$) and the ending point ($p \to 0$), the order can exhibit a number of different behaviors, as shown schematically in figure 31. If k was very large initially, but much less than 1 at the end of life[1], there could be singularity which disappeared as $\widetilde{k}$ approached 1, and then reappeared as this ratio became much less than one. Because Poisson's ratio is not independent of time, this brief discussion does not give the full picture, but the same behaviors are still possible. Generally, though, the order of singularity changes little during the life of a component composed of "real" materials. This will be shown in the following chapter.

[1]This case occurs when one material is much stiffer than the other, but it creeps much faster.

Figure 31: Schematic of possible time dependencies of the order of singularity for a structure with or without an initial singularity

Chapter 8

Fusion Applications

8.1 Materials

8.1.1 Choice of Materials

The materials generally considered for use in duplex fusion components are described briefly in chapter 2. For this study, two substrate materials and two surface materials have been chosen to study the impact of interface singularities in fusion components. The two substrate materials are vanadium and copper; vanadium was picked for its resistance to swelling and its relatively good high-temperature strength and copper for its high thermal conductivity. The surface materials are tungsten, a high-Z material, and graphite, a low-Z material. Because graphite has a relatively short lifetime (due to radiation damage), it is not, in its present state, a viable candidate for a steady-state, commercial fusion reactor. It can be used for near-term machines, though, because they will not accumulate nearly as much radiation damage as a commercial reactor. NET, for instance, is expected to accumulate a lifetime radiation dose of only 2.5 MWy/m^2[81], which is less than the expected lifetime of most graphites, as will be shown in a following section. Hence,

one material combination chosen for this study is graphite on copper, which can be considered a near-term choice. For long term machines, the material combination chosen is tungsten on vanadium. These two combinations are certainly not the only possible choices for either near- or far-term machines, but they are representative of the spectrum of choices and will be used to demonstrate the problems that interface stresses and edge singularities can present.

8.1.2 Alloys

Each of the materials considered here are used in a number of different alloys and forms. Some of these variants are discussed in this section. In many cases, a particular alloy is chosen over another because of its resistance to radiation damage. The criteria which govern the various choices will be mentioned in the following discussions, but the radiation effects themselves will not be discussed in detail until later in this chapter.

COPPER

Some of the copper alloys considered for fusion are:

- pure copper
- OFHC, (Oxygen-Free, High Conductivity copper)
- CuBe (precipitation hardened)
- MZC (precipitation hardened)
- AMZIRC (precipitation hardened)

- Glidcop Al-20 (dispersion hardened)

The Cu-Be alloy is chosen because of its low (or non-existent) swelling [82]. This choice is consistent with the choice for the INTOR design [10].

GRAPHITE

Graphites can be divided into three different categories:

1. highly anisotropic (like pyrolitic graphite)
2. near-isotropic
3. highly isotropic

Pyrolitic graphites are materials in which the hexagonal crystals are all aligned. This leads to thermal expansion coefficients and swelling rates which are quite different in different directions. The other two graphites have increasingly finer grains and higher strengths. The highly isotropic graphite (POCO Graphite Corporation's grade AXF-5Q, for example) is chosen for this study because of its low swelling and high strength[83].

VANADIUM

The vanadium alloys receiving the most consideration for fusion applications are V-15Cr-5Ti and V-3Ti-1Si. The former of these alloys has a larger database than the latter, but the V-3Ti-1Si alloy seems to be a more attractive candidate for fusion (as demonstrated by its use in the TITAN study [12]). This is primarily due to the reduced susceptibility to helium embrittlement induced by radiation damage [84]. The alloy of choice for this study is the V-3Ti-1Si.

Table 5: Material Properties Used for Fusion Applications

	Graphite[83]	CuBe[86]	V-3Ti-1Si[87, 86]	W-3Re[85]
elastic modulus (GPa)	11	116	118	430
expansion coefficient (K^{-1})	7.5×10^{-6}	17×10^{-6}	10×10^{-6}	4.8×10^{-6}
Poisson's ratio	0.11	0.34	0.36	0.36
thermal conductivity (W/m/K)	110	135	28	67
melting point (°C)	–	1100	1900	3400

TUNGSTEN

The choices for tungsten are more limited than those for vanadium or copper. In general the choice to be made is between either pure tungsten or a tungsten-rhenium alloy. Pure tungsten has a large ductile-to-brittle-transition (DBTT) temperature (100-400° C [85]), so it tends to be brittle at room temperature. This can be a problem for duplex structures because the residual stresses generated by the fabrication process and those that occur during shutdown after significant creep relaxation has occurred, can cause failure in the brittle surface material. Hence, rhenium is often added to the pure tungsten because it decreases the DBTT [85]. Unfortunately, rhenium causes long-term activation problems, so W-5Re, which has relatively little rhenium content, is chosen. This choice is consistent with the one of the choices of the NET team [33].

8.1.3 Unirradiated Properties

The materials considered in this dissertation are all assumed to be homogeneous and isotropic. Hence, the important material properties for this study are Young's modulus E, Poisson's ratio ν, the thermal conductivity k_T, and the thermal expansion coefficient α. The unirradiated properties used in this dissertation are given in Table 5.

8.1.4 Radiation Effects

SUMMARY OF IMPORTANT EFFECTS

hardening and embrittlement[78] Many metals are susceptible to both hardening and embrittlement by irradiation. Hardening is an increase in the yield and ultimate stresses as a function of radiation dose, while embrittlement is a decrease in the amount of plastic deformation that occurs before fracture. Hardening is generally attributed to defects such as vacancies, interstitials, impurities (by transmutation), voids and precipitates. These defects increase the stress required to start dislocation motion and they impede subsequent dislocation movement, thus increasing the stresses required to induce yielding. Embrittlement, on the other hand, is generally caused by the production of helium by (n,α) reactions with the constituents of the metal. This helium, which is generally found in the form of bubbles on the grain boundary, is thought to lead to intergranular fracture as the bubbles interconnect.

Hardening can be included in this type of analysis once the associated changes in the elastic properties are known. Also, both the hardening and embrittlement can be related to the allowable boundary-layer stress intensity factor for the initiation of failure. Since the hardening data are limited, and no data for the critical boundary-layer stress intensity are available, these effects will not be included in this study.

void swelling [78] Most metals irradiated by neutrons exhibit density decreases brought about by the formation of voids within the material. The voids are formed by a sequence of events, beginning with the production of a vacancy-interstitial pair by the collision of the neutron with a lattice atom. Often, the interstitial is absorbed by a sink, such as a dislocation, so the vacancies are essentially free to combine. Under certain conditions, the vacancies can form a large number of voids,

causing a decrease in the density. The associated volume increase can be more than 10-100%, which is much greater than the 0.1% increase associated with the thermal expansion of a 100 °C temperature increase in a typical steel. Needless to say, this kind of deformation can lead to very large stresses in a bonded structure.

The temperature dependence of swelling is quite important. Most metals exhibit a bell-shaped curve on a swelling vs. temperature plot, as illustrated in figure 32. At low temperatures, there is very little diffusion, so the vacancies are not free to move and voids do not form. At high temperatures, the thermal emission of vacancies from the voids is quite high, so the voids do not grow. Therefore, the swelling peaks at an intermediate temperature, usually in the neighborhood of 400-600 °C in steels. This temperature dependence can be a problem for bonded structures, even if the two materials swell at similar rates, because temperature gradients in the structure can put different layers on different points of the swelling curve. The resulting differential expansion creates the same problems as a thermal problem in which the expansion coefficients are not matched.

irradiation creep Neutron irradiation of a metal or ceramic can affect creep in two ways. Irradiation-induced creep refers to the development of creep at temperatures and stresses for which thermal creep would not otherwise occur, while irradiation-enhanced creep refer to the augmentation of thermal creep by irradiation. Only irradiation-induced creep will be considered here.

Irradiation creep in swelling materials is generally ascribed to two phenomena, the stress orientation of dislocation loops and accelerated dislocation climb followed by glide [78]. In most materials irradiation-induced creep can be represented by the

Figure 32: Temperature dependence of swelling for a typical steel irradiated to a fluence of $5 \times 10^{22} n/cm^2$

following equation [88]:

$$\dot{\epsilon_c} = (B\sigma + C\sigma^3)\phi + D\sigma\dot{S} \tag{167}$$

where B, C, and D are constants, ϕ is the neutron flux, and $\dot{S}$ is the swelling rate. Except at high stresses, the second term in this equation is small as compared to the first, so the creep rate is given approximately by:

$$\dot{\epsilon_c} = (B\phi + D\dot{S})\sigma. \tag{168}$$

Hence, the steady-state creep rate is linear in the stress. Many irradiation creep theories, considering both stress-oriented dislocation loops and climb-controlled glide mechanisms, predict creep rate constants that are independent of temperature. Hence, considering the lack of data for the materials discussed in this dissertation, the creep coefficients will be assumed to be independent of temperature.

As indicated in equation 167, there is a connection between swelling and creep rates. Gittus [88] states that this swelling rate term in the creep rate equation can, in many cases, be quite large compared with the other terms. In addition, Olander [78] hypothesizes a climb-controlled glide mechanism in which the irradiation creep rate is proportional to the swelling rate. Because of these theories and the lack of creep data for this study, it is assumed that materials which show little swelling will also exhibit relatively low creep rates.

COPPER-BERYLLIUM

void swelling Pure copper is prone to high swelling rates, with measured values of up to 7% after only 15 dpa[89][1]. This is clearly undesirable, but the addition of

[1]The dpa (displacement per atom) is a measure of the radiation damage which has occurred in a material. It refers to the number of times (on average) that each lattice atom is displaced from its original lattice site.

beryllium to copper reduces the swelling by forming CuBe precipitates and enhancing diffusion[82]. Copper-beryllium alloys are actually found to densify, at a rate of up to 0.66% per 16 dpa at 450 °C [82].

irradiation creep As mentioned previously, there appears to be a correlation between swelling and creep. Therefore, it is assumed that there is negligible irradiation creep in copper-beryllium alloys because they exhibit extremely small swelling rates. This should be verified by experimentation.

property changes The elastic property changes in copper alloys are not well understood, but the effect of irradiation on the thermal conductivity is known. The conductivity of pure copper and many of its alloys is reduced because of the voids produced by irradiation and because of transmutations (primarily nickel and zinc). This reduction can be as much as 19% of the unirradiated value after 16 dpa at 385 °C. Fortunately, beryllium again prevents this undesirable property change. No conductivity change is expected in the Cu-Be alloys [82].

ISOTROPIC GRAPHITE

void swelling The swelling of graphite is quite different from metals [90]. Under irradiation the hexagonal crystals tend to grow perpendicular to the basal plane and shrink within this plane. In the initial stages of irradiation, the growth of the crystals along the basal plane tends to fill voids in the material produced by the fabrication process, while the transverse shrinkage tends to yield an overall shrinkage of the material. Therefore, the material densifies. When the voids are filled, the growth along the basal plane dominates the process and the graphite begins to swell rapidly. This unique behavior is illustrated in figure 33, which shows the swelling

Figure 33: Swelling of graphite[86]

of graphite as a function of neutron fluence. This densification is actually desirable, because the Cu-Be substrate chosen for this material combination also densifies, so their expansions will, in some sense, match. For this study, the initial shrinkage rate is taken to be 0.2 %/dpa.

It is generally believed[91] that the end-of-life of a graphite component is reached when the density returns to its original value. At this point the pores generated by the radiation damage in the crystals interconnect to such a degree that the graphite all but disintegrates. This leads to lifetimes shorter than a year[2], as shown in Table 6. As indicated in this table, the graphite temperature should be kept as low as possible.

[2]A typical plasma-facing component will be subjected to ∼50 dpa/year.

Table 6: Lifetime of Graphite for Different Temperatures

Temperature (°C)	lifetime (dpa)
400	37
600	23
800	13
1000	8
1200	14

irradiation creep Like the copper-beryllium alloys, it is assumed that graphite exhibits negligible irradiation creep rates.

property changes The elastic properties of isotropic graphites are relatively unchanged by irradiation [83], but the thermal conductivity is degraded significantly by the damage. This leads to an increasing temperature gradient with irradiation. The conductivity is assumed to drop linearly to one quarter of its unirradiated value by the end-of-life [91].

V-3Ti-1Si

void swelling The swelling rate of pure vanadium can be quite high ($\sim$0.05 % /dpa), but the addition of titanium decreases this rate considerably by enhancing point-defect recombination at titanium precipitates. Braski [84] has measured the swelling rate in V-3Ti-1Si to be about 0.002 % /dpa at 420 °C with an implanted helium level of 82 appm. The helium is implanted to simulate the helium that will be produced by the (n,α) reactions and will stabilize the voids that lead to swelling. The Blanket Comparison and Selection Study (BCSS) [29] estimates that swelling rates of this order will occur until about 175 dpa, after which the swelling rate will increase to about 1% /dpa. There are no data to substantiate this claim.

Figure 34: Temperature dependence of the swelling of TZM (E_n >0.1 MeV) [92]

There also are no data on the temperature dependence of swelling in vanadium, so this information will be taken from other refractory metals. The temperature dependence of swelling in TZM, a molybdenum-based alloy, is shown in figure 34 [92]. The peak swelling temperature of both tungsten and V-3Ti-1Si will be assumed to occur at the same homologous temperature T_h[3] as the TZM. The homologous temperature for the peak swelling of TZM is 0.3. This is translated into peak swelling temperatures of 660 °C for the vanadium alloy and 840 °C for the tungsten.

irradiation creep As with the materials discussed above, there are no irradiation creep data for any vanadium alloys. Fortunately, there are some data for another

[3]The homologous temperature is defined as the ratio of a given temperature to the melting temperature of the solid.

Table 7: Swelling and Creep Rates of Several Titanium Alloys

Alloy	Swelling ($5 \times 10^{22} n/cm^2$) %	Creep Rate (B) $10^{-27} MPa^{-1}(n/cm^2)^{-1}$
Ti-6242S	0.27	1.9
Ti-5621S	.66	0.9
Ti-64 Mill Anneal	1.50	4.2
Ti-64 Duplex Anneal	3.52	6.5
Ti-64 Beta Anneal	4.85	3.8

refractory alloy: titanium. Therefore, it will be assumed that the irradiation creep rates for both the V-3Ti- 1Si and tungsten-rhenium alloys is identical to that of titanium. Nygren [93] has determined the irradiation creep rates of several titanium alloys up to fluences of 4×10^{21} n/cm^2 at 450°C. Using the equation

$$\dot{\epsilon_c} = B\phi\sigma, \tag{169}$$

where $\dot{\epsilon_c}$ is the irradiation creep rate, ϕ is the neutron flux, and σ is the stress, he found the creep coefficients B to vary from 0.9 to 6.9 $\times 10^{-27}$MPa^{-1}(n/cm^2)$^{-1}$. This is a wide range, but the swelling rates of these materials also vary widely [94], with higher swelling rates corresponding to higher creep rates. This is shown in Table 7. These alloys seem to fall into two distinct groups, one with high swelling and creep rates and another with lower rates. Since the swelling of vanadium is expected to be low, it is assumed to behave like the low-rate group. Hence, an average value of 1.5×10^{-27}MPa^{-1}(n/cm^2)$^{-1}$ will be used for this study. The sensitivity of this value will be studied by varying the creep rates of both the tungsten and the vanadium. The chosen value is equivalent to a creep rate of roughly 2×10^{-6}/MPa/dpa.

property changes Other than the strength and ductility, the elastic and thermal properties of V-3Ti-1Si are not expected to be changed by irradiation.

TUNGSTEN

It is well known that the irradiation of tungsten increases its DBTT. One experiment [95] indicates an increase of about 200°C after a dose of about 5 dpa at 371 °C. Hence irradiated tungsten will be brittle at temperatures well above room temperature.

Unfortunately, no data exist for other behaviors, such as swelling and irradiation creep. Hence, tungsten-rhenium alloys will be assumed to behave like vanadium alloys, with corrections made for the different melting temperatures (through the homologous temperature).

8.1.5 Uncertainties

It is apparent from the previous discussions of irradiated behavior that the database for the damage of the candidate materials for plasma-facing components is very sparse. The data for the property changes and swelling rates are limited to a few data points from a few experiments, while there are no data for the irradiation creep rates of any of the materials. There also are no data for the swelling of tungsten or its alloys. This, of course, leads to major uncertainties in the behaviors assumed for this study.

Uncertainties in structural problems can be dealt with in a number of ways, including response surfaces and Monte Carlo methods [96]. One such study, by Blanchard and Ghoniem [97], used a Monte Carlo technique to determine the effects of uncertainties in swelling and creep rates on the lifetime of a simple, tubular fusion blanket and found them to be quite significant. This study, though, dealt only with ferritic steels, which have a relatively extensive database of irradiated behaviors.

Because the materials considered in this dissertation have little or no data, the aforementioned techniques of uncertainty analysis have little meaning. One must be at least mildly aware of the expected distribution of material properties and behaviors before such analyses are useful. Hopefully, this dissertation will motivate the experimentation necessary to provide this information.

8.1.6 Bonding

Brazing is the bonding method of choice for several existing and proposed fusion components, including those in ASDEX Upgrade [19], NET [25], and FER [31]. Brazing is accomplished by heating the two pieces that are to be bonded to above 450°C, wetting the surfaces with a molten filler metal, then cooling to room temperature [98]. The advantage of brazing over welding is that the base metals needn't melt, so dissimilar materials with vastly different melting temperatures can be brazed. Hence, the braze temperature lies below the melting temperatures of either base metals. Based on data given by Schwartz [98], the following braze temperatures have been assumed for the two material combination considered in this dissertation:

- Graphite on Copper : $T_{braze} = 800°C$
- Tungsten on Vanadium : $T_{braze} = 1000°C$

8.2 Stress Intensities in Graphite-Copper Duplexes

In this section, the boundary layer stress intensity factors for a graphite-on-copper component are calculated for the fabrication, start-up, and full power operation under typical reactor conditions. The component is assumed to consist of a 5 mm

thick graphite tile on a 5 mm thick Cu-Be substrate, and the length of each is taken to be 40 mm. Plane strain conditions are assumed. This idealization is not a model of an actual divertor or limiter, but it should represent the influence of various loadings on the crack-free stress intensities near the edge of the interface in such components. The peak, full-power heat flux is taken to be 5 MW/m^2. The order of the stress singularity for this duplex is 0.07.

8.2.1 Fabrication

The tortuous history that a fusion component experiences begins with its fabrication. For a brazed part, fabrication takes place at high temperatures, leading to differential contraction during cool-down if the constituents have different thermal expansion coefficients. As the component is cooled from its brazing temperature, the filler metal will freeze at some point, thus locking the interface and causing thermal stresses as the part cools further. For the copper-graphite component described in the previous sections, this lockup is assumed to occur at 700 °C, a difference of 100 °C from the brazing temperature. The stresses caused by this process can be calculated from the model developed for this dissertation by considering a uniform temperature change of -670 °C, assuming that room temperature is 30 °C. This leads to the following boundary layer stress intensity factors:

$$\begin{aligned} K'_{rr} &= 136\ MPa(m)^{0.07} \\ K''_{rr} &= -49\ MPa(m)^{0.07} \\ K_{r\theta} &= -52\ MPa(m)^{0.07} \\ K_{\theta\theta} &= -764\ MPa(m)^{0.07} \end{aligned} \tag{170}$$

The azimuthal stress intensity $K_{\theta\theta}$ is negative here indicating that the "tear-away" stress (the stress normal to the interface) at the edge of the interface is compressive, thus, perhaps, preventing delamination. Also, the magnitude of this boundary-layer stress intensity than any of the other four. This is insignificant, though, because the ratios of the different stress intensities depends only on the materials involved, so this would be automatically accounted for by any measurements of the critical boundary-layer stress intensity factors. Experiments must be conducted to determine whether these stress intensities are sufficiently high to cause crack initiation or failure.

8.2.2 Start-Up

The start-up of a fusion machine consists of several steps. First, the whole machine is heated fairly uniformly to a temperature near the operating temperature for full power operation. This brings the component nearer to the brazing temperature, so, assuming that the residual stresses resulting from the fabrication process have not relaxed, this initial heating will actually reduce the stresses in the component. In this case, heating to an equilibrium temperature of 300 °C would reduce the residual stresses by more than 50% (actually, by 400/670).

After bringing the reactor to a uniform temperature near the operating temperature, the reactor power is ramped up to full power, pausing at different levels for any required testing. When the power is initiated, the associated heat flux sets up thermal gradients in the plasma-facing components, thus introducing an additional source of stresses. For a copper surface temperature of 300 °C, the 5 MW/m^2 heat flux, pictured in figure 35, leads to an interface temperature T_i of 485 °C and a graphite surface temperature T_s of 715 °C. This gradient, on top of the stresses remaining from the fabrication and uniform heating, leads to the following stress

Figure 35: Model for analysis of stress intensities in fusion components

intensities:

$$\begin{aligned} K'_{rr} &= 74\ MPa(m)^{0.07} \\ K''_{rr} &= -27\ MPa(m)^{0.07} \\ K_{r\theta} &= -28\ MPa(m)^{0.07} \\ K_{\theta\theta} &= -412\ MPa(m)^{0.07} \end{aligned} \tag{171}$$

Still, the tear-away stress is compressive and is dominated by the residual stresses caused by fabrication.

8.2.3 Radiation Effects

For high heat flux components, the most critical aspect of full power operation is the radiation damage. In the copper graphite combination, this entails the degradation of the thermal conductivity of the graphite and the swelling (or, rather, the densification) of both materials. The decreasing graphite thermal conductivity leads to an increasing thermal gradient in the top layer, so the surface temperature increases with the dose. This is shown in figure 36, which shows the surface temperature and swelling as a function of dose. As shown in this figure, the surface temperature goes from a low of 715 °C to a high of 1053 °C, but the interface temperature doesn't change because the thermal conductivity of the copper is not affected by the damage. The percent volume change caused by the damage is also shown in figure 36, which shows that the graphite initially densifies much quicker than the copper. After about 15 dpa, though, the graphite volume begins to increase, while the copper-beryllium alloy continues to densify. The graphite is assumed to fail at about 24 dpa, where its density returns to its original value.

Because the damage behavior of the graphite is highly nonlinear, the resulting boundary-layer stress intensities vary significantly as a function of the dose. This is shown in figure 37, which plots the four stress intensities as a function of radiation dose. The largest of the four, $K_{\theta\theta}$, begins at about half its peak value, changes sign at about 5 dpa and again at about 19 dpa. This stress intensity reaches it peak magnitude at the end-of-life dose of 24 dpa. Failure could occur before this time, if the allowable stress intensity is degraded by the damage, or if tensile tear-away stresses are more severe than compressive values of equal magnitude.

Figure 36: Surface temperature and swelling of both materials for the copper-graphite component

Figure 37: Boundary-layer stress intensities in copper-graphite component during full power operation

8.2.4 Discussion

The time-dependent boundary-layer stress intensity $K_{\theta\theta}$ for the copper-graphite component is shown in figure 38, which includes the fabrication and startup responses. During the cool-down from the brazing temperature, the copper shrinks faster than the graphite and the resulting stress intensity is negative. The subsequent uniform heating prior to start-up reduces the magnitude of this residual stress intensity as shown, while the heat fluxes associated with the onset of power causes a slight increase. As the radiation damage begins, the graphite densifies faster than the copper alloy, so the magnitude of the stress intensity is reduced because the graphite had shrunk slower during the cool-down from the braze temperature. This phenomenon causes the stress intensity to become positive, until the graphite begins to expand again at around 15 dpa. This leads to an end-of-life stress intensity of about the same value as the initial residual stress intensity resulting from fabrication.

Determining the point of failure (or crack initiation) from this curve is impossible because there is no data for the critical value, so only comparisons of one time relative to another are possible. The following points about this stress intensity history can be made:

- The stress intensity never has a larger magnitude than the initial fabrication value, so, assuming the critical stress intensity doesn't change and only the magnitude is important, failure will not occur in a part that is fabricable.

- If the critical stress intensity is degraded by the radiation damage, then failure or crack initiation prior to the assumed graphite lifetime of 24 dpa is possible.

- Because $K_{\theta\theta}$ represents the stress intensity associated with the stress perpendicular to interface near the edge, then crack initiation could be more likely when it is positive (indicating a positive tear-away stress). This would indicate a possible failure in the dose range of 5 to 20 dpa, where $K_{\theta\theta}$ is positive

- In cases where the azimuthal stress intensity is negative, it is not clear which of the other intensities, such as the shear stress intensity, would be most significant.

- The stress intensities in figure 38 could be significantly different if either the Cu-Be or graphite exhibits irradiation creep strains. The effects of creep on stress intensities in bonded structures is shown in the following section.

8.3 Tungsten on Vanadium

8.3.1 Fabrication

For the analysis of the tungsten-vanadium duplex, the model consists of a 2 mm thick tungsten layer on a 1.5 mm vanadium strip. The length is taken to be 10 mm and only the plane strain state is considered. The heat flux is again chosen to be 5 MW/m^2. For this duplex the calculated order of singularity is 0.12, which is slightly stronger than that for the graphite/copper combination. Given the assumed brazing temperature of 1000 °C and an assumed "lock-up" temperature of 900 °C, the boundary-layer stress intensities are given by:

$$\begin{aligned} K'_{rr} &= 180\ MPa(m)^{0.12} \\ K''_{rr} &= -331\ MPa(m)^{0.12} \end{aligned}$$

Figure 38: Boundary-layer stress intensities in copper-graphite component

$$
\begin{aligned}
K_{r\theta} &= -131\ MPa(m)^{0.12} \\
K_{\theta\theta} &= 664\ MPa(m)^{0.12}.
\end{aligned} \tag{172}
$$

As before, these numbers have little meaning by themselves, because there are no measured data for the critical values for failure initiation. These numbers are useful only for comparison with the stress intensities during other portions of the life of the component. It is significant, though, that the tear-away stress intensity $K_{\theta\theta}$ is positive in this case.

8.3.2 Start-Up

To begin the start-up process, the component is heated uniformly to the coolant temperature, which is assumed to be 450 °C for this analysis. This again reduces the residual stresses by more than half. The onset of reactor power provides the assumed heat flux of 5 MW/m^2, which produces an interface temperature of 718 °C and a peak tungsten surface temperature of 867 °C. These thermal fields give average temperatures of 584 °C and 793 °C in the vanadium and tungsten, respectively. These temperatures are important because they determine where the two layers lie within the bell-shaped temperature dependence of the swelling curves of the two materials. The residual stresses, combined with the thermal gradients, lead to the following initial stress intensities:

$$
\begin{aligned}
K'_{rr} &= 51\ MPa(m)^{0.12} \\
K''_{rr} &= -110\ MPa(m)^{0.12} \\
K_{r\theta} &= -38\ MPa(m)^{0.12} \\
K_{\theta\theta} &= 189\ MPa(m)^{0.12}.
\end{aligned} \tag{173}
$$

8.3.3 Radiation Effects

The analysis of the tungsten/vanadium duplex under irradiation is complicated by the occurrence of irradiation creep in both materials. This deformation relaxes the thermal stresses and reduces the effects of the swelling. As mentioned earlier in this chapter, the swelling of both materials is expected to be quite small, so the initial analysis consists of relaxation of the initial stresses. Assuming identical creep constants $\tau_0' = \tau_0'' = 1.3$ dpa, the relaxation is shown in figure 39. The azimuthal stress intensity $K_{\theta\theta}$ is shown to relax to zero within just a few dpa. Considering the expected lifetime of a typical reactor component is over 100 dpa, this relaxation takes place very early in the life of the component, and the stresses would be essentially zero as long as the reactor power continues. Of course, a portion of these stresses would be recovered (as residual stresses) when the power is turned off.

A companion curve in figure 39 shows an exponential decay of the initial stress intensity. The relaxation for a Maxwell material is exponential, so under most conditions, the decay of an initial thermal stress would also be exponential. For a bonded material, though, the order of singularity also changes, so the decay is altered. As shown in the same figure, the order of singularity actually increases slightly, causing a faster-than-expected decay of the stress intensity. The implications of this behavior on the initiation of failure are unclear.

If the assumed swelling behaviors of the materials in this component are in error, there may be a significant swelling in one or both of the layers. To study this, the tungsten will be assumed to swell at the rate of 0.2 %/dpa. Because the material is continuously expanding as the radiation damage occurs, the stresses in the component reach a steady-state level, where the swelling rate is balanced by the creep relaxation rate. This level is determined by the two rates, and by the

Figure 39: Dose dependence of the order of the singularity and the boundary-layer stress intensity without swelling

Figure 40: Relaxation of boundary-layer stress intensity with and without swelling

amount of self-constraint associated with the structure. This phenomenon is shown in figure 40, where the boundary-layer stress intensity relaxes to a steady value of around 106 MPa $(\mathrm{m})^{s_1+2}$ within just a few time constants. The fact that the stress intensity decreases, even in the high swelling case, is a result of several factors and is not a general result. If the swelling rate was higher, all other factors being equal, then the stress intensity could increase, despite the creep relaxation.

Besides the swelling rates, another uncertainty in this analysis is the irradiation creep rates of the two materials. If the two rates are in error, but still equal, then the scaling in figure 39 would still hold, so that the relaxation time scale would just increase or decrease as the creep rate changes. In the swelling case, both the relaxation time and the steady state stress intensity level would change.

The relaxation time would scale with the time constant τ_0 and the steady state stress intensity would be proportional to the relaxation constant. One complication arises, though, when one creep rate changes relative to the other. In this case the scaling arguments, which are possible if both rates change, are not valid. Figure 41 Shows the dose-dependence of the boundary-layer stress intensity for two different creep rates. When the creep constant τ_0 is identical for both materials, the stress intensity is shown to relax to a steady-state value of about 106 MPA m^{s_1+2}. When the creep constant in the vanadium τ_0' is increased to 2.6 dpa (indicating a slower creep rate), the steady state stress intensity increases to about 240 MPa m^{s_1+2}. As a companion to this figure, the time dependence of the order of the singularity is shown in figure 42. Whereas the steady state stress intensity increased with the increased creep constant, the end-of-life order of singularity decreases. The change, though, is quite small over the life of the component.

The effects of changing a single creep rate are again shown in figure 43, which shows how both the steady state stress intensity and the order of singularity vary with the vanadium creep constant for a fixed tungsten creep rate. The steady state stress intensity is seen to increase as the creep in the vanadium slows, while the order of singularity decreases. The increase in the stress intensity is quite strong, but if τ_0' were increased sufficiently, the order would go to zero and the singularity would disappear. This occurs because as the creep in the vanadium slows, it becomes effectively stiff, relative to the tungsten. Because the tungsten was originally stiffer than the vanadium, the effective stiffening of the vanadium brings the ratio of the stiffnesses closer, thus eliminating the singularity.

Figure 41: Dose dependence of the boundary-layer stress intensity for two different vanadium creep constants

Figure 42: Dose dependence of the order of the singularity for two different vanadium creep constants

Figure 43: Steady-State stress intensity and order of singularity for varying vanadium creep rate

8.3.4 Discussion

The history of the boundary-layer stress intensity for the tungsten-vanadium duplex is shown in figure 44. Two important points are readily apparent: first, the stress intensity is always positive, providing a positive tear-away stress at the edge of the interface throughout the life of the component, and, second, the stress intensity history is dominated by the fabrication process. This latter observation is a result of the high creep rates and low swelling rates in both materials. Therefore, the primary concerns during full power operation are the dimensional changes caused by any swelling that occurs. In many cases, high-heat-flux fusion components are shaped to reduce the peak heat flux on the component, so relatively small dimensional changes, resulting from typical swelling rates, can lead to large changes in the heat flux as the profile of the component changes. This could be a severe problem, but it is design dependent and won't be treated further here.

Figure 44: Boundary-layer stress intensities in tungsten-vanadium component

Chapter 9

Conclusions

Duplex structures occur in many different fields of high technology. Of particular interest to this dissertation are bi-layer components for fusion reactors. Plasma-facing components in fusion machines are generally designed with a layer of either very low-Z or very high-Z material facing the plasma and some other material as a backing plate. In existing machines, the pulse length is short, so the two layers can be loosely connected, with little conductance of heat through the interface. Between pulses, the plasma-facing layer can radiate the heat to the vacuum vessel, thus preventing excessive temperatures in either material. In a steady state machine, though, all the heat must be conducted through the interface and a rigid attachment is required.

When components consisting of two rigidly attached layers of different materials are loaded, either mechanically or thermally, two-dimensional elasticity theory often predicts a stress singularity at the edge of the interface. For certain materials, plane (stress or strain) conditions, and loadings, this singularity can be logarithmic, but it is generally a weak algebraic singularity, which is weaker than the square root singularity seen in crack problems. The order of the singularity is material

dependent. By analogy to fracture mechanics, the strength of this singularity is assumed to control the initiation of delamination at the edge. Once delamination occurs, the problem becomes one of an interface crack, with an associated square root singularity. In light of the elastic singularity in crack-free problems, beam and plate theories, which ignore end effects, and standard finite element techniques, which do not admit singularities (directly), are both of suspect utility. Either a global approach, using series solutions for the field quantities, or finite element techniques which incorporate singular elements, must be used to study these edge singularities.

A series solution, derived using the Airy stress function, was used to determine the asymptotic stresses near the edge of the interface in a bonded component. A global approach, using point collocation to determine the series coefficients that would satisfy the traction conditions on the boundaries not adjacent to the interface, was inadequate for determining stress fields either in the bulk or near the edge of the interface. A similar approach, which minimizes the integral of the boundary conditions along these same boundaries, adequately predicted the boundary-layer stress intensity factor in a finite body, although significant errors occurred on the boundary. Hence, this method was useful only for studies of stress fields near the edge of the interface. The reliability of this method was verified using a semi-analytical solution of a half-space problem, so it can be used to study stress singularities in finite bodies for fusion structures.

Two points are apparent from a generic investigation of singularities caused by inelastic strain fields in finite bodies. First, the particular solution of the thermal problem indicates ways in which the stresses in finite bodies can be reduced. For a uniform temperature change, this is accomplished by matching the thermal expansion of the two layers, while prudent choice of both the expansion coefficients and

the thermal conductivities can reduce the stresses when a thermal gradient exists. A designer rarely has complete flexibility in choosing the material properties, but these guidelines may be helpful. The second point to be made here is that reducing the length of the component, which is often proposed as a stress reduction strategy, may be ineffective unless the length is reduced to the order of the layer thicknesses. This increases the number of tiles required to cover a given area, so this type of design may not be desirable, particularly if the tiles are to be handled remotely.

The fabrication of bonded structures can create residual stress intensities, due to differential contraction of the two strips as they are cooled from the fabrication temperature. In some cases, these intensities can exceed those produced by full-power, in-reactor operation. This would indicate that a fabricable component may survive the life of the reactor, but this may not be true if the sign of the stress intensity changes, or if radiation reduces the critical boundary layer stress intensity for crack initiation. Another conclusion based on the high stress intensities induced by fabrication would indicate that cracks are most likely to initiate before final reactor assembly. Therefore, post-fabrication inspection to remove those components which already show some delamination, could significantly increase the component life.

Assuming that a component is fabricable, it must then exhibit resistance to radiation damage in order to provide a sufficient lifetime. In fusion machines the most critical damage phenomena are embrittlement, void swelling, and irradiation creep. Although embrittlement cannot be treated without experimental data relevant to delamination problems, the interaction between swelling and creep can be treated using existing data and some parametric analysis. For a duplex of copper and graphite, which are assumed to undergo no irradiation creep, the densification caused by the radiation damage causes the stress intensity to first increase, and

then decrease to a value that is on the order of the original residual stress caused by fabrication. If the crack initiation depends on the sign of the stress intensity, *i.e.*, if a tensile delamination stress is more likely to cause crack initiation than one that is compressive, then this material combination could present an interesting case for further study because the sign of the boundary layer stress intensity $K_{\theta\theta}$ changes twice during the component life. Correlating the time of crack initiation (or failure) to the sign change could provide important results.

Because of the simple nature of the irradiation creep correlations available for most materials, viscoelastic analyses can be used to study the relaxation of stresses during in-reactor operation. Experimentation is required to verify the linearity of irradiation creep, and to determine the actual relaxation behavior. For duplex structures, the material property dependence of the order of the singularity at the edge of the interface leads to a time-dependent order in viscoelastic materials. This order can change significantly during the lifetime of a reactor, but the variation is shown to be slight in the two material combinations chosen for further study.

For the vanadium-tungsten duplex, the high irradiation creep rates in each material reduce the stress intensity during full power operation considerably. Even assuming swelling rates which are 100 times the measured rates, the interaction of the swelling and the creep relaxation lead to stress intensities that are of the same order as the intensities just following start up. These intensities are of the same sign and much lower than the boundary layer stress intensities expected after fabrication, so a fabricable part should have a reasonable life.

Inherent in the analyses of these two duplexes for fusion machines are uncertainties in the radiation damage characteristics of the materials and of the interpretation of the boundary layer stress intensities in terms of crack initiation or delamination.

The uncertainties in the radiation damage parameters is immense, with very little (if any) irradiation creep data available for any of the materials and no swelling data available for some materials. In terms of crack initiation, experimental study is needed to explore the importance of boundary layer stress intensity factors, and to determine the critical values for the onset of delamination. The applicability of this work to design will be limited only by the availability of relevant thermo-mechanical testing for fusion duplex structures.

Bibliography

[1] R. M. Hussein **Composite Panels/Plates**, Technomic Publishing Co., Inc., 1986, 1.

[2] N. L. Hancox, **Fibre Composite Hybrid Materials**, Macmillan Publishing Co., Inc., 1981, 221.

[3] C. G. Peattie, *et al.*, *Elements of Semiconductor-Device Reliability*, Proc. IEEE, 62, (2), 1974, 149.

[4] P. A. Cooper and P. F. Holloway, *The Shuttle Tile Story*, Astro. and Aero., January 1981, 24.

[5] T. McKelvey, *et al.*, *Final Design and Fabrication of the Moveable Limiter for TFTR*, Proceedings IEEE, 1983, 1043.

[6] M. Huguet, *et al.*, *The JET Machine: Design, Construction, and Operation of the Major Systems*, Fusion Tech., 11, 1987, 43.

[7] H. Nakamura, *et al.*, *Evaluation of Operation Region of JT-60 First Wall Related to High Heat Flux*, Proceedings IEEE, 1983, 1022.

[8] G. Listvinsky, *et al.*, *First Wall and Vacuum Vessel Analysis for Compact Ignition Tokamaks*, Fusion Tech., 10, 1986, 514.

[9] C. Baker, *et al.*, *STARFIRE–A Commercial Tokamak Fusion Power Plant Study*, ANL/FPP-80-1, Vol. I, 1980.

[10] R. F. Mattas, *et al. Divertor Engineering Studies for the International Tokamak Reactor*, Fusion Tech., 10, 1986, 364.

[11] D. R. Harries and E. Zolti *Structural and Material Aspects of the Next European Torus*, Nuc. Eng. Des./Fusion, 3, 1986, 331.

[12] Najmabadi, F. *et al.*, *The TITAN Reversed-Field Pinch Fusion Reactor Study – Final Report*, to be published.

[13] McGrath, R., *Technical Issues and Testing Requirements for Plasma Interactive Components in Magnetic Fusion Applications*, Sandia Report SAND87-0418, UC-20, 1988.

[14] W. M. Stacey, Jr. **Fusion**, John Wiley and Sons, New York, 1981, 88.

[15] B. Badger, *et al.*, *A Wisconsin Toroidal Fusion Power Reactor Design*, University of Wisconsin Report, UWFDM-68, 1974.

[16] B. Badger, *et al.*, *A Noncircular Tokamak Power Reactor Design*, University of Wisconsin Report, UWFDM-150, 1976.

[17] E. Johnson, *et al.*, *Doublet- III Limiter/Armor Update*, Proceedings IEEE, 1983, 1014.

[18] G. Hopkins, P. Trester, and J. Kaae, *Experience with C+SiC Alloy Coated Armor/Limiter Tiles in Doublet-III*, J. Nuc. Mat., 122&123, 1984, 1165.

[19] Kotzlowski, H.E., *Actively Cooled Graphite Limiter*, Proc. 13th Symposium on Fusion Technology, Varese (Italy), Sept. 1984, 1253.

[20] R. Hasiguti and K. Tomabechi, *The Material Aspects of the JT-60 Device*, J. Nuc. Mat., 122&123 1984, 1099.

[21] G. W. Brown, J. A. Koski, and R. D. Watson, *Engineering Design of the ALT-II Pump Limiter for Textor*, Fusion Tech., 8, 1985, 1729.

[22] C. Flanagan, *et al.*, *Overview of the Compact Ignition Tokamak*, Fusion Tech., 10, 1986, 491.

[23] T. Tone, *et al.*, *Conceptual Design of Fusion Experimental Reactor(FER)*, Nuc. Tech./Fusion, 4, 1983, 573.

[24] K. Sumita and Y. Seki, *Status of Blanket and First-Wall Engineering Works in Japan*,Fusion Tech., 8, 1985, 501.

[25] G. Casini and F. Farfaletti-Casali, *Engineering Solutions for Components Facing the Plasma in Experimental Fusion Power Reactors*, Nuc. Eng. Des., 3, 1986, 399.

[26] B. Lipschultz, *et al.* , *Alcator-DCT: Heat Removal and Impurity Control in a Proposed Long Pulse Tokamak*, J. Nuc. Mat., 121, 1984, 441.

[27] G. Fuller, B. Cramer, and J. Haines, *DCT-8 Pumped Limiter Design Considerations*, J. Nuc. Mat., 121, 1984, 435.

[28] J. Brooks *et al.*, *Pumped Limiter Study for Alcator-DCT*, ANL/FPP/PM-175, June 1983.

[29] D. Smith *et al.*, *Blanket Comparison and Selection Study: Final Report*, ANL/FPP- 84-1, Vol. 3, 1984, 7-123.

[30] Glasgow, B. and W. Wolfer, *Stress Analysis of a Duplex Structure in a High Heat Flux Fusion Environment*, Fusion Tech., 8, 1985, 596-600.

[31] Horie, T., *et al.*, *Lifetime Analysis for Fusion Reactor First Walls and Divertor Plates*, Fusion Eng. Design, 5, 1987, 221-231.

[32] Oomura, H., M. Akiyama, and K. Miya, *Lifetime Evaluation Study for Fusion Reactor First Wall*, Fusion Tech., 8, 1985, 625-629.

[33] Renda, V., G. Federici, and L. Papa, *Feasibility Study for an Engineering Concept of a Stainless Steel/Copper Divertor Plate Protected by W-5Re Alloy or Graphite Armor*, Fusion Tech., 13, 1988, 473-483.

[34] Timoshenko, S., *Analysis of Bi-Metal Thermostats*, J. Strain Analysis, 11, 1925, 233-255.

[35] Goland, M. and E. Reissner, *The Stresses in Cemented Joints*, J. Appl. Mech, 1, 1944, A17-A27.

[36] Grimado P.B., *Interlaminar Thermoelastic Stresses in Layered Beams*, J. Thermal Stresses, 1, 1978, 75-86.

[37] Chen, W.T. and C.W. Nelson, *Thermal Stresses in Bonded Joints*, IBM J. Res. Develop., 23, 1979, 179-188.

[38] Ochoa, O.O., and V.M. Marcano, *Thermal Stresses in Laminated Beams*, Int. J. Solids Structures, 20, 1984, 579-587.

[39] Weitsmann, Y., *Stresses in Adhesive Joints Due to Moisture and Temperature*, J. Composite Materials, 11, 1977, 378-394.

[40] Chen, D., S. Cheng, and T. Gerhardt, *Thermal Stresses in Laminated Beams*, J. Thermal Stresses, 5, 1982, 67-84.

[41] Chen, D. and S. Cheng, *An Analysis of Adhesive-Bonded Single-Lap Joints*, J. Applied Mechanics, 50, 1983, 109-115.

[42] Williams, H.E., *Thermal Stresses in Bonded Solar Cells-the Effect of the Adhesive Layer*, J. Thermal Stresses, 6, 1983, 231-252.

[43] Williams, H.E., *Asymptotic Analysis of the Thermal Stresses in a Two-Layer Composite with an Adhesive Layer*, J. Thermal Stresses, 8, 1985, 183-203.

[44] Williams, M. L., *On the Stress Distribution at the base of a Stationary Crack*, J. Appl. Mech., 24, 1957, 109-114.

[45] Bogy, D.B., *Edge-Bonded Dissimilar Orthogonal Elastic Wedges Under Normal and Shear Loading*, J. Appl. Mech., 35, 1968, 460-466.

[46] Bogy, D.B. and E. Sternberg, *The Effect of Couple-Stresses on the Corner Singularity Due to an Asymmetric Shear Loading*, Int. J. Solids Structures, 4, 1968, 159-174.

[47] Bogy, D.B., *On the Problem of Edge-Bonded Elastic Quarter-Planes Loaded at the Boundary*, Int. J. Solids Structures, 6, 1970, 1287-313.

[48] Wang, S. S. and I. Choi, *Boundary-Layer Effects in Composite Laminates:Part 1–Free-Edge Stress Singularities*, J. Appl. Mech., 49, 1982, 549.

[49] Wang, S. S. and I. Choi, *Boundary-Layer Effects in Composite Laminates:Part 2–Free-Edge Stress Solutions and Basic Characteristics*, J. Appl. Mech., 49, 1982, 549.

[50] Fadle, J., *Die Selbstspannungs-Eigenwertfunktionen der quadratischen Scheibe*, *Ingenieur-Archiv*, 11, 1941, 125-149.

[51] Hein, V.L. and F. Erdogan, *Stress Singularities in a Two-Material Wedge*, Int. J. Fracture Mech., 7, 1971, 317-330.

[52] Dempsey, J.P. and G.B. Sinclair, *On the singularities in the Plane Elasticity of the Composite Wedge*, J. Elasticity, 9, 1979, 373-391

[53] Dempsey, J.P. and G.B. Sinclair, *On the Singular Behavior at the Vertex of a Bi-Material Wedge*, J. Elasticity, 11, 1981, 317-327.

[54] Erdogan, F., *Stress Distribution in Bonded Dissimilar Materials with Cracks*, J. Appl. Mech., 32, 1965, 403-410.

[55] Comninou, M., *The Interface Crack*, J. Applied Mechanics, 44, 1977, 631-636.

[56] Bartholomew, P., *Solution of Elastic Crack Problems by Superposition of Finite Elements and Singular Fields*, Comp. Meth. in Appl. Mech., 13, 1978, 59-78.

[57] Lin, K.Y. and J.W. Mar, *Finite Element Analysis of Stress Intensity Factors for Cracks at a Bi-Material Interface*, Int. J. Fracture, 12, 1976, 521-531.

[58] Wang, S. S. and F. Yuan, *A Singular Hybrid Finite Element Analysis of Boundary-Layer Stresses in Composite Laminates*, Int. J. Solids Struct., 19, 1983, 825-837.

[59] Raju, I.S. and J.H. Crews, Jr., *Interlaminar Stress Singularities at a Straight Free Edge in Composite Laminates*, Computers and Structures, 14, 1981, 21-28.

[60] Kokini, K. and R.W. Perkins, *Free Edge Thermal Stress Singularities in Finite Concentric Cylinders*, Computers and Structures, 19, 1984, 531-534.

[61] Trantina, G.G., *Fracture Mechanics Approach to Adhesive Joints*, J. Composite Materials, 6, 1972, 192-207.

[62] Ishai, O. and S. Gali, *Two-Dimensional Interlaminar Stress Distribution Within the Adhesive Layer of a Symmetrical Doubler Model*, J. Adhesion, 8, 1977, 301-312.

[63] Gali, S. and O. Ishai, *Interlaminar Stress Distribution Within an Adhesive Layer in the Nonlinear Range*, J. Adhesion, 9, 1978, 253-266.

[64] Gulati, S. T. and H.E. Hagy, *Analysis and Measurement of Glue-Spall Stresses in Glass-Epoxy Bonds*, J. Amer. Ceramic Society, 65, 1982, 1-5.

[65] Griffin, O.H. and J.C. Roberts, *Numerical/Experimental Correlation of Three-Dimensional Thermal Stress Distributions in Graphite/Epoxy laminates*, J. Comp. Matl., 17, 1983, 539-548.

[66] Herakovich, C.T. and D. Post, *Free Edge Strain Concentrations in Real Composite Laminates: Experimental-Theoretical Correlation*, J. Appl. Mech., 52, 1985, 787-793.

[67] Blanchard, J.P. and R.D. Watson, *Residual Stresses in Bonded Armor Tiles for In-Vessel Fusion Components*, Nuc. Eng. Design/Fusion, 4, 1986, 61-66.

[68] Bauld, N.R., Jr. and J.G. Goree, *Comparison of Finite-Difference and Finite-Element Methods for Calculating Free Edge Stresses in Composites*, Computers and Structures, 20, 1985, 897-914.

[69] Whitcomb, J.D. and I. S. Raju, *Reliability of the Finite Element Method for Calculating Free Edge Stresses in Composite Laminates*, Computers and Structures, 15, 1982, 23-37.

[70] Delale, F. and F. Erdogan, *Viscoelastic Analysis of Adhesively Bonded Structures*, J. Appl. Mech., 48, 1981, 331-338.

[71] Delale, F. and F. Erdogan, *Time-Temperature Effect in Adhesively Bonded Joints*, J. Composite Materials, 5, 1981, 561-581.

[72] Pagano, N.J. and S. R. Soni, *Global-Local Laminate Variational Model*, Int. J. Solids Structures, 19, 1983, 207-228.

[73] Dundurs, J., *Discussion*, J. Appl. Mech., 35, 1968, 460.

[74] Muller, D., *A Method for Solving Algebraic Equations Using and Automatic Computer*, Math. Tables and Other Aids to Computation, 10, 1956, 208.

[75] Hildebrand, F., **Methods of Applied Mathematics**, Prentice-Hall, Inc., New Jersey, 1965.

[76] Zwiers, R. , T. Ting, and R. Spilker, *On the Logarithmic Singularity of Free-edge Stress in Laminated Composites Under Uniform Extension*, J. Appl. Mech., 49, 1982, 561-569.

[77] Rabotnov, Y.N., **Creep Problems in Structural Members**, Elsevier, New York, 1969, 122.

[78] Olander, D., *Fundamental Aspects of Nuclear Reactor Fuel Elements*, Technical Information Center, TIC-26711-P1, U.S. Department of Energy, 1976, 590.

[79] Miller, M.K., and W.T.Guy, Jr., *Numerical Inversion of the Laplace Transform by Use of the Jacobi Polynomials*, SIAM J. Numer. Anal., 3, 1966, 624-635.

[80] Papoulis, A., *A New Method of Inversion of the Laplace Transform*, Quart. Appl. Math., 14, 1957, 405-414.

[81] Harries, D., and S. Dupouy, *Materials Selection for NET and the Associated R & D Programme*, Proceedings 13^{th} Symposium on Fusion Technology, Varese, Italy, 1984, 147-152.

[82] Brager, H., H. Heinisch, and F. Garner, *Effects of Neutron Irradiation at 450 $^{\circ}C$ and 16 dpa on the Properties of Various Commercial Copper Alloys*, ADIP Semi-Annual Progress Report DOE/ER-0045/13, 1984, 183-187.

[83] Engle, G. and B. Kelly, *Radiation Damage of Graphite in Fission and Fusion Reactor Systems*, J. Nuc. Mat., 122 & 123, 1984, 122-129.

[84] Braski, D., *The Effect of Neutron Irradiation on Vanadium Alloys*, J. Nuc. Mat., 141-143, 1986, 1125-1131.

[85] Corporate Publication, Metallwerk Plansee GmbH, A-6600 Reutte.

[86] Mattas, R., D. Smith, and M. Abdou, *Materials for Impurity Control*, J. Nuc. Mat., 122 & 123, 1984, 66-79.

[87] Smith, D., B. Loomis, and D. Diercks, *Vanadium-Base Alloys for Fusion Reactor Applications - a Review*, J. Nuc. Mat., 135, 1985, 125-139.

[88] Gittus, J., **Creep, Viscoelasticity and Creep Fracture in Solids**, Applied Science Publishers, Ltd., London, 1975.

[89] Livak, R., T. Zocco, and J. Kennedy, *Swelling and Tensile Properties of Neutron Irradiated Copper Alloys for High Heat Flux Applications*, ADIP Semi-Annual Progress Report, DOE/ER-0045/14, 1985, 152-160.

[90] Watson, R. and W. Wolfer, *Mechanical Constitutive Laws for the Irradiation Behavior of Graphite*, J. Nuc. Mat., 85 & 86, 1979, 159-164.

[91] Green, W., D. Smith, and B. Kelly, *Summary of the Rigi-Kaltbad Workshop on Graphite*, J. Nuc. Mat., 122 & 123, 1984, 14-16.

[92] Powell, R., *et al.*, *Swelling of Several Commercial Alloys Following High Fluence Neutron Irradiation*, J. Nuc. Mat., 103 & 104, 1981, 969-974.

[93] Nygren, R., *Irradiation Creep Studies of Titanium Alloys*, J. Nuc. Mat., 85 & 86, 1979, 861-865.

[94] Peterson, D., *Swelling in Neutron Irradiated Titanium Alloys*, ADIP Semi-Annual Progress Report, DOE/ER-0045/8, 1982, 304-311.

[95] Steichen, J., *Tensile Properties of Neutron Irradiated TZM and Tungsten*, J. Nuc. Mat., 60, 1976, 13-19.

[96] Hart, G., **Uncertainty Analysis, Loads, and Safety in Structural Engineering**, Prentice-Hall, Inc., Englewood Cliffs, New Jersey, 1982.

[97] Blanchard, J., and N. Ghoniem, *The Influence of Uncertainties in Material Properties and the Effects of Dimensional Scaling on the Prediction of Fusion Structure Lifetimes*, Nuc. Eng. Des./Fusion, 4, 1986, 67-74.

[98] Schwartz, M., **Brazing**, ASM International, Metals Park, Ohio, 1987.

Appendix A

List of Symbols

α thermal expansion coefficient

α_D first Dundurs parameter, $\alpha_D = k_2/k_3$

β_D first Dundurs parameter, $\beta_D = (k_2 - k_1)/k_3$

$\delta = 2(s+1) + 2\cos 2\xi$

δ order of stress singularity

δ measure of radiation damage – units: displacements per atom (dpa)

ϵ strain

ϵ_{in} inelastic strain, $\epsilon_{in} = \alpha T + \frac{\Delta V}{3V}$

$\eta = -2\sin\xi\cos\xi$

$\gamma = 2(s+1) - 2\cos 2\xi$

κ bulk modulus

μ shear modulus

ν Poisson's ratio

ϕ neutron fluence, (n/cm^2/s)

Φ Airy stress function

σ stress

τ generalized time variable used in Laplace transform inversion

τ_0 creep constant in viscoelastic constitutive equation for Maxwell material

$\xi = s\pi/2$

$\{a\}$ vector of unknown constants in homogeneous solution

a, b, c, d constants in homogeneous solution for stress function

A, B, C, D constants in homogeneous solution, normalized to a'

B, C, D constants used in general irradiation creep equation,
$\dot{\epsilon} = (B + C\sigma^2)\sigma\phi + D\sigma\dot{S}$

B irradiation creep coefficient, $\dot{\epsilon} = B\phi\sigma$

C uniform axial displacement (u_x) on symmetry line of model

C_0, C_1 constants representing time dependence of inelastic strain, $\epsilon_{in} = C_0 + C_1 t$

C_{kn} general constant in viscoelastic constitutive equation

e strain deviator

E elastic modulus (Young's modulus)

$\{f\}$ loading vector in equation for constants in homogeneous solution

f_{ij} θ -dependence of σ_{ij}

$f_{ij(k)}$ θ -dependence of σ_{ij} for k^{th} eigenvalue

f_{u_r} θ -dependence of u_r

$f_{u_r(k)}$ θ -dependence of u_r for k^{th} eigenvalue

f_{u_θ} θ -dependence of u_θ

$f_{u_\theta(k)}$ θ -dependence of u_θ for k^{th} eigenvalue

$F(\theta)$ θ -dependence of Airy stress function, $\Phi = r^{-s}F(\theta)$

h denominator in solution for constants A, B, C,and D

i, j integer roots of determinant

I residual integral of squares of difference between prescribed surface tractions and symmetry conditions around boundary of strip model

k ratio of shear moduli, $k = \mu''/\mu'$

$k_1 = 2(k-1)$

$k_2 = km'' - m'$

$k_3 = km'' + m'$

k_T thermal conductivity

k_∞ end-of-life value of ratio of shear moduli

K kernel representing viscoelastic material behavior

K stress intensity factor, $\sigma \sim Kr^{-0.5}$

K boundary-layer stress intensity factor, $\sigma \sim Kr^{-\delta}$

l length of strip

m material property depending on Poisson's ratio and plane (stress or strain) conditions

$[\mathbf{M}]$ matrix in linear system formed by minimizing residual integral I

n material property depending on Poisson's ratio and plane (stress or strain) conditions

N number of equations used in numerical inversion of Laplace transform

p Laplace transform parameter, $\widetilde{f} = \int_0^\infty f(t)e^{-pt}dt$

$P = 2k_1 - k_2$

$P_k(D)$ differential operator representing general viscoelastic constitutive equations

q uniform heat flux on plasma-facing surface of model

$\{q\}$ forcing vector in linear system formed by minimizing residual integral I

Q material property depending on Poisson's ratio and plane (stress or strain) conditions

Q_3, Q_4 functions of s that determine azimuthal interface stresses in half-space benchmark problem

R_0 radius of temperature field used in half-space benchmark problem

R_1 first non-zero term in $\{f\}$

R_2 second non-zero term in $\{f\}$

s stress deviator

s exponent in assumed solution for Airy stress function, $\Phi = r^{-s}F(\theta)$

s Mellin Transform parameter

$\dot{S}$ swelling rate

t thickness of strip

t time

T difference between temperature and stress-free reference temperature

ΔT temperature difference across strip, $T = \Delta T \left(\frac{y}{t}\right)$

T_c surface temperature on coolant side of model

T_h homologous temperature, $T_h = T/T_{melt}$

T_i interface temperature

T_s surface temperature

T_0 temperature change within semi-circle of radius R_0 used in half-space benchmark problem

u displacement

u_0, v_0 axial and transverse rigid body displacements

w_{ij} weighting factors used for minimization of residual integral I

w_{ij} weighting factors used for gaussian quadrature

$[\mathbf{X}]$ matrix in equation for constants in homogeneous solution

$\|\mathbf{X}\|$ determinant of $[\mathbf{X}]$

Z atomic number

$\overline{\Phi}$, $\overline{\sigma}$, $\overline{T}$, $\overline{u}$ Mellin transforms of respective functions

$\widetilde{f}$, $\widetilde{\epsilon}$, $\widetilde{\nu}$, $\widetilde{E}$, $\widetilde{k}$, $\widetilde{\sigma}$ Laplace transforms of respective functions