A Beam Size Monitor Based on Appearance Intensities for Multiple Gas Ionization

T. Katsouleas and J. Yoshii
Department of Electrical Engineering–Electrophysics,
University of Southern California
Los Angeles, CA 90089

W. B. Mori, C. Joshi and C. Clayton
UCLA Department of Electrical Engineering
Los Angeles, CA 90024

Abstract

A method of measuring the spot size or bunch length of intense charged particle beams is proposed. The relation between the size (widths and length) of a charged particle beam and the beam’s electric field forms the basis for a sub-micron beam size monitor. When the beam passes through a low pressure gas of high Z atoms, the collective field of the beam causes multiple ionizations of the gas atoms. Measurement of the appearance of ionized atoms in a given charge state gives information about the field of the beam and hence its size. Sample calculations show that appearance thresholds can indicate the spot size of round beams with 10 nm accuracy or the bunch length of round or flat beams with up to 10 μm accuracy.

INTRODUCTION

The designs now being proposed for future colliders for high energy physics call for beams of incredibly small dimensions, both in their width and their length. For example, flat beams with transverse dimensions as small as 3 nm and bunch lengths of order 100 μm have been proposed. There is an inherent need, then, to develop new diagnostic techniques capable of resolving such small scales.

Conventional beam size measurements based on fine wires in the beam path have been used at the Stanford Linear Collider (SLC) to measure round beam spot sizes above one micron. Recently, two new techniques for measuring sub-micron beams have been tested. A laser interference pattern used to produce synchrotron radiation from a test beam at the Final Focus Test Beam (FFTB) at Stanford enabled measurement of a 70 nm flat beam spot size. Another technique proposed by a group from Orsay measured the time-of-flight of gas ions ionized by the beam and accelerated by its space charge. The scheme we propose here is complementary to these two. It is similar to the Orsay monitor in that it is based on beam ionization of a gas; however, it differs in that it uses the information about the charge states of the ionized atoms. Multiple ionization has already been observed in the testing of the Orsay monitor, but the information was parasitic and not used to determine spot size. The scheme proposed here can alternatively be used to measure bunch length, typically...
down to about 100 μm, if spot size is known. Conventional techniques for measuring bunch length, such as streak cameras, are limited to 3 ps time scales or mm length scales.

Our approach for beam size measurement is a modification of the appearance intensity diagnostic developed for use with lasers. Augst et al. have conducted experiments using tunnel ionization to measure the intensity of laser beams. Their work correlates the appearance of the various charge states of a given target gas with the laser intensities required for successive, multiple tunnel ionization of the gas. The appearance intensity technique is useful for measuring laser intensities on the order of 10$^{16}$ to 10$^{18}$ W/cm$^2$. This diagnostic could be applied to the highly focused particle beams considered for future collider research, which have beam space charge electric fields corresponding to intensities of the same orders$^{4,5}$.

![Figure 1: Proposed experimental setup for beam diagnostic.](image)

The beam size diagnostic setup we propose is as shown in Figure 1: After the final focusing, the particle beam is passed through a gas cell. The ions produced from the tunnel ionization are accelerated, by means of an electric field applied across the cell, to detectors which make time of flight measurements. Since the time of flight for an ion depends on the ion’s charge state as well as the applied field, the ionization yield for each charge state can be determined. For a round Gaussian beam where the beam number and bunch length are known, the beam spot size can then be found from calculations relating the spot size to the expected numbers of atoms in the charge states. For flat beams (i.e., transverse height $\sigma_y$ much less than transverse width $\sigma_x$), the peak beam electric field is independent of $\sigma_y$ so that this diagnostic does not provide information about the small spot size dimension. However, if the spot size is known from some other measurement technique, the appearance of charge states can be used to compute the bunch length of either flat or round beams. In this work we calculate ionization yields as a function of spot size by modelling the interaction of the electric field of a bi-Gaussian (in r and z) round beam with various gases. For typical parameters, this yields spot size information down to 10 nm. We also calculate ionization
yields vs. bunch length for a sample flat beam and show that this can give bunch length information down to 0.1 mm.

THEORETICAL MODEL

The appearance intensity is related to the total number of atoms reaching each charge state during the passage of the beam. Any given ionization state cannot occur until the previous charge state has occurred, and the number of ions in each state also depends on the probability of tunnel ionization into that state. Thus the equations governing the population of each charge species are

$$\frac{\partial N_j}{\partial t} = w_j N_{j-1}$$  \hspace{1cm} (1)

The $N_j$ represent the fraction of the total gas atoms which are in the $j$th charge state. The ionization probability, per unit time, is given by the Keldysh formula

$$w_j = w_0 \frac{E_j^2}{E} e^{-\frac{2E_j}{18}}$$  \hspace{1cm} (2)

where $E_j$ = ionization potential for the $j$th ionization state, normalized by 13.6 eV; $E$ = transverse beam electric field ($E_r(r)$ for round beams), normalized by the atomic field $5.1453 \times 10^{11}$ V/m; $w_0 = 1.635 \times 10^5$ /ps. The total ionization yields are obtained by integrating Equations (1) in time.

Round Beams

The round beam is modelled by a cylindrical bi-Gaussian distribution,

$$N_b(r, z) = N_{b0} e^{-\frac{r^2}{2\sigma_r^2}} e^{-\frac{z^2}{2\sigma_z^2}}$$  \hspace{1cm} (3)

where $N_{b0}$ = maximum particle density, $\sigma_r$ = beam spot size, and $\sigma_z$ = bunch length. The beam number $N_b$ is then given by

$$N_b = \int_{z=-\infty}^{\infty} \int_{r=0}^{\infty} N_b(r, z) 2\pi rdrdz$$ \hspace{1cm} (4)

from which the relation

$$N_{b0} = \frac{N_b}{(2\pi)^{1/2} \sigma_r^2 \sigma_z}$$ \hspace{1cm} (5)

is found. Applying Gauss' Law to obtain the radial electric field yields

$$E_r = \frac{2eN_b}{(2\pi)^{1/2}} \frac{e^{-\frac{1}{4}(\frac{r}{\sigma_r})^2}}{\sigma_z} \cdot \frac{1}{r} \left(1 - e^{-\frac{1}{4}(\frac{z}{\sigma_z})^2}\right)$$  \hspace{1cm} (6)

In deriving Eq. (6), use has been made of Eq. (5). Since the beam is travelling at nearly the speed of light, the time dependence of Eq. (6) is brought in through
\[ z = c(t - t_p) \]  

where \( t_p \) is the time when the beam particle density is a maximum.

To generate the ionization data in the Figures, Eqs. (1) were time-integrated numerically and averaged over \( r \) for a number of values of either \( \sigma_r \) and/or \( \sigma_z \), depending on the data to be computed. \( N_b \) was taken to be \( 5.5 \times 10^{10} \). For the calculations, the gas sample is assumed to have a radius \( r_{\text{max}} = 30 \) microns here; for all cases, ionization is negligible beyond this radius. The \( r \)-dependence of the electric field is included by dividing the sample volume into cylindrical shells and performing the time-integration at the radial midpoint of each shell. Since the radial electric field varies slowly beyond \( r = 5\sigma_r \), the amount of computation can be reduced by selecting thicker shells for that region. The shell thicknesses were chosen to be \( 0.1\sigma_r \) for \( r \leq 5\sigma_r \) and \( 1.0\sigma_r \) for \( 5\sigma_r < r \leq r_{\text{max}} \). After integrating in time at each \( r \)-point, the resulting ionization yields are weighted by the volume of the shell at radius \( r \) and summed over \( r \).

**Flat Beams**

The flat beam is modelled by a tri-Gaussian distribution,

\[ Nb(x, y, z) = N_{b0} e^{-\frac{x^2}{2\sigma_x^2}} e^{-\frac{z^2}{2\sigma_z^2}} \]  

where \( \sigma_x = \) beam width and \( \sigma_y = \) beam height \( \ll \sigma_x \); \( N_{b0} \) and \( \sigma_z \) are defined as in the round beam case, and the same relation between \( z \) and \( t \) applies. \( N_b \) is defined by

\[ N_b = \int_{x=-\infty}^{\infty} \int_{y=-\infty}^{\infty} \int_{z=-\infty}^{\infty} Nb(x, y, z) dx dy dz \]  

\[ N_{b0} = \frac{N_b}{(2\pi)^{\frac{3}{2}}\sigma_x\sigma_y\sigma_z} \]  

For these calculations, the beam is treated as a 2-D problem (\( \partial(\hat{y}) = 0 \)). This assumes that the gas cell is shorter than the beam beta functions \( \beta_x^* \) or \( \beta_y^* \). The gas cell is divided into rectangular boxes in the region \(-\sigma_x \leq x \leq \sigma_x \) where the \( x \)-increments \( \Delta x \) were small enough that \( \frac{\partial E_x}{\partial x} \ll \frac{E_x}{\Delta x} \). Also, since \( \sigma_y \ll \sigma_x \) over most of the ionization volume \( E_x \ll E_y \) and we neglect it. With these simplifications, Gauss' Law becomes

\[ \frac{\partial E_y}{\partial y} = 4\pi e N_b(x, y, z) \]  

Integrating Eq. (11) gives \( E_y \):

\[ E_y = \frac{Ne}{\sigma_x\sigma_y} e^{-\frac{x^2}{2\sigma_x^2}} e^{-\frac{z^2}{2\sigma_z^2}} erf \left( \frac{y}{\sqrt{2}\sigma_y} \right) \]  

Eqs. (1) with Eq. (12) were time-integrated at the center of each box. The ionization yields are then weighted by the charge contained within the box and summed.
RESULTS

Spot size

Figure 2 plots the fraction of the total Ne atoms in the gas cell which reach the $N_{e}^{1+}$, $N_{e}^{2+}$, $N_{e}^{3+}$, and $N_{e}^{4+}$ charge states versus the spot size for $\sigma_{z} = 0.05$ cm and $N_{b} = 5.5 \times 10^{10}$. Two points should be noted. First, a charge state will not appear unless $\sigma_{r}$ is below some threshold value which allows tunnelling to occur. If $\sigma_{r}$ is below the threshold, the ionization yield increases rapidly, then begins to level off. Second, as each successive charge state appears, the yield for the preceding state is decreased by that of the new charge state.

![Figure 2: Fraction of total Ne atoms reaching charge states $N_{e}^{1+}$ ($N_{1}$) through $N_{e}^{4+}$ ($N_{4}$) as a function of $\sigma_{r}$. $N_{b} = 5.5 \times 10^{10}$ and $\sigma_{z} = 0.05$ cm. The effects of successive ionizations are evident. Similar plots have been made for He and Ar.](image)

For example, the fraction of atoms which are in the $N_{e}^{1+}$ state, $N_{1}$, increases rapidly as $\sigma_{r}$ decreases from $1.5 \times 10^{-4}$ cm and begins levelling off near $\sigma_{r} \approx 1 \times 10^{-4}$ cm. $N_{1}$ decreases as the $N_{e}^{2+}$ ions appear; the amount of the decrease, from the maximum $N_{1}$, is approximately $N_{2}$. As $\sigma_{r}$ continues to decrease and $N_{2}$ begins to level off, $N_{1}$ reaches a nearly constant value.

The ionization yields have been plotted logarithmically in Figure 3; the horizontal scale has been expanded to distinguish the curves for the $N_{e}^{4+}$ through $N_{e}^{7+}$ states. It is clear that no significant production of $N_{e}^{2+}$ occurs until the spot size is below 1 $\mu$m; thus the appearance of $N_{e}^{2+}$ indicates a spot size below this value. Similarly, the appearance of other charge states gives spot size information from 1 $\mu$m to 0.07 $\mu$m with accuracy up to about 0.03 $\mu$m.

In an experimental setting, these plots would be generated numerically for each set of beam number and bunch length; there are no simple, analytic solutions for the final $N_{i}$. However, to gain some insight into the variation of the ionization yields with $\sigma_{z}$, the calculations were repeated for $N_{b} = 5.5 \times 10^{10}$ and several values of $\sigma_{z}$ ranging from 0.01...
Figure 3: Logarithm of fractional charge state densities for $Ne^{1+}$ through $Ne^{8+}$ vs. $\sigma_r$ for $N_b = 5.5 \times 10^{10}$ and $\sigma_z = 0.05$ cm.

Figure 4: Logarithmic plot showing the variation in ionization yield for $Ne^{1+}$ vs. $\sigma_r$ as $\sigma_z$ is varied from 0.01 cm to 0.3 cm. Notice the reduction in $N_1$ and the threshold value of $\sigma_r$ for higher values of $\sigma_z$. $N_b = 5.5 \times 10^{10}$ in these calculations. The data for the higher charge states show a similar variation.
In(Ni) vs \(\sigma_z\), \(i=4..8\); \(N_e, N_b = 5.5 \times 10^{10}\), \(\sigma_r = 1\mu m\). At these small values of \(\sigma_z\), there are clear appearance thresholds for \(Ne^{5+}\) and higher charge states.

Figure 5: Logarithmic plot of round beam ionization yields for Ne plotted against small values of bunch length. \(N_b = 5.5 \times 10^{10}\) and \(\sigma_r = 1\mu m\). At these small values of \(\sigma_z\), there are clear appearance thresholds for \(Ne^{5+}\) and higher charge states.

Bunch length

Round beams

Two sets of calculations were done for the round beam. Ne was chosen as the sample gas, with \(N_b = 5.5 \times 10^{10}\), \(\sigma_r = 1\mu m\). The first set of data, displayed in Figure 5, was computed with \(1 \times 10^{-4} \text{ cm} \leq \sigma_z \leq 0.01 \text{ cm}\). These bunch lengths are small enough that significant amounts of \(Ne^{4+}\) and higher charge states are produced. The second set of data represents longer bunch lengths in the range \(0.01 \text{ cm} \leq \sigma_z \leq 0.1 \text{ cm}\) and is plotted in Figure 6. Here, only \(Ne^{1+}\) through \(Ne^{4+}\) appear in significant numbers.

Similar to the data for spot size determination, there are threshold values of bunch length for the appearance of successively higher charge states. For example, the bunch length must be below 0.01 cm for significant production of \(Ne^{5+}\), while \(Ne^{6+}\) requires a bunch length less than 0.007 cm. The charge states from \(Ne^{2+}\) up give information on bunch lengths ranging from 0.05 cm down to 0.0035 cm with accuracy up to about 0.001 cm.

Flat beams

The flat beam data were calculated for a beam containing \(10^{10}\) electrons with \(\sigma_x = 1\mu m\), \(\sigma_y = 70\) nm, and Ne in the gas cell. These data are shown in Figure 7. As in the round beam case, there are well-defined bunch length thresholds below which charge states will not occur. Bunch length information from 0.09 cm down to about 0.035 cm may be obtained...
Figure 6: Logarithmic plot of round beam ionization yields for Ne plotted against larger values of bunch length. $N_b = 5.5 \times 10^{10}$ and $\sigma_r = 1\mu m$. The appearance thresholds for $Ne^{1+}$ through $Ne^{4+}$ are visible on this scale.

Figure 7: Logarithmic plot of flat beam ionization yields vs. bunch length. The sample gas is Ne, $N_b = 10^{10}$, $\sigma_x = 1\mu m$, $\sigma_y = 70nm$. Appearance thresholds for charge states $Ne^{5+}$ and higher are evident.
with accuracy up to about 0.01 cm within the limitations of the simplifications made in the calculations.

**Conclusion**

A diagnostic for particle beam size based on field ionization has been described. Calculations of the ionization yields versus spot size for round beams show fairly sharp thresholds for the appearance of the various charge states involved. Similar calculations using different bunch lengths yield curves with the same form, with maxima and threshold spot size decreasing as the bunch length is increased. Plots of ionization yields versus bunch length for both round and flat beams also display thresholds for the appearance of successive charge states. These thresholds can be used to determine the beam size given the beam number and either a known spot size or a known beam length.

For a gas cell of density-length product equal to $10^{15}$ cm$^{-2}$, there are $3 \times 10^{10}$ atoms in the modelled region. Thus an ionization fraction of $10^{-5}$ would yield an absolute number of $3 \times 10^5$ ions and should be readily detectable. The possible effects of collisional ionization in the gas cell-beam system need to be addressed. In addition, further simulations which take into account the effects of fringe fields in the flat beam case should be performed.

In addition to its application to accelerators, an experiment to test this diagnostic could also be useful in atomic physics. Previous tests of DC atomic tunneling theory with lasers\(^4\) have yielded discrepancies between theory and experiment attributed to the AC nature of the laser (i.e., ionized electrons quiver in the laser and return to enhance secondary ionization). The gas cell-beam apparatus described here could be used, with beam intensities comparable to those of the laser, to do a cleaner test of atomic theory. As the particle beam does not oscillate, the data obtained through such an experiment would be useful to separate the DC and AC effects in the atomic physics.

Work supported by US DOE grant DE-FG03-92ER40745.
REFERENCES

6. H. Wiedemann, Particle Accelerator Physics, New York: Springer-Verlag, 1993, pp. 159-164.