Threshold for Trapping Positrons in the Wake Driven by a Ultra-relativistic Electron Bunch

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Abstract. We have recently proposed a new concept for generating, injecting and accelerating positrons in a plasma using a double-pulse electron bunch. Monte Carlo simulations show that the number of the positrons produced in a foil target has an exponentially decay energy spectrum. The energy threshold for the trapping of these positrons in a ultra-relativistic electron wake is investigated numerically. For a typical 28.5 GeV electron drive bunch, the trapping threshold for the positrons is a few MeV, and therefore a majority of positrons generated in the foil target are focused and accelerated by the plasma wake.

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INTRODUCTION

To realize a future plasma-based linear collider, high-gradient positron and electron acceleration is equally important. Recent plasma wakefield accelerator (PWFA) experiments have demonstrated high-gradient electron acceleration in the highly nonlinear plasma wake. Compared to electron acceleration, positron acceleration in a plasma is more challenging. Experimentally, positron acceleration has been less studied due to the lack of suitable relativistic positron beams. Nevertheless, focusing \cite{1, 2} and acceleration \cite{3} of positron beams in a plasma has been demonstrated in the proof-of-principle experiments at Stanford Linear Accelerator Center (SLAC). However, the plasma wake created by the positron beam is not desirable for particle acceleration. This is because the suck-in plasma electrons do not reach at the axis at the same time, and phase-mix in the first oscillation. As a result, the accelerating field is found to be two to five times smaller \cite{4} and the focusing field is nonlinear. However, in the plasma wake driven by an electron beam, the space charge force of the suck-in plasma electrons can provide a large accelerating field and focusing field for the positrons right behind the suck-in region. Therefore, we propose to inject a short positron bunch into the phase of a plasma wake driven by an electron bunch for both focusing and accelerating positrons.
FIGURE 1. Schematic of positron focusing and accelerating in the plasma wake driven by an electron beam or a laser beam. The positron bunch is injected into a phase where the plasma wakefield is both focusing and accelerating positrons, right behind the plasma electron density spike.

However, it is difficult to transport an electron bunch and a positron bunch with a separation of a few hundred microns along the same beam line in conventional RF accelerators because the separation distance is much less than a RF bucket length. To solve this technological issue, we propose a new approach to generate and accelerate a positron bunch in a plasma using a double-pulse electron bunch [5]. The idea is described as follows (Figure 2): two closely-spaced electron bunches are focused on a high Z thin foil target and produce positrons through pair creation. The foil target is placed inside the plasma where the plasma density is uniform to avoid positron loss due to variation of plasma wavelength. The resulting positron bunches are superimposed in space and time with the original electron bunches. Then the four bunches propagate into the plasma. We can either change the spacing between the two electron bunches or the plasma density to ensure that the trailing positron bunch is riding on the right phase of plasma wake. The strong transverse wakefield in the blowout region focus the drive electron bunch but defocus the drive positron bunch. The situation is reversed in the suck-in region: the trailing positron bunch is focused but the trailing electron bunch is defocused. Only the drive electron bunch and the trailing positron bunch remain in the plasma after a short propagation distance. The desirable accelerating structure for positrons (Figure 1) is obtained.

FIGURE 2. Schematic of positron generation, injection and acceleration in a plasma. The two electron bunches (1, 2) generate two positron bunches (1’, 2’) in a thin foil target. The drive electron bunch (1) and the trailing positron bunch (2’) remain in the plasma once the transverse wakefield have blown out the drive positron bunch (1’) and the trailing electron bunch (2).

**POSITRON GENERATION**

Interaction between particle beam and foil target is modeled using the Monte-Carlo code Electron Gamma Shower 5 (EGS5) [6]. Figure 3 show the full phase space of the electron bunch and the generated positron bunch after a 0.5 mm Tantalum target. The
incoming electron bunch suffers relatively small energy loss in the target (Figure 3a), and is still able to drive a large amplitude plasma wake. The energy spectrum of the generated positron bunch (Figure 3b) has approximately an exponential distribution. The positron yield for this target is about 5%.

![Graph showing energy spectra of electrons and positrons](image)

**FIGURE 3.** EGS5 simulation results of the energy spectra of the electrons (a) and the produced positrons (b).

**POSITRON ENERGY TRAPPING THRESHOLD**

To systematically examine positron injection in a reasonable time, we use the 2D cylindrically symmetric version OSIRIS. The electron and positron beam parameters after the target are induced from the full phase space of the 3D EGS5 simulation. In this example, the energy spectrum for the positron bunches is fitted into half-Gaussian distribution with r.m.s. energy of 120 MeV. The normalized emittances in the r and θ directions are deduced according to $e_{N_r} = e_{N_θ} = [(e_{N_r}^2 + e_{N_θ}^2)/2]^{1/2}$ by assuming $e_{N_r} = e_{N_θ}$. The number of particle in the drive electron bunch is $1.8 \times 10^9$ since the created modestly nonlinear plasma wave is favorable for positron injection. The rest of parameters are listed in Table 1.

**TABLE 1** 2D OSIRIS simulation parameters for optimizing positron injection and acceleration. $N_b$ is the total number of particles in the bunch.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Electron beam energy (GeV)</td>
<td>28.5</td>
</tr>
<tr>
<td>Electron bunches $e_{N_r}/e_{N_θ}$ (mm rad)</td>
<td>75/75</td>
</tr>
<tr>
<td>Positron bunches $e_{N_r}/e_{N_θ}$ (mm rad)</td>
<td>5/5</td>
</tr>
<tr>
<td>Drive electron bunch length (μm) &amp; $N_b$</td>
<td>26.0 &amp; 1.8$\times$10$^9$</td>
</tr>
<tr>
<td>Trailing electron bunch length (μm) &amp; $N_b$</td>
<td>6.5 &amp; 4.5$\times$10$^8$</td>
</tr>
<tr>
<td>Drive positron bunch length (μm) &amp; $N_b$</td>
<td>26.0 &amp; 9.0$\times$10$^7$</td>
</tr>
<tr>
<td>Trailing positron bunch length (μm) &amp; $N_b$</td>
<td>6.5 &amp; 2.3$\times$10$^7$</td>
</tr>
<tr>
<td>Transverse spotsizes $σ_r$ (μm)</td>
<td>10</td>
</tr>
<tr>
<td>Distance between the bunches (μm)</td>
<td>130</td>
</tr>
<tr>
<td>Preformed plasma density (cm$^{-3}$)</td>
<td>5$\times$10$^{16}$</td>
</tr>
<tr>
<td>Simulation box size (c/ω$_p$)</td>
<td>20x4</td>
</tr>
<tr>
<td>Cell size (c/ω$_p$)</td>
<td>$dz=0.05$, $dr=0.02$</td>
</tr>
<tr>
<td>Number of particles per cell</td>
<td>Plasma: 4, Beam: 25</td>
</tr>
<tr>
<td>Time step dt (1/ω$_p$)</td>
<td>0.017</td>
</tr>
</tbody>
</table>
In a ultra-relativistic phase wake driven by a high-energy electron bunch, the phase velocity of the plasma wave is equal to that of the drive bunch, and close to the speed of light. Therefore the initial positrons need to have a significant forward momentum or energy to be trapped by the plasma wave. The EGS5 simulation shows that the number of positrons produced in the foils decreases exponentially with energy, and there are many low-energy positrons. Therefore, we calculate how many initial positrons can be trapped by the plasma wake and deduce an energy threshold for positron trapping. In the simulations, we inject positron bunches with various single-value energies. All the other beam and plasma parameters are kept fixed. The trapping efficiency here is defined as the number of positrons remained in the plasma divided by the number of initial positrons after 12 cm propagation into plasma. Figure 4 shows that the positron injection efficiency is close to 100% for injection energies larger than 5 MeV. In this example, the relativistic factor for plasma wave ($\gamma_{ph}$) is 56,000 (the drive bunch incoming energy of 28.5GeV), the wakefield amplitude normalized by the cold wave breaking amplitude ($\epsilon = cE/\gamma mcw_0$) is 0.25. According to the 1D linear particle injection theory [7], the relativistic factor for trapping threshold is given by:

$$\gamma = \gamma_{ph}^2 \left\{ \epsilon + 1/\gamma_{ph} - \beta_{ph} \left[ (\epsilon + 2/\gamma_{ph}) \epsilon \right]^{1/2} \right\}$$

where $\beta_{ph} = \sqrt{1 - 1/\gamma_{ph}^2}$. For the beam and plasma parameters considered here, $\gamma$ is 5 or an energy of ~2 MeV, which is close to the energy injection threshold deduced from the OSIRIS simulations. EGS5 simulation results (Figure 3b) show that more than 99% of the positrons produced by the 28.5 GeV electron bunch in the 0.5 mm Tantalum target are born with energy larger than 2 MeV. Therefore, a majority of positrons born in the target are able to inject into the relativistic propagating plasma wake, the positron injection loss is not a big issue.

![FIGURE 4. Positron injection efficiency versus the positron beam energy.](image)

**SUMMARY**

The four-bunch scheme recently proposed provides a promising approach for realizing high-gradient positron acceleration in a plasma. It also provides an opportunity to experimentally study positron injection and acceleration on an electron wake before external injection of a positron beam loaded on a plasma wake becomes available. We determined that the energy threshold for trapping of the positrons in the
wake driven by the ultra-relativistic electron bunch is around 5 MeV, corresponding to a trapping efficiency larger than 95%. This value is in good agreement with the value derived from the trapping theory in linear wakes. We also determined from the simulations of the energy spectrum of the positrons produced by pair creation in a thin Ta target that more than 99% of the positrons have a forward energy larger than the trapping threshold. Therefore, large overall trapping efficiencies can be expected in the four-bunch scheme recently proposed.

ACKNOWLEDGMENTS

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REFERENCES