Contribution to half-harmonic splitting resulting from Raman backscattering in a ripple plasma

H. Figueroa and C. Joshi
University of California, Los Angeles, California 90024

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The dispersion relation of electron plasma waves in a ripple plasma is analyzed. The ripple couples the plasma wave to its spatial harmonics thereby introducing “forbidden” frequency bands. It is shown that the stimulated Brillouin scattering (SBS) induced ripple can directly influence the frequency matching condition for Raman near the quarter-critical density and thereby lead to an asymmetric splitting of the half-harmonic radiation. The predicted blue-shifted peak is found to be more sensitive to the ripple amplitude than the red-shifted peak. This mechanism may contribute to the half-harmonic splitting that is observed in many experiments.

Recently, there has been a resurgence of interest in the stimulated Raman scattering (SRS) and the two-plasmon decay (TPD) instabilities when an intense laser beam interacts with plasma whose density is close to the quarter-critical density. In laser fusion, the plasma waves associated with these instabilities may produce significant nonthermal heating of the electrons leading to pellet preheat. Evidence for these instabilities comes from detailed measurements of the odd integer harmonic ($\omega_0/2$ and $3\omega_0/2$) radiation, hot-electron emission, and Thomson scattering. In particular, several groups have observed an asymmetric splitting of the half-harmonic radiation, which depends on target composition and laser intensity.1-3 This splitting has been attributed to direct and indirect mode conversion from TPD or the presence of magnetic fields in the megagauss range in the vicinity of the $n_c/4$ layer. In this paper we suggest another mechanism that may contribute to the observed splitting of the $\omega_0/2$ radiation; the result of SRS occurring in the presence of stimulated Brillouin scattering (SBS) near the quarter-critical layer. SBS effectively introduces a ripple on the plasma. The plasma wave from SRS can now couple to its spatial harmonics thereby introducing “forbidden” frequency bands. As a result the frequency matching for Raman near $n_c/4$ can take place at two densities, leading to the splitting of the $\omega_0/2$ radiation. The predicted blue-shifted peak is found to be more sensitive to the ripple amplitude than the red-shifted peak.

We begin with the analysis of the dispersion relation of the electron plasma wave in a rippled plasma. Consider an infinite plasma whose density varies sinusoidally in space about an average $n_0$ as

$$n_0(x) = n_0(1 + \epsilon \cos k_x x).$$  

Here $\epsilon$ is the ripple height. The dispersion relation of the plasma wave in the presence of such a ripple is approximated by

$$\left(\alpha^2 - q^2\right) - \frac{\beta^2}{4} \left(\frac{E_1}{E_0} + \frac{E_{-1}}{E_0}\right) = 0,$$

where

$$\alpha^2 = b - \frac{\beta^2}{2}, \quad \beta^2 = \frac{\omega_0^2}{v^2_e} k_x^2,$$

$$b = 4(\omega^2 - \omega_0^2 + \epsilon \omega_0^2)/3v^2_e k_x^2,$$

and $\omega_0^2 = 4\pi n e^2/m$, $q = 2k/k_c$, and $E_{\pm 1}$ are the amplitudes of the first spatial harmonics that in turn are coupled to the second spatial harmonics and so on. Equation (2) can be solved numerically by carrying out a continued fraction expansion in $E_{\pm 1}$, which has a rapid convergence. Notice that as $\epsilon$ goes to zero, we obtain $\alpha^2 - q^2 = 0$, which is the usual dispersion relation for the plasma wave. Equation (2) is a simplified version of Eq. (17) of Ref. 4 with $n = 0$.

When a ripple is present, the wave can couple to its spatial harmonics which have the same frequency but their wave vectors are shifted by multiple integers of $\pm k_c$. Figure 1 shows the numerical solution for the dispersion relation for $\beta^2 = 2$ and $q$ varying between $-4$ and $+4$. The real solutions for $\alpha^2 < 5$ are shown in solid lines. The dispersion relation is seen to break at $\alpha^2 = n^2$ ($n = 1, 2, ...$) giving rise to “forbidden” frequency bands. The bands result from Bragg reflections of the waves at intervals separated by $k = k_c/2$. On this same plot, the imaginary parts of $q$ have been plotted (dotted curve) at the first forbidden band. Since waves with frequencies between $\alpha^2$ and $\alpha^2$ possess complex wave vectors, these waves are damped. This phenomenon is analogous to the forbidden bands in solids, in distributed feedback lasers, and in periodic structures; such as a disk loaded waveguide. Clearly, the width of the forbidden bands as well as the attenuation coefficient (Im $q$) de-

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pends on the ripple amplitude. This is shown in Fig. 2. Notice that for high enough $\epsilon$ the wave can decay one $e$-folding in a distance much less than the ripple wavelength.

Now we consider the effects of a rrippled plasma on SRS occurring close to $n_e/4$. Figure 3 shows the details of the dispersion relation of the primary electron plasma wave (which is the strongest mode) showing the frequency discontinuity at $k_p = k_0$ (or $k_0 = 0$) when the ripple is present. The dashed curve in this figure represents the plasma wave dispersion relation in a warm plasma in the absence of a ripple. The plasma density increases in the vertical direction.

We have also shown the points where $\omega$ and $k$ matching conditions may be satisfied. For the Raman instability matching can be achieved at points (a) in an unrippled plasma or at points (b) if the plasma is rippled. Here we assume that the effect of the ripple on the em waves can be neglected because $\beta^2$ now scales at $1/c^2$ instead of $1/3\omega_p^2$. The ripple introduces a negative shift $\Delta k$ in the wave vector and a positive shift $\Delta \omega$ in the frequency of the excited plasma wave. As a consequence the scattered spectrum is red shifted. In regions near the quarter-critical density ($k_p = k_0$) there is a continuous range of solutions for frequency matching, but only the solutions at each end of the frequency step are undamped. These two solutions introduce two different shifts in the frequency away from $\omega_0/2$ in the backscattered spectrum.

These shifts have been estimated by using the frequency matching conditions and give the result

$$\Delta \omega = -\frac{9}{8} \alpha_+^2 \frac{\omega_0^2}{c^2} \omega_0 = -\frac{9}{8} \frac{T(\text{keV})}{511} \alpha_+^2 \omega_0.$$  (3)

Thus, the shift is just the thermal shift multiplied by $\alpha_+^2$.

Since $\alpha_+^2$ is larger than 1, the shift produced by this solution will be positive and the scattered spectrum will be red shifted by an amount given by Eq. (3). The shift produced by $\alpha_-^2$ is more involved since $\alpha_-^2$ can be positive or negative. The maximum value $\alpha_-^2$ can take is 1 (no ripple), where we obtain the usual red shift as a result of thermal corrections. As the ripple develops the value for $\alpha_-^2$ becomes negative, creating a blue-shifted spectrum.

We now apply this new source of splitting of the $\omega_0/2$ line to a specific example of $0.35 \mu$m laser and a plasma temperature of 1 keV. The wavelength shift of the backscattered light from the cold plasma case ($\lambda_s = 2\lambda_0$) is shown for both satellites as a function of the ripple amplitude in Fig. 4. It can be seen that the blue-shifted line is more

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**FIG. 1.** Dispersion relation of a plasma wave. The solid line represents the dispersion relation in the presence of a ripple with $\beta^2 = 2$. For comparison, the dispersion relation in the absence of a ripple (dashed curve) is shown. Also, the imaginary part of $q$ in the first forbidden band (occurring at $Re(q) = \pm 1, 3, \ldots$) is shown at $q = 1$.

**FIG. 2.** Plots of $\alpha^2_+$, $\alpha^2_-$, and $\text{Im}(q)$ vs $\beta^2$. Notice that the width of the first forbidden band and the damping of the wave increase with the ripple height.

**FIG. 3.** The $\omega$ vs $k$ diagrams of the plasma wave in the presence of a ripple in the region where $k_0 \approx k_0$. Four different cases corresponding to four different plasma densities are shown.

We have also shown the points where $\omega$ and $k$ matching conditions may be satisfied. For the Raman instability matching can be achieved at points (a) in an unrippled plasma or at points (b) if the plasma is rippled. Here we assume that the effect of the ripple on the em waves can be neglected because $\beta^2$ now scales at $1/c^2$ instead of $1/3\omega_p^2$. The ripple introduces a negative shift $\Delta k$ in the wave vector and a positive shift $\Delta \omega$ in the frequency of the excited plasma wave. As a consequence the scattered spectrum is red shifted. In regions near the quarter-critical density ($k_p = k_0$) there is a continuous range of solutions for frequency matching, but only the solutions at each end of the frequency step are undamped. These two solutions introduce two different shifts in the frequency away from $\omega_0/2$ in the backscattered spectrum.

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We now apply this new source of splitting of the $\omega_0/2$ line to a specific example of $0.35 \mu$m laser and a plasma temperature of 1 keV. The wavelength shift of the backscattered light from the cold plasma case ($\lambda_s = 2\lambda_0$) is shown for both satellites as a function of the ripple amplitude in Fig. 4. It can be seen that the blue-shifted line is more
sensitive to the variations of the ripple height than the redshifted counterpart. If the first spatial harmonic is taken into account, there is yet another point at which the locus for \((\omega, k)\) matching intercepts the plasma wave dispersion relation and this contributes further to the blue side of the \(\omega_{0}/2\) spectrum. At this point it is worth mentioning that the coupling of energy from the fundamental plasma mode (the dominant mode) to the spatial frequencies represents a further damping and could raise the threshold for SRS in regions below the quarter-critical density in the plasma.

In Fig. 5 we show experimental data taken with an aluminum target that shows the behavior of the backscattered spectrum around \(\omega_{0}/2\) as the laser intensity is varied.\(^1\) This data shows all the qualitative features predicted by the above model. At a laser intensity of \(2.5 \times 10^{14}\) W/cm\(^2\), one observes a red satellite that is shifted from the \(\omega_{0}/2\) position by 60 Å whereas the blue satellite is weak and is shifted by only 20 Å. As the laser intensity is increased the spacing of both satellites is further increased but the blue satellite is more sensitive to laser intensity than the red. An average wavelength shift of 50 Å for \(I = 5 \times 10^{14}\) W/cm\(^2\) (not shown in the figure) implies a ripple \(\epsilon = 5\%\) assuming a 1 keV temperature. We speculate that the amplitude of the SBS induced ion wave is a function of laser intensity in these experiments leading to the observed behavior. The width of the red and the blue satellites may be related to a time-dependent ripple, two-dimensional effects, or contributions to this splitting from other mechanisms such as the \(2\omega_p\) decay. To quantitatively compare the observed shifts with those predicted by this model requires the knowledge of both the local temperature and ripple height. We note that similar data on half-harmonic splitting has been obtained on CH targets.\(^2\) Clearly, further studies using time-resolved measurements of SBS and \(\omega_{0}/2\) are needed for more detailed comparison with this model.

In summary, we have discussed how the dispersion relation for plasma waves is altered in the presence of a zero-order density ripple. We find that the SBS induced density ripple can directly influence the energy and the momentum conservation for Raman near the \(n_e/4\) layer thereby leading to an asymmetric splitting of the half-harmonic radiation. The influence of the ripple on sub-quarter-critical Raman, which has a higher threshold than Raman at \(n_e/4\), and TPD needs to be calculated. This mechanism may also contribute to the observed splitting of the half-harmonic radiation in many experiments\(^1,2\) where such a splitting was attributed to the two-plasmon decay.

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