Motion of relativistic electrons through transverse relativistic plasma waves

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We examine computationally the interaction of a pulse of relativistic electrons that is injected perpendicular to the direction of propagation of a relativistic plasma wave. Such a study is useful for evaluating the feasibility of using transverse beam injection as a diagnostic of the amplitude of electric fields in a plasma wave, as well as for determining the suitability of using such waves as extremely short wavelength undulators for generating radiation from free electrons. We use 2D and 3D models which include the effects of transverse and longitudinal fields of the plasma wave, its finite size and electron beam emittance. We find that the change in the spot size of a short electron bunch as it traverses a relativistic plasma wave can indeed be a sensitive diagnostic of its local amplitude-width product.

I. INTRODUCTION

The measurement of large amplitude relativistic plasma waves is a difficult task and, aside from Thomson scattering, there are no well-established diagnostic techniques. We consider the measurement of plasma waves that are excited by optical mixing of two laser pulses in a dense plasma. In our laboratory, the excited plasma waves have typical density fluctuations of up to approximately 10% of the background density (10^{16}-10^{17} cm^{-3}). Such high densities imply that the wavelengths of the plasma waves are submillimeter and lifetimes are on the order of 100 ps. The laser wavelength combinations used in beat excitation are either 9.6 and 10.6, 9.6 and 10.3 or 10.3 and 10.6 μm. In this article we consider the scattering of an injected relativistic electron bunch by the plasma wave as a diagnostic technique which may complement Thomson scattering. We herein report the results of numerical simulations of the injected electron beam deflection and emittance increase due to interaction with the electric fields of the plasma wave.

We note that recently, large amplitude plasma waves have been under intense study because of their potential in accelerating particles at a very rapid rate, and their potential as plasma wigglers for generating high frequency radiation. Some ways in which large amplitude plasma waves may be excited are (a) by the beating of two lasers in a plasma which have a difference frequency equal to the background plasma frequency (optical mixing) as in the beat wave accelerator scheme, (b) by a dense compact relativistic electron bunch drifting through a plasma which leaves behind a relativistic plasma wave in its wake as in the wake field accelerator scheme, or (c) by a short intense laser pulse passing through a plasma which leaves behind a relativistic plasma wave in its wake as in the laser wake field acceleration scheme. We previously used 2D and 3D numerical models to predict the electron energy spectrum in a plasma wave accelerator, wherein the electrons were injected parallel to the plasma wave velocity. For the present study, the same numerical models are used to study the motion of electron bunches which are injected transverse to the plasma wave velocity.

This model is also used to study the radiation emitted by an electron as it wiggles in the periodic plasma wave fields, called the “plasma wiggler” scheme. The spontaneously emitted radiation is of interest since its wavelength, \lambda_r, can be many times shorter than the wavelength of the plasma wave, or \lambda_r = \lambda_p/2\gamma^2, where \gamma is the electron’s total energy in units of its rest energy and \lambda_p is the wavelength of the plasma wave. The electric fields in the plasma wave may also deflect the electrons to the extent that the radiation is affected. The present paper deals with the spatial effects on the electron beam due to the plasma wave fields and a future paper will report on the radiated frequency spectrum and power distribution due to the fields.

II. MODEL AND ASSUMPTIONS

The model assumes that the longitudinal fields of the plasma wave are periodic in the z (wave propagation) direction, and that the maximum value of the field is constant. The transverse fields have a gaussian profile in the directions perpendicular to z. The fields vary in time at the plasma frequency. A wide plasma wave (tens of plasma wavelengths wide) was used in this simulation because many undulations were desired for the plasma wiggler radiation study. The velocities of the electrons and the phase velocity of the plasma wave are approximately equal to the speed of light. (In a previous study the plasma wave’s full gaussian width was one plasma wave wavelength, and the resulting large radial field had a strong effect on the electron trajectories, which approximated the situations found in plasma wave acceleration experiments. Later we will briefly comment on transverse electron trajectories through narrow plasma waves.) The space-charge repulsion among the electrons is considered negligible, so that the electrons are noninteracting, which is a consequence of the number of injected electrons being less than the beam loading limit. The fractional plasma wave amplitude, a_{wiggler} is considered small (<0.15), the wave is linear and the maximum

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FIG. 1. Typical two-dimensional trajectory of a single electron drifting through a plasma wave. The electron moves from left to right and the plasma waves move from bottom to top in the figure.

longitudinal fields scale as \( \sqrt{n_e} \) V/cm, where \( n_e \) is the background plasma density in cm\(^{-3}\).

The theoretical derivation of the longitudinal and transverse plasma wave electric fields, \( E_x, E_y, \) and \( E_z \), and a discussion of the numerical solution of the 2D and 3D relativistic equations of motion appear in earlier publications.\(^9\)\(^{10}\)

III. 2D MODEL RESULTS

An example of the 2D trajectory of a single electron drifting transversely through a plasma wave is shown in Fig. 1. The plasma wave moves from bottom to top and the electron is injected from left to right in the figure. The major features of the trajectory are that (a) the electron is angularly deflected away from its initial direction of motion either upward or downward in the figure depending on its phase of injection, (b) the electron has a small drift always in the direction of the plasma wave velocity, and (c) the electron is wiggled while inside the plasma wave, all due to the plasma wave’s longitudinal electric field. As the plasma wave is relatively wide, the plasma wave’s radial electric field is negligibly small.

The electron bunch drift, which is generally just a fraction of a micron, would be difficult to measure in the laboratory. However, the deflection angle, which could result in large changes in the electron bunch spot size as it drifts through the plasma wave, varies with the electron’s injection radius and phase. In an experimental situation, electrons in a pulse would enter the plasma wave at all phases and radii, and there would be a distribution of deflections, which is more conveniently studied using the 3D model.

IV. 3D MODEL RESULTS

For the 3D simulations results shown below the electron bunch contains 5000 particles which have a uniform random distribution along the direction \( r \) of the bunch’s velocity and have a gaussian random distribution in the directions transverse \( (x, z) \) to the bunch’s velocity. The plasma wave’s velocity is in the \( z \) direction. The electron bunch’s and plasma wave’s Gaussian half-widths are approximately 0.050 and 0.50 cm, respectively, which correspond to measured values in an ongoing plasma wave experiment.\(^5\) The electron bunch is 10 mm long, which corresponds to an electron pulse length of 33 ps. At the start of the simulation, the front of the electron bunch is 0.75 cm from the centerline of the plasma wave, and at the end the front of the bunch is approximately 10 cm past the centerline.

Figure 2 shows the effects of variations of plasma wave amplitude, or \( A_{\text{wiggler}} \), on the electron bunch density distribution, when the emittance is zero. In this figure the pattern of scattered dots represents the electron distribution after the bunch has passed perpendicularly through the plasma wave and has drifted approximately 10 cm away from the wave’s centerline. The plots show the end-on view of the bunch with the electrons moving up out of the plane of the figure while the plasma wave moves from left to right. Figure 2(a) shows the electron distribution for the case in which the fractional plasma wave amplitude is zero, which corresponds to no plasma wave present. This distribution is the original injected electron distribution, since the electrons travel in straight lines when the emittance is zero. Figures 2(b), (c), and (d) show the electron distributions when the plasma wave’s amplitude was increased to 5%, 10%, and 20%, respectively. We see that the bunch has been elongated (from about 2 to about 6 mm wide) in going from 0% to 20% fractional plasma wave amplitude. The elongation is in the \( z \) direction, parallel to the plasma wave motion, indicating that the spreading is due to the
FIG. 3. Variation of electron bunch transverse density distribution as function of the electron beam emittance. The electron beam injection energy is \( \gamma = 4 \). The plasma wave gamma phase equals 9.7, and density fluctuation, \( a_w \gamma \), equals 0.10.

longitudinal plasma wave field. This elongation may have potential as a probe of the electron wave amplitude.

We also examined the effect on the electron bunch distribution due to changes in the wavelength of the plasma wave, and found that longitudinal spreading increased approximately linearly with wavelength, for the case of no emittance. The distribution varied inversely with the electron energy, and was insensitive to changes in the width of the plasma wave for widths greater than 10 \( \lambda_p \).

Separate graphical studies not shown indicated that the radial plasma wave electric fields, \( E_x \) and \( E_y \), had no effect on the longitudinal spreading of the electron bunch in the \( z \) direction, even for the very narrow plasma waves having 0.005 cm (0.5 \( \lambda_p \)) Gaussian half-width, for the case of no emittance. For wide plasma waves the transverse \( E_x \) and \( E_y \) fields had no effect on the electron bunch in the \( x \) and \( y \) directions, but for the very narrow wave in which \( E_x \) and \( E_y \) are large, changes in the transverse directions were observed. For narrow plasma waves, having \( E_x \) and \( E_y \) large, the electron bunch's distribution spreads in the \( z \) direction (vertical in Fig. 2) and in the \( y \) direction, but remains very dense near the center. An electron bunch having a squared leading edge at injection obtains a conical leading edge after drifting through a narrow plasma wave having large radial fields.

The effect of changes in the electron bunch emittance on the bunch distribution was also examined. Figure 3 shows the bunch distributions after passing through a plasma wave having \( a_w \gamma = 10\% \) and \( \gamma = 9.7 \), and four different values of beam emittance \( [\gamma_{ph} = (1.0 - \beta_{ph})^{-1/2}, \beta_{ph} = \text{phase velocity of plasma wave}/c] \). Figure 3(a) is the bunch distribution when the initial emittance is zero, and is the same as Fig. 2(c) but on a different scale. In Figs. 2(b), (c), and (d) the emittance is increased to 10, 20, and 40 mm mrad, respectively. Current plasma wave experiments utilize electron bunches having emittance on the order of 20 mm mrad. We see that the emittance of this magnitude randomizes away any spatial structure in the bunch's distribution. By reducing the emittance we found that the structure remained visible up until the emittance increases above 1.0 mm mrad, for our sample parameters. Electron beam sources utilizing laser driven photocathodes would be required if this scheme is to be used as a diagnostic.

The change in the energies of the electrons in the bunch after passing transversely through the plasma wave was also determined using the model, for the case of no emittance. In one instance, all electrons were injected having \( \gamma = 4.00 \) exactly and after drifting transversely through the plasma wave the energy spread was found to be 3.96 < \( \gamma < 4.04 \), or 2%.

V. SUMMARY

We have found that the scattering of a single bunch of injected relativistic electrons by a large amplitude relativistic plasma wave can be a sensitive diagnostic of the plasma wave electric fields. Based on the numerical model we summarize the angular deflection, \( \Delta \theta \), in the \( z \) direction of a cold beam of energy, \( \gamma \), injected across a plasma wave of wavelength, \( \lambda_p \), and amplitude \( a_w \):

\[
\Delta \theta = \kappa \lambda_p a_w \gamma / \gamma,
\]

where \( \kappa \) is a constant. Thus, the angular deflection is greater than the angular spread of an incident beam of radius \( \sigma \) for normalized beam emittances:

\[
\varepsilon_n = \gamma \sigma < \kappa \lambda_p a_w \sigma.
\]

If larger beams are used, the allowable emittance before the structure is lost should increase in proportion to the beam size.

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