

## Nonlinear Evolution of the Modified Simon-Hoh Instability via a Cascade of Sideband Instabilities in a Weak Beam Plasma System

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The modified Simon-Hoh instability is observed in a collisionless cylindrical plasma, produced by a weak electron beam, in which electrons are strongly magnetized and the ions are essentially unmagnetized. The nonlinear evolution of this instability occurs through a sequence of sideband instabilities, thought to be induced by trapped ions, and can lead to a chaotic state.

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Turbulence abounds in plasmas [1] as is evident from measurements on laboratory, fusion, space, and astrophysical plasmas. However, it is extremely difficult to study the evolution of turbulence in a plasma because, in addition to having a rich variety of collective modes *a* of oscillation, a plasma also has many nonlinear coupling mechanisms. In this Letter we document the nonlinear evolution of a single coherent mode (a modified Simon-Hoh instability) in a plasma produced by a weak electron beam via a sequence of sideband instabilities that are thought to arise because of ion trapping effects. While the eventual frequency-locked states observed in these experiments appear to be of the generic period-doubling type [2], the transitions are not. As in some fluid experiments [2] each transition begins with the appearance of an apparently new oscillation mode at a very low frequency after which the frequency rises to half the value of the previous lowest frequency where it meets and locks with the decreasing-frequency local lower sideband frequency. We believe that such a sequence of successive sideband instabilities and locking may be common to plasma systems in which large ion orbit dynamics are important and may lead to understanding their transition to turbulence.

To be able to exhibit the fine details of the transitions such as those shown here, the plasma must be extremely controllable. The low electron density ( $n_e \sim 10^6$ – $10^9$  cm<sup>-3</sup>) plasma is generated by ionization of background gas by a very-low-density electron beam ( $n_b \sim 10^4$ – $10^7$  cm<sup>-3</sup>). A Gaussian, temperature-limited, electron beam ( $I_b \sim 1$   $\mu$ A–1 mA, 250 V) with 5 mm (FWHM) diameter is injected axially from one end of a 10-cm-diam, 180-cm-long stainless-steel tube immersed in a dc axial magnetic field ( $B \sim 50$ –320 G). The beam and the plasma are terminated by a grounded collector located at 80 cm distance from the gun. The plasma density can be controlled by varying the ionization rate either by changing the gas fill pressure ( $5 \times 10^{-6}$  to  $5 \times 10^{-5}$  torr) or by varying the beam current over its range. At these low values of pressure and current, the system is always below the beam-plasma discharge [3] threshold and thus can be kept remarkably free of noise.

In this experiment ( $B = 160$  G, Ar plasma) the electrons are strongly magnetized ( $r_{Le} = 0.04$  cm for  $T_e \approx 4$  eV) whereas the ions are essentially unmagnetized ( $r_{Li}$

$\sim 5.6$ – $17.7$  cm for  $T_{i\perp} \sim 1$ – $10$  eV). Here  $r_{Le}$  and  $r_{Li}$  are the electron and ion Larmor radius, respectively,  $T_e$  is the plasma electron temperature, and  $T_{i\perp}$  is the characteristic ion energy in the radial direction, as will be discussed later. The steady-state electron and ion radial density profiles were measured using a Langmuir probe. These show that the ions have a broader profile than the electrons; i.e., the center region of the chamber containing the electron beam is electron rich, whereas the outer region is seen to be ion rich [4]. These steady-state electron and ion density profiles imply the existence of a significant radial electric field  $E_{r0}$ , which was deduced by directly measuring the radial plasma potential profile  $\Phi(r)$  [Fig. 1(a)] with an emissive probe. It is this potential profile that confines the ions in the radial direction. The plasma electron temperature  $T_e$  measured by a Langmuir probe is  $\approx 4$  eV, whereas in the radial direction the characteristic ion energy  $T_{i\perp}$ , measured by an energy analyzer, monotonically decreases as the ions bounce back and forth almost radially in the radial potential well [4].

By varying either the gas pressure  $P$  or the beam current  $I_b$ , a stable (persisting for many hours in its linear stage) mode  $M_1$  with a frequency  $f_1$  ( $f_{ci} < f_1 < f_{pi}$ ) can be excited [5]. Here  $f_{ci}$  and  $f_{pi}$  are the ion cyclotron and ion plasma frequencies, respectively. This mode has four basic experimental characteristics [6]. First, it is a mode with  $m = 1$  azimuthal behavior, no radial variation in phase or frequency, and with a spectral amplitude peak at a radius of 3.5 mm from the beam axis [Fig. 1(b)]. By spectral amplitude we mean the peak spectral power density obtained by Fourier transforming the real time probe signal using a spectrum analyzer [5]. Second, the measured value of  $f_1$  is consistent with a calculated effective ion  $\mathbf{E} \times \mathbf{B}$  drift frequency  $f_{Ei}$ , which takes into account the large ion Larmor radius effect [Fig. 1(c)] [6]. Third, the measured value of the mean azimuthal velocity  $v_{\theta i}$  of the ions (using a one-sided probe [7]) gives a frequency  $f_{\theta i} = v_{\theta i}/2\pi r$ , which shows a reasonable correlation with  $f_1$  over a wide range of  $I_b$  [Fig. 1(c)]. Finally,  $f_1$  scales as  $\sqrt{M}$ , where  $M$  is the ion mass for four different gases, N<sub>2</sub>, Ar, Kr, and Xe.

All the above observations are consistent with the notion that the basic instability being observed is a saturat-

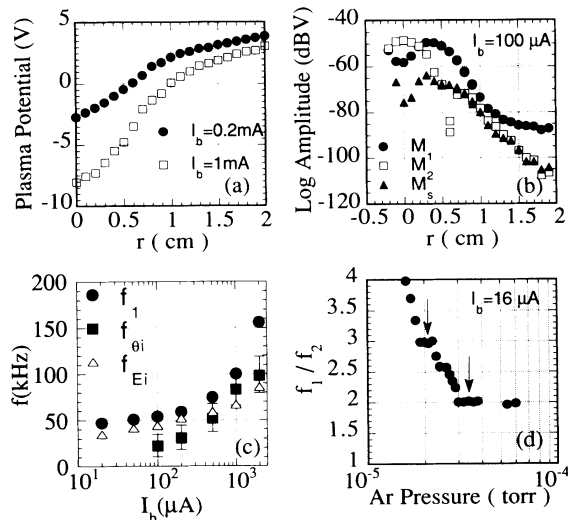


FIG. 1. (a) Radial plasma potential profile measured by an emissive probe for two different values of  $I_b$  in mA and  $P=2 \times 10^{-5}$  torr. (b) Radial profiles of the log of spectral amplitudes (in dBV) of the  $M_1$ ,  $M_2$ , and  $M_s$  modes, respectively.  $I_b=100 \mu\text{A}$  and  $P=1 \times 10^{-5}$  torr. (c)  $I_b$  dependence of  $f_1$ ,  $f_{\theta i}$ , and  $f_{Ei}$ . Here,  $f_{\theta i}$  is the measured ion rotation frequency by the one-sided probe and expressed as  $f_{\theta i} = v_{\theta i} k_{\theta} / 2\pi$ , where  $k_{\theta} = 1/r_1$  is  $2 \text{ cm}^{-1}$  and  $r_1 = 0.5 \text{ cm}$  is the radius at which the spectral amplitude of the  $M_1$  mode is maximum.  $f_{Ei}$  is the effective ion  $\mathbf{E} \times \mathbf{B}$  drift frequency for the large ion Larmor radius [6]. The electron  $\mathbf{E} \times \mathbf{B}$  drift frequency is nearly a factor of 20 larger than  $f_{Ei}$ . Here,  $f_{ci} = 6.7 \text{ kHz}$  and  $f_{pi} = 46\text{--}460 \text{ kHz}$  for  $I_b = 20\text{--}2000 \mu\text{A}$ .  $P = 2 \times 10^{-5}$  torr,  $r = 0.5 \text{ cm}$ . (d)  $f_1/f_2$  vs pressure  $P$ .  $I_b = 16 \mu\text{A}$ ,  $r = 0.5 \text{ cm}$ . The ranges of  $P$  over which  $f_1$  and  $f_2$  are frequency locked are shown by arrows. It is over this range that a cascade of sideband instabilities is observed. The uncertainties are smaller than the size of the symbols where no error bars are shown.

ed single mode of the modified Simon-Hoh instability (MSHI) [6], which can arise in a cylindrical, collisionless plasma if a radial dc electric field exists and if this dc electric field and the radial density gradient are in the same direction. In such a plasma, if the ions are weakly magnetized while the electrons are strongly magnetized, a velocity difference in the  $\theta$  direction drift can arise between the electrons and the ions because of the finite ion Larmor radius effect. If azimuthal symmetry is broken, this leads to a space charge separation in the  $\theta$  direction. The resulting azimuthal electric field  $E_{\theta i}$  and the enhancement of the density perturbation occur in the same manner as in the usual Simon-Hoh instability, seen in collisional plasmas with magnetized ions. Although the collisional Simon-Hoh instability has been investigated both theoretically [8] and experimentally [8], this is the first documentation, to our knowledge, of the MSHI dominated by kinetic effects rather than collisions.

In an initial state with azimuthal symmetry, writing (i) the electron-fluid density perturbation as  $\tilde{n}_e/n_0 = (e\phi_1/$

$T_e)\omega^*/(\omega - \omega_E)$ , where  $\phi_1$  is the fluctuating potential,  $\omega^* = -k_{\theta}(cT_e/eB_0)(1/n_0)dn_0/dr$  is the electron diamagnetic drift frequency, and  $\omega_E = -k_{\theta}cE_{r0}/B_0$  is the electron  $\mathbf{E} \times \mathbf{B}$  drift frequency, and (ii) the unmagnetized ion-fluid density perturbation (with azimuthal Doppler shift and neglecting radial variations) as  $\tilde{n}_i/n_0 = k_{\theta}^2 e\phi_1 / M(\omega - k_{\theta}v_{\theta i})^2$ , where  $v_{\theta i}$  is the mean azimuthal ion drift speed, we obtain, assuming that perturbations in the electron and ion densities are about the same, the dispersion relation

$$\omega^*/(\omega - \omega_E) = k_{\theta}^2 c_s^2 / (\omega - k_{\theta}v_{\theta i})^2, \quad (1)$$

where  $c_s = (T_e/M)^{1/2}$ . This quadratic in  $\omega$  is easily solved and one finds the real frequency to be very close to  $v_{\theta i}k_{\theta} \approx v_{\theta i}/r$ , while the growth rate is nearly equal to  $[c_s^2 k_{\theta}^2 (\omega_E - v_{\theta i}k_{\theta}) / \omega^*]^{1/2}$ . This shows that when  $cE_{r0}/B_0 > v_{\theta i}$  and  $\omega_E/\omega^* > 0$ , i.e., the dc electric field and the radial density gradient are in the same direction, a fluid instability (MSHI) can be excited. In fact, radial variation is important but examination of the orbit kinetics leads to the same conclusion with an appropriately modified definition for an averaged  $v_{\theta i}$  [6].

Experimentally, we find that at low values of beam current  $I_b$  ( $< 2 \mu\text{A}$ ) and pressure  $P$  ( $< 1.5 \times 10^{-5}$  torr), the MSHI or  $M_1$  mode is the only coherent mode that exists in the plasma. There are no other intermediate frequency [9,10], flutelike modes [10], or any electron plasma and cyclotron oscillations observable in this system. The typical signal-to-noise ratio before the MSHI begins to interact nonlinearly with the plasma is  $10^6$ , at this point the perturbed ion density level of the MSHI is typically  $\tilde{n}_i/n_0 \approx 30\%$ .

As  $I_b$  or  $P$  is increased, the spectral amplitude of the MSHI ( $M_1$  mode) increases and the harmonics of  $f_1$  up to the seventh harmonic can be observed. For a beam current greater than  $3 \mu\text{A}$ , the nonlinear sequence begins with the appearance of a second mode  $M_2$  with a frequency  $f_2$  ( $f_2 \ll f_1$ ) together with sideband modes  $M_s$  at frequencies  $f_s = f_1 \pm n f_2$  ( $n = 1, 2, \dots$ ). As  $I_b$  is increased  $f_2$  increases, and  $f_2$  and  $f_s$  migrate towards one another until they mode lock at  $f_1/n$  and  $(n+1)f_1/n$ , respectively. The onset of  $M_2$  and  $M_s$  modes is characterized by a sudden decrease in the spectral amplitude of the  $M_1$  mode. Thereafter as  $I_b$  further increases, the spectral amplitude of  $M_1$  continues to increase monotonically whereas the spectral amplitudes of  $M_2$  and  $M_s$  show a strong increase when  $f_1/f_2$  are frequency locked at 3 or 2. In Fig. 1(d) the variation of  $f_1/f_2$  vs  $P$  is presented which shows the range of  $P$  over which locking occurs. A similar frequency locking sequence can also be obtained at a given value of  $P$  by increasing  $I_b$ . The range of  $I_b$  or  $P$  over which the two frequencies are locked becomes broader when  $f_1/f_2 = 2$  (where  $f_2$  and the lower sideband of  $f_s$  become indistinguishable) compared to when  $f_1/f_2 = 3$ . It is over this parameter range where  $f_1$  and  $f_2$  are locked in this way that a whole cascade of sideband instabilities can be observed.

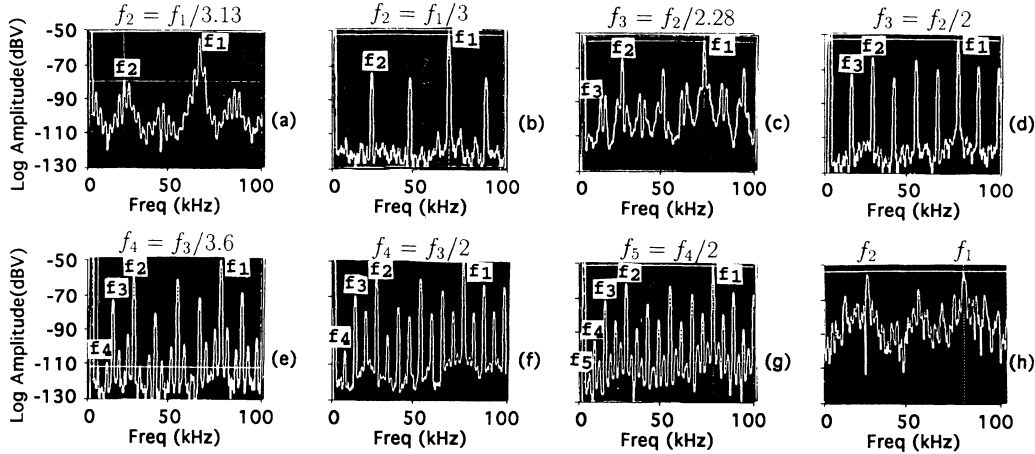


FIG. 2. Observation of a cascade of sideband instabilities at  $r=0.5$  cm when  $f_1/f_2$  is locked at 3 and  $P$  is gradually increased from (a)  $1.9 \times 10^{-5}$  to (h)  $2.15 \times 10^{-5}$  torr. The vertical scale in (a)-(h) is  $\log_{10}$  of spectral amplitude, in dBV. 0 dBV = 1 V. We identify the state observed in (h) as the chaotic state.

One such sequence, showing a great range of cascading, is shown in Fig. 2, where  $f_1/f_2 \approx 3$  [Figs. 2(a) and 2(b)]. An increase in  $P$  leads to the appearance of a third low-frequency mode  $M_3$  which has a frequency  $f_3$ , which is less than  $f_2$  and the beat components  $mf_1 + nf_2 + pf_3$  ( $m, n, p = \pm 1, \pm 2, \dots$ ) [Fig. 2(c)]. With further increase in  $P$ ,  $f_3$  migrates towards  $f_2$  until  $f_3$  locks with  $f_2$  such that  $f_3 = f_2/2 = f_1/6$  [Fig. 2(d)]. At this point with a very slight increase in  $P$  yet another mode  $M_4$ , which has a frequency  $f_4$  (which is less than  $f_3$ ), appears [Fig. 2(e)], migrates toward  $f_3$ , and locks with its lower sideband at  $f_3/2$  [Fig. 2(f)]. We have seen clearly up to five cascades of decays by extremely fine control of the pressure which gives frequencies  $f_5 = f_4/2 = f_3/4 = f_2/8 = f_1/24$  (i.e.,  $3 \times 2^3$  discrete frequency subharmonic components) [Fig. 2(g)]. Any further increase in the control parameter ( $P$  or  $I_b$ ) leads to frequency unlocking of all the modes as shown in Fig. 2(h) and the spectrum becomes quite chaotic where various peaks are no longer identifiable as a linear combination of the modes, their harmonics, and their beat frequencies. Here we point out that the nonlinear sequence described above is also applicable to the harmonics of  $f_1$  which are present over the entire pressure range shown in Fig. 1(d).

Now we discuss characteristics of the  $M_2$  and the  $M_s$  modes. While  $M_1$  is an  $m=1$  mode with an off-axis spectral amplitude peak,  $M_2$  is an  $m=0$  mode whose spectral amplitude peaks on axis [Fig. 1(b)] and has a rather deep null at about the beam edge, together with a rapid radial phase change of  $180^\circ$ , across the near-null with nearly uniform radial phase elsewhere. Like  $M_1$ ,  $M_s$  is also an  $m=1$  mode whose spectral amplitude peaks near the beam edge [Fig. 1(b)]. It is also found that after the onset of the  $M_2$  mode, its frequency  $f_2$  is proportional to the square root of the electron-density fluctuation level associated with  $M_1$  which is  $\propto \tilde{n}_e^{1/2}$ . Furthermore, the fre-

quency  $f_2$  of the  $M_2$  mode shows a reasonable correlation (Fig. 3) with the calculated value of the azimuthal ( $m=1$ ) bounce frequency of the ions. Here the bounce or trapping frequency  $f_b$  is given by  $(1/2\pi)(e\phi_1/Mr^2)^{1/2}$  and  $\phi_1$  is the fluctuating potential of the  $M_1$  mode measured at  $r=0.2$  cm and at  $r=0.5$  cm by using an emissive probe. We also found that, while the dc plasma potential showed no dependence on the ion mass,  $\phi_1$  was found to be  $\propto M$ . Since  $f_b \propto (\phi_1/M)^{1/2}$ , and  $f_2 \approx f_b$ ,  $f_2$  was thus found to be independent of the ion mass.

Now we propose the following interpretation of the above-mentioned observations. When the  $M_1$  mode attains a sufficiently large amplitude, it can trap a significant number of ions in the wave potential. These trapped ions are not, however, resonant with the  $M_1$  mode in the usual sense of executing only small oscillations around the mode angular velocity. As a result of the large radial excursions, the trapping lies in the fact that these particles are resonant over the whole cycle and

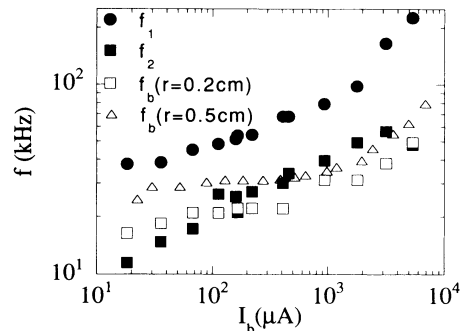


FIG. 3. Dependences of  $f_1$ ,  $f_2$ , and  $f_b$  on  $I_b$ .  $P=2 \times 10^{-5}$  torr,  $r=0.2$  cm. Values of  $f_b$  calculated from  $\phi_1$  measured at two different radii are shown.

slowly gain or lose energy and angular momentum as a result, even when making large excursions about their mean velocity and radius. A proper calculation of  $f_b$  should take the details of ion orbits in the dc as well as fluctuating fields into account. However, given that there is a reasonable correlation between the measured  $f_2$  and calculated  $f_b$ , we postulate that, as a result of trapping, the  $M_1$  mode may now be driven modulationally unstable to sideband modes  $M_s$  [11]. The sideband modes will have Doppler-shifted frequencies in the laboratory frame of  $f_1 \pm f_b = f_s$ , via a nonlinear coupling introduced by these trapped particles. Indeed, the real time probe signals do show an amplitude-modulated wave form at the beat frequencies,  $f_1 \pm f_2$ . Simultaneously, the  $M_2$  mode with a frequency  $f_2 = f_1 - f_s$  and azimuthal mode number  $m = 0$  is excited in the plasma. Although the physical origin of the  $M_2$  mode is at present not understood, there can exist a whole continuum of sound wave modes with finite  $k_{\parallel}$  which can be simultaneously excited with the strong sideband modes. However, in the experiments we were unable to measure the necessary  $k_{\parallel}$  for these modes. An alternate explanation for the  $M_2$  mode is that it is some sort of kinetic hybrid mode between the electron-rich inner core and the ion-rich (kinetic) outer halo, all driven by the azimuthally averaged modulation of the ion orbits.

As  $\phi_1$  increases,  $f_2$  migrates towards  $f_1$  ( $f_2 = f_b \propto \phi_1^{1/2}$ ) and the final state can be a new periodic oscillation, the mode-locked state with  $f_2 = f_1/n$  ( $n = 1, 2, \dots$ ). Once  $f_2$  is mode locked with  $f_1$ , this new periodic state is likely to be further modulationally unstable to a low-frequency mode  $M_3$  with a frequency  $f_3$  which will then migrate and mode lock at  $f_2/2$  with the local lower sideband frequency and so on. Thus, what at first sight appears to be the subharmonic route to a chaotic state, follows a rather complex path starting at the low-frequency modulational end and finally mode locking at the subharmonic frequency.

In conclusion, we have observed a modified Simon-Hoh instability in a collisionless plasma column that is produced by a weak electron beam. The nonlinear evolution of this instability occurs through a sequence of sideband instabilities, thought to be induced by trapped ions, and

can lead to a chaotic plasma state.

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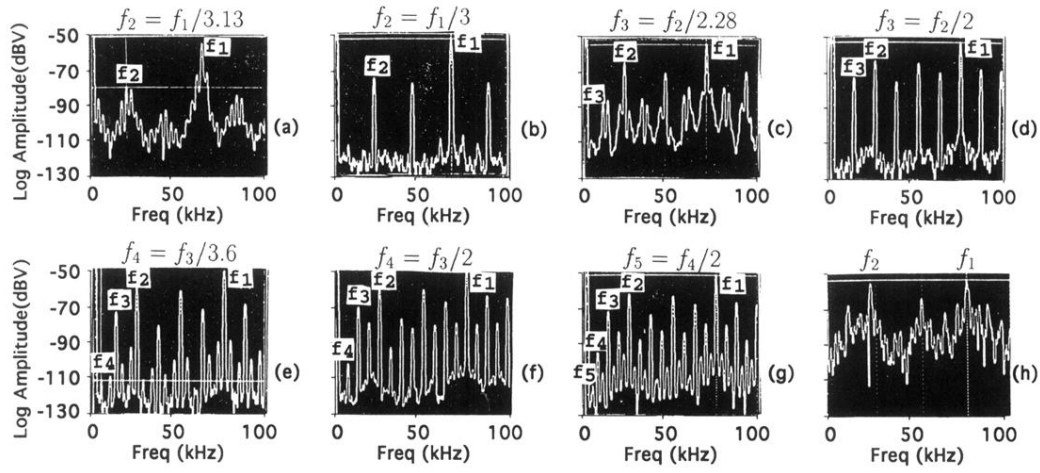


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