The Prospects for a GeV Plasma Beat Wave Accelerator

In view of the recent progress on the Plasma Beat Wave Accelerator (PBWA), the prospects of accelerating a significant number of electrons to one GeV energy are discussed. The laser, plasma and injector technologies appear to be at hand; furthermore, key issues associated with plasma production, wave excitation, instabilities and acceleration are sufficiently well understood to propose such an undertaking.

Key Words: beat wave, accelerator, laser plasma, relativistic plasma waves

1. INTRODUCTION

Relativistic plasma waves, having phase velocity \( v_{ph} \) very close to the speed of light \( c \), have been proposed as ultrahigh-gradient structures for charged particles.\(^1\) Such plasma waves are either laser driven\(^2\) as in the Plasma Beat Wave Accelerator (PBWA)\(^2,3\) or the Laser Wake Field Accelerator (LWFA)\(^2,4\) or driven by an electron bunch\(^5\) as in the Plasma Wake Field Accelerator (PWFA). Over the years many groups around the world have succeeded in exciting such plasma waves.\(^6\)–\(^8\) More recently, electron acceleration of externally injected\(^9\) as well as background plasma electrons\(^10\) has also been shown. To date experiments on the PBWA scheme have resulted in both the highest gradients and significant energy gains for the injected electrons.

In this Comment we therefore ask the question, "What are the
prospects for a 1 GeV collective accelerator based on the Plasma Beat Wave Acceleration scheme?" We believe that key physics issues associated with plasma formation, plasma wave excitation, plasma instabilities (which affect the saturation amplitude of the plasma wave), and electron acceleration are now sufficiently well understood to propose such an experiment. After reviewing the basic principles and key experimental issues associated with the PBWA, we propose an optimal set of self-consistent parameters for a GeV range experiment and argue that laser, plasma and injector technologies necessary to accomplish this in a single stage, without the need to rely on laser guiding, are now at hand.

II. BASIC PRINCIPLES

In 1979 T. Tajima and J. M. Dawson\textsuperscript{2} suggested that relativistically propagating plasma oscillations might be useful for the acceleration of charged particles to very high energies in a very short distance. In the Plasma Beat Wave Accelerator\textsuperscript{2,3} such waves are excited by collinearly propagating two laser beams ($\omega_0$, $k_0$) and ($\omega_1$, $k_1$) through the plasma such that their frequency difference matches the plasma frequency. Thus

$$\omega_0 - \omega_1 = \Delta \omega = \omega_p,$$

$$k_0 - k_1 = \Delta k = k_p.$$  \hfill (1)

The phase velocity $v_{ph} = \omega_p/k_p$ is then equal to the group velocity of light in plasma $v_g \equiv \Delta \omega/\Delta k \equiv c(1 - \omega_p^2/\omega_0^2)^{1/2}$ if $\omega_0$, $\omega_1 >> \omega_p$. The longitudinal or accelerating electric field $E$ of a plasma wave with an oscillating density $n_1$ can be estimated from Gauss' Law by approximately $\nabla \cdot E \approx ik_pE$,

$$E = |k_p\phi| = \epsilon mc\omega_p/e$$  \hfill (2)

where $\phi$ is the wave potential and $\epsilon$ is the plasma wave amplitude or fractional density bunching $n_1/n_0$. Numerically, this gives a field of $E = 0.96\epsilon\sqrt{n_0}$ in V/cm for $n_0$ expressed in cm$^{-3}$. An electron trapped at $\phi = 0$ in a plasma wave of amplitude $\epsilon$ and falling to
the bottom of the potential well will gain an energy $\Delta W$ given by

$$\Delta W \equiv 2\varepsilon \gamma_{ph}^2 mc^2.$$  \hspace{1cm} (3)

Here $\gamma_{ph}$ is the relativistic Lorentz factor, associated with the normalized phase velocity of the wave $\beta_{ph} = \nu_{ph}/c$, given by

$$\gamma_{ph} = \left[1 - \beta_{ph}^2\right]^{-1/2} \equiv \omega_0 \omega_{\rho}/\omega_{\rho}.$$  \hspace{1cm} (4)

It is clear that for given values of $\omega_0$, there is a trade-off between the accelerating gradient $E$ and the maximum energy gain $\Delta W$ in choosing the density, and therefore $\omega_1$ and $\omega_{\rho}$. As an electron accelerates, it slips forward in phase with respect to the plasma wave. The maximum energy gain is limited by this dephasing which occurs over a length $L$ given by

$$L = \frac{\Delta W}{eE} = 2\gamma_{ph}^3/k_0.$$  \hspace{1cm} (5)

Plasma wave excitation by collinear optical mixing was originally addressed by Rosenbluth and Liu.\textsuperscript{11} The growth of the plasma wave is described by the equation

$$\epsilon = \int_0^t dt' \alpha_0 x_{1} \omega_{\rho}/4$$  \hspace{1cm} (6)

where $\alpha_{0,1} = eE_{0,i}/m\omega_{0,1}c$ is the normalized oscillatory velocity in the laser field. As the plasma oscillations grow, $\omega_{\rho}$ suffers a small red shift $\Delta \omega_{\rho} = -(3/16)\epsilon^2$ due to the relativistic increase in the electron mass causing the wave to saturate at\textsuperscript{11,12,14–17}

$$\epsilon_{sat} = \left(\frac{16}{3} \alpha_0 \alpha_1\right)^{1/3}$$  \hspace{1cm} (7)

for a constant amplitude pump. This saturation occurs in

$$\omega_{\rho}^2 \tau_{sat} = 8(2/3)^{1/3}(\alpha_0 \alpha_1)^{-2/3}.$$  \hspace{1cm} (8)
For pulses shorter than $\tau_{\text{sat}}$, relativistic detuning is avoided and the plasma wave amplitude is given by Eq. (6) integrated over the pulse length.

Thus, if $\alpha_{0,1} = 0.2$, $\omega_p \tau_{\text{sat}} = 60$. This is only ten plasma periods long. Such short pulses are also helpful in minimizing other plasma instabilities, especially those involving ion motion.

The energy gain of particles injected into the plasma beat wave depends on the integral of the accelerating field amplitude (given by Eq. (6)) and the interaction length. Aside from phase slippage, the interaction length can be limited by diffraction of the laser beams (which limits the depth of focus or Rayleigh length of the lasers) or pump depletion of the laser beams. The approximate scaling laws for energy gain for each of these are as follows:

$$\Delta \gamma mc^2 \approx 1 \text{ GeV} \left( \frac{10^{18} \text{ cm}^{-3}}{n_0} \right) \left( \frac{1}{\lambda^2} \right) \text{ dephasing},$$

$$\approx 1 \text{ GeV} \left( \frac{n_0}{10^{18} \text{ cm}^{-3}} \right)^{1/2} \left( \frac{\sigma}{33 \mu \text{m}} \right)^2 \left( \frac{1}{\lambda} \right) \text{ diffraction}, \quad (9)$$

$$\approx 1 \text{ GeV} \left( \frac{10^{18} \text{ cm}^{-3}}{n_0} \right) \left( \frac{1}{\lambda^2} \right) \text{ pump depletion}.$$

Here $n_0$ is electron plasma density in cm$^{-3}$, $\lambda$ is the laser wavelength in microns and $\sigma$ is the width of the plasma wave in microns (for a Gaussian laser intensity profile of $e^{-2\sigma^2}$). There are a number of advanced concepts for overcoming these limitations; however, we do not discuss any of these in this Comment.

III. KEY ISSUES AND LESSONS LEARNED

1. Plasma Formation

In the PBWA the plasma density has to be tuned close to the resonance value over the entire length of an acceleration stage, which may be hundreds of wavelengths long. Previous theoretical
and simulation work has shown that the density homogeneity required is

\[
\frac{\delta n}{n} \leq \frac{2\pi}{\omega_p \tau_{\text{sat}}}
\]

(10)

This is a severe condition on lasers with long risetimes (many plasma periods). However, for short laser pulses this turns out to be not so restrictive; for example, in the case of \(\omega_p \tau_{\text{sat}} = 60\), we find that \(\delta n/n \approx 0.1\).

Secondly, the plasma must be free from any trapped or self-generated \(B\) fields. This forbids the use of conventional discharge-produced plasmas. However, it was shown by the Imperial College/Rutherford group\(^7\) that multi-photon ionization could produce fully ionized, sufficiently homogeneous plasmas at intensities much lower than needed for excitation of large \(\epsilon\) plasma waves. Shortly thereafter, tunnel ionization\(^{18}\) was shown to produce similar plasmas using intense and short (100 psec) \(\text{CO}_2\) laser pulses. This meant that a single laser pulse can be used to both ionize the plasma and thereafter excite the plasma wave.

The anisotropic electron distribution functions associated with MPI/tunnel ionized plasmas do give rise to large \(B\) fields via the Weibel\(^{19}\) instability. However, electron acceleration can be accomplished before the Weibel \(B\) field builds up. Another problem is ionization induced refraction.\(^{20}\) This clamps the plasma density at \(n/n_e < 2 \times 10^{-3}\) for cm-scale plasmas and \(< 10^{-4}\) for 10 cm-scale plasmas, thus restricting the parameter regime for the experiments.

The determination of plasma density is relatively easy for \(n > 10^{17}\ \text{cm}^{-3}\) using Raman backscattering or collective Thomson scattering.\(^{21}\) At lower densities (i.e., high values of \(k_p \lambda_{De}\)) one is typically in the Compton regime;\(^{22}\) therefore density measurement accuracy is reduced. The measurement of plasma uniformity is even more difficult. Even at higher densities the uncertainty in measuring the plasma uniformity is on the order of the thermal correction to the dispersion relation which is often greater than the limitation imposed by Eq. (10). For very short pulses, the ions are essentially inertially confined, and one can rely on knowledge of the neutral gas density variation prior to full ionization.
2. Excitation and Diagnostics of Relativistic Plasma Waves

This is a truly challenging problem in plasma diagnostics. None of the groups have been able to use the most obvious diagnostic in a quantitative way. This is the generation of Stokes (anti-Stokes) sidebands,\textsuperscript{23} frequency down (up) shifted by $\omega_p$ from the incident e.m. frequencies. This is because these sidebands are readily produced quite efficiently as the laser traverses any (non-linear) medium including air, vacuum-windows, lenses, etc.\textsuperscript{24} Only when interaction lengths become a substantial fraction of the pump depletion length will the sidebands become a more quantitative diagnostic.

In our opinion the most successful diagnostic is small angle Thomson scattering\textsuperscript{6,25} carried out at 90° to the plasma wave. It has given quantitative information about the plasma wave growth, frequency detuning due to combined hydrodynamic and relativistic effects, plasma wave steepening, lifetime and saturation amplitude.\textsuperscript{26}

Another diagnostic which gives qualitative information about the relativistic plasma wave is the collective Thomson scattering probe of the quasi-resonant modes\textsuperscript{27}(QRM) that are excited when the plasma wave couples to an ion acoustic wave at $2k_0$. In order to differentiate between QRM and similar wavenumber modes excited from counter-propagating optical mixing,\textsuperscript{23} the frequency spectrum near $2k_0$ can be wavenumber resolved.\textsuperscript{28} The backscatter e.m. spectrum similarly shows a clear peak near $\Delta \omega$ when a plasma wave is excited through a higher order mode-coupling process.\textsuperscript{9}

The two main lessons confirmed by the experiments on the excitation process are that to obtain significant amplitude waves the $\alpha_{0,1}$ of the two waves should be greater than 0.1 each, and that the laser risetime should be less than a few ion plasma periods. This avoids the plasma instabilities and competing processes discussed next.

3. Plasma Instabilities and Competing Processes

Over the past 15 years a number of other competing effects that can limit beat wave growth have been identified. These include plasma noise,\textsuperscript{12,17} parametric decay and modulational instabilities,\textsuperscript{12,29} mode coupling,\textsuperscript{27} hydrodynamic expansion and blowout
and transverse break-up due to the nonlinear frequency shifts of narrow plasma waves.\textsuperscript{12} As stated above, all of these can be minimized by using short and intense laser pulses. Although the Raman and Compton instabilities have hitherto been found to be energetically unimportant but useful for diagnostics, in future experiments at even higher $\gamma_{ph}$ (i.e., higher $\omega_y/\omega_p$) they will become energetically less important. Near forward Raman scatter and the related mechanism of cascade focusing\textsuperscript{30} have not been important in experiments until now. In future experiments the propagation distance of the laser beams will be much greater than the pulse length/c and these mechanisms may become important. It is not clear yet whether these effects may be beneficial, in that they could guide the lasers overcoming diffraction, or deleterious. The relativistic instabilities of the electromagnetic waves are not important since $\alpha_{0,1} << 1$. The modulational instability\textsuperscript{12,29} can saturate the plasma wave at a lower level than the relativistic detuning. However, at higher pump strengths ($\alpha_{0,1} > 0.1$) and pulses shorter than an ion period the modulational instabilities can be avoided. The plasma wave eventually breaks up transversely\textsuperscript{31} due to plasma blowout associated with its ponderomotive force and due to nonlinear frequency shifts.

4. Electron Injection and Beam Quality

Although several types of electron sources are used (or proposed) as injectors in various experiments (laser-produced electrons,\textsuperscript{32} pulsed diode\textsuperscript{33,34} and linacs\textsuperscript{32,35}) linacs are the most suitable for injecting preaccelerated electrons into a plasma accelerator. This is because linac beams are of high enough quality (low emittance and energy spread) that they can be focused to small enough dimensions and matched into the plasma accelerating structure. This requires the emittance ($\epsilon'$) of the beam to be less than

$$\epsilon' \leq 0.3 \left( \frac{\epsilon \sin \phi}{\gamma} \right)^{1/2} \sigma$$  \hspace{1cm} (11)

where $\phi$ is the phase of the particles in the plasma wave relative to the peak accelerating field. This relationship is obtained by
balancing the angular spread of the particle beam to the focusing angle of the plasma wave and by assuming that $\gamma$ is the injection energy. Also, the electrons in linacs are bunched into a few degrees of the RF phase which helps to match the time scales of the electron bunch and the plasma wave fields. This gives an improvement in signal-to-noise on the accelerated electron detectors by minimizing the number of noninteracting electrons. Linacs also readily give output energies in the few MeV range which means that the electrons are above the trapping threshold for even modest amplitude ($\epsilon \sim$ few percent) plasma waves.$^{14,36}$

The number of electrons ultimately accelerated to the highest energy in any experiment may be only a small fraction of those injected. It is therefore desirable to have at least two independent electron diagnostics so that these small signals can be confirmed to be due to electrons and not due to X-rays.

IV. PROPOSAL FOR A 1 GeV PBWA

In view of the tremendous progress in the plasma acceleration field in general and in the PBWA scheme in particular it is natural to ask, “What should be the next phase of the PBWA program?” We believe that the next phase of experiments should aim to demonstrate these two important milestones:

(1) Demonstration of substantial energy gain, say 1 GeV, for the injected electrons at an accelerating gradient of at least 1 GeV/m.

(2) Acceleration of a substantial number ($\sim 10^8$) of electrons in a modest (few percent) energy spread.

These goals can be realized using laser, plasma and injector technologies that have already been developed. Here we give (Table 1) one set of self-consistent parameters that can give 1 GeV maximum electron energy in a single acceleration stage without the need for laser beam guiding.

We have carried out numerical modeling of the plasma formation, wave excitation and injected electron acceleration for the above example. The model has been described in detail else-
where.\textsuperscript{37} Basically, we solve for the time histories of plasma density and plasma wave amplitude on a two-dimensional grid by integrating the tunnel ionization (TI) rate equation\textsuperscript{20} simultaneously with the ponderomotively driven plasma wave equation.\textsuperscript{15} For the parameter set shown in Table I, we show in Fig. 1(a) the plasma wave amplitude vs. axial position when Gaussian beam optics are used for laser propagation. At these intensities, tunneling ionization produces a fully ionized plasma over more than \( \pm 4 Z_0 \). Here \( Z_0 \) is the Rayleigh length. Over the short time scale of the laser pulse, hydrodynamic effects (plasma expansion) can be neglected. From Fig. 1(a) we see that the plasma wave axial profile is far from constant in amplitude. The peak accelerating gradient is at the best focus 16 (GeV/m) corresponding to \( \epsilon = 0.52 \). Plasma formation and excitation of the plasma wave does not produce any significant pump depletion. Nevertheless, a substantial modification of the electromagnetic spectrum is expected because of cascading.\textsuperscript{23} We have compared the predictions of this model with those obtained using a fully relativistic and electromagnetic two-dimensional particle-in-cell (PIC) code and found that the two agree rather well for the conditions shown in Table I. In Fig. 1(b) we show the 2-D plots of the plasma wave potential from the PIC simulations. One can see that these contours are mostly planar with some curvature becoming apparent due to a slight nonlinear frequency shift\textsuperscript{12} towards the end of the laser pulse.

The expected spectrum of the accelerated electrons was estimated (Fig. 1(c)) from a numerical simulation where the acceleration (longitudinal) and focusing (radial) fields were taken from

<table>
<thead>
<tr>
<th>Table I</th>
</tr>
</thead>
<tbody>
<tr>
<td>Laser Wavelengths</td>
</tr>
<tr>
<td>Plasma Density</td>
</tr>
<tr>
<td>Plasma Source</td>
</tr>
<tr>
<td>Laser Pulselength</td>
</tr>
<tr>
<td>Laser Power</td>
</tr>
<tr>
<td>Laser Spot Size (2( \sigma ))</td>
</tr>
<tr>
<td>Rayleigh Length (( Z_0 ))</td>
</tr>
<tr>
<td>Plasma Homogeneity</td>
</tr>
<tr>
<td>Peak Plasma Wave Amplitude</td>
</tr>
<tr>
<td>Peak Gradient</td>
</tr>
<tr>
<td>Final Energy</td>
</tr>
</tbody>
</table>
FIGURE 1 (a) Peak plasma wave amplitude vs. axial position calculated for a 4 ps FWHM laser pulse with an axial intensity variation given by Gaussian optics. The level at $z = 0$ is limited by the finite duration of the pump rather than relativistic detuning. (b) Output of a two-dimensional PIC simulation of the proposed experiment. The simulation box is roughly 0.5 mm (transverse or $y$ dimension) by 2 mm (longitudinal or $z$ dimension). The simulation has evolved for 120 $\omega_p^{-1}$ which is also the full duration of the laser pulse so that the pulse has just reached the right-hand boundary, as shown by the line graph at the bottom of the figure. (c) Output energy spectrum of the electrons from a numerical simulation where an electron bunch (100 A at 10 MeV with an emittance of 0.5 $\pi$-mm-mrad) uniformly filled two buckets of plasma wave and the plasma extended from $-2Z_0$ to $+6Z_0$, corresponding to $-6.1 \text{ cm} < z < 18.6 \text{ cm}$. (d) Longitudinal phase space showing strong bunching in the two accelerated buckets at the exit of the plasma.

The analytic solution to the cylindrically symmetric, linear-fluid beat wave equations for a Gaussian laser beam.\textsuperscript{38} Relativistic effects have been neglected here. The simulation assumes the injected electron current is 100 A at 10 MeV with an emittance of
0.5 π mm-mrad. The pulse duration was chosen to uniformly fill only two buckets. The plasma extended from \(-2Z_0\) to \(+6Z_0\) corresponding to \(-6.1\) cm \(< z < 18.6\) cm. Here, \(Z_0 = 3.1\) cm. The electrons were focused to a \(\approx 100\) μm spot at the edge of the plasma at \(z = -2Z_0\). We find from the simulations that better beam quality results when the injected beam \(\sigma\) is less than half of the laser beam \(\sigma\). In this simulation the centroid of the accelerated particles is at 900 MeV with approximately \(10^8\) electrons in the \(\pm 10\%\) energy spread. The energy spread has contributions from two factors; first, the injected electrons are not pre-bunched, and second, the energy changes bunch-to-bunch due to the finite growth rate of the plasma wave. Even though the electrons are injected uniformly at all phases of the plasma wave, they quickly bunch due to trapping. This bunching can be seen in Fig. 1(d) where the longitudinal phase-space is shown for the two accelerated buckets at the end of the run. The wave is moving to the right. The energy difference between the buckets is due to the fact that the plasma wave was still rising in time when the electrons were injected. The sudden drop of the current in the tail of each bunch is due to defocusing fields in those phases of the wave. The bunch duration is about \(\frac{1}{3}\) of the plasma period corresponding to about 60 fs while the spacing is \(\approx 350\) fs. The acceleration length is only about \(\frac{1}{3}\) of the dephasing length and so phase dynamics leading to energy-bunching are not playing a substantial role for these design parameters. To our knowledge such short electron pulses have never before been produced in a laboratory. In fact, one of the challenges of this next phase of experiments will be to develop diagnostic techniques to show this bunching.

This experiment requires a 14 TW, 4 ps, two frequency Nd:glass laser. While 14 TW in 4 ps appears to be rather straightforward using the chirped pulse amplification technique, two frequency operation in such a laser has not yet been shown. Rutherford Laboratory¹ and Ecole Polytechnique² groups, however, have shown two frequency operation at somewhat longer pulse widths. Therefore the laser technology should require little new research and development. If the quality of the laser beam is not good enough to give a near diffraction limited focus, the beams would have to be Raman shifted before compression.

A deterministically synchronized, short pulse, high current in-
jector is needed for this experiment. The ideal injector is a photo-injector driven RF linac giving an output bunch containing at least 1 nC in a pulse less than 4 ps long. The same laser that produces the plasma can be used (after frequency up-conversion) to produce the photoelectrons to give deterministic synchronization. Such linacs have recently been developed by many groups worldwide.\textsuperscript{39} Acceleration of $10^8$ electrons to 1 GeV needs 18 mJ of energy. The total energy content of plasma wave is on the order of 2 J so that beam loading\textsuperscript{40} will not be a problem. In future experiments it might be worthwhile to consider an inverse free electron laser (IFEL) as a pre-buncher so that precisely synchronized microbunches can be injected into the plasma wave. It might be more meaningful to study emittance growth in plasma accelerators at that point.

V. CONCLUSIONS

The key physics issues associated with plasma generation, plasma wave excitation, plasma instabilities and electron acceleration are now sufficiently well understood and the technology appears to be in hand to propose a PBWA in the GeV range. The main goal of such an experiment would be to demonstrate the acceleration of a substantial number of electrons ($O(10^9)$) to about 1 GeV energy with a reasonable energy spread, without the need to employ laser beam guiding.

Acknowledgments

This work was supported by DOE grants numbers DE-FG03-92ER40727 and DE-FG03-91-ER12114.

C. JOSHI, C. E. CLAYTON, W. B. MORI and J. M. DAWSON

University of California, Los Angeles, Los Angeles, California 90024

T. KATSOULEAS

University of Southern California, Los Angeles, California 90089

Received May 3, 1994
Contributed by T. Katsouleas
References

26. M. Everett et al., to be published.
28. M. Everett et al., to be published.
33. Y. Kitagawa, private communication.