Efficient tools for the simulation of flapping wing flows

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The development of novel strategies for lift and propulsion using flapping wings requires the use of computational tools that are at once efficient and capable of handling complex deforming boundary motion. In this work we present the use of a viscous vortex particle method for the simulation of the flow produced by a two-dimensional rigid wing in pitching and plunging motion of moderate Reynolds number. By its Lagrangian nature, this method is able to automatically adapt to important flow structures. Efficiency is ensured by using vorticity-bearing computational elements that are distributed only to the extent that vorticity itself is spread through its convection and diffusion; no effort is needed for irrotational regions of flow. Moreover, the correct behavior of the velocity at infinity is automatically satisfied, obviating the need for an artificial boundary treatment. Results of the dynamically shed vorticity and the forces exerted are presented for a single stroke of a flapping elliptical wing.

I. Motivation

Research in biologically-inspired mechanisms for propulsion has increased considerably in recent years, due in large part to a growing interest in developing micro air vehicles (MAVs) with capabilities that exceed those of conventional aircraft: for example, carrying out remote sensing while flying in a confined space. Airborne insects are a natural paradigm for such vehicles, as they are equipped for an amazing range of maneuvers: hovering, flying backward, and rapidly changing trajectory.

Placing such demands on a technological equivalent requires a thorough knowledge of the fluid dynamics involved. To develop an understanding of the physics of a flapping wing, as well as to provide a basis with which control strategies can be developed, a set of efficient and accurate computational tools is essential. In this spirit, we present here the application of a vortex particle method for solving the viscous incompressible flow around a two-dimensional flapping wing.

Computational investigations of a single flapping wing have been carried out by a number of researchers, for instance by Ramamurti & Sandberg,1,2 Liu & Kawachi3 and Wang.4 The flow produced by a flapping wing is highly unsteady and dominated by vortex shedding at its leading and trailing edges. Consequently, many investigators use a grid-based approach in vorticity-streamfunction formulation. However, conventional grid-based simulations are difficult, due to the complication of (perhaps multiple) moving boundaries, which requires that the grid be continually regenerated in response to such motion. A notable exception is the recent work of Russell & Wang,5 in which the surfaces of moving objects impose conditions that are interpolated

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to the control points of a regular Cartesian grid. Even with this capability, the use of a grid requires that a choice be made about its outer truncation, as well as the explicit introduction of an artificial boundary condition to ensure the correct behavior at infinity.

The vortex particle method, in contrast, is based on a Lagrangian approach, wherein computational elements—or particles—are identified with material elements and thus move with the flow. These particles each carry a distribution of vorticity; the velocity field of the flow is the superposition of all particles’ contributions through the Biot-Savart integral. Since particles, in a sense, define the flow rather than merely sample it, they are able to automatically adapt to the important features. They are only present in regions of non-zero vorticity, so no computational effort is wasted on irrotational regions. The computation of velocity through the Biot-Savart integral ensures that the boundary condition at infinity is automatically satisfied. Finally, the simulation of flows produced by complex moving, deformable boundaries presents no significant challenge beyond those encountered in the flow around a rigid, stationary object.

In this work, a viscous vortex particle method is applied to simulate the flow produced by two-dimensional bodies in arbitrary motion. In section II, the methodology and practical aspects of the simulation tool are briefly described. Results from applying the tool to a pitching, plunging elliptical wing are presented in section III. Finally, future directions are discussed in section IV.

II. Methodology

The use of the vortex particle method to compute viscous flows around objects was first presented by Chorin,6 in whose method the vorticity produced by the body was carried away by newly-created particles, and diffusion was simulated by a random walk. The more recent work of Koumoutsakos, Leonard and Pépin7 exhibited its utility as a tool for high fidelity simulation. Instead of creating new particles at the body, vorticity is introduced by diffusion onto existing particles. The techniques developed in their work have since been refined, and the state of the art expounded in detail in the recent book by Cottet and Koumoutsakos.8

The basic principles of the method are merely outlined here, with specialty to two dimensional flow.

A. Vortex particles

The vorticity field is regarded as the composition of a set of regularized vortex particles,

$$\omega(x, t) = \sum_p \Gamma_p(t) \zeta_r(x - x_p(t)),$$  

where $\Gamma_p$ is the strength of the particle and $x_p$ is its position. The vorticity distribution of each particle is given by $\zeta$, which is generally radially symmetric, and $\epsilon$ is the radius of the distribution. Particles are initially laid on a uniform Cartesian grid of spacing $\delta x$, and the particle strengths set to $\Gamma_p(0) = \delta x^2 \omega(x_p(0), 0)$. The particles then move with the local value of the velocity field,

$$\frac{dx_p}{dt} = u(x_p, t).$$

This velocity field is related to the vorticity through the inversion of $\omega = \nabla \times u$, which is provided by the Biot-Savart integral, plus any additional potential flow (e.g. uniform flow)

$$u(x, t) = \frac{1}{2\pi} \int \frac{\omega(x', t)}{|x - x'|^2} \left( \begin{array}{c} -(y - y') \\ x - x' \end{array} \right) \, dx' + \nabla \phi.$$
When the regularized vorticity (1) is introduced to this integral, the velocity field is expressed as a sum of all particles’ contributions, so each particle evolves according to

$$\frac{d\mathbf{x}_p}{dt} = \sum_q \Gamma_q(t) \mathbf{K}_\varepsilon(\mathbf{x}_p(t) - \mathbf{x}_q(t)) + \nabla \phi,$$

(4)

where $\mathbf{K}_\varepsilon$ is a regularized, discrete Biot-Savart kernel. Thus, the regularizing function $\zeta_\varepsilon$ serves the dual purpose of providing a smooth vorticity distribution for each particle, and eliminating the velocity singularity between two particles whose separation shrinks to zero. For stability, the method requires that particles overlap (i.e., $\varepsilon \geq \delta x$). It is important to note that, by computing the velocity via (4), its correct behavior at infinity is automatically satisfied.

For an inviscid flow, equation (4) is closed due to the invariance of vorticity of material elements. However, in a viscous incompressible flow the vorticity of a material element is governed by

$$\frac{D\omega}{Dt} = \nu \nabla^2 \omega.$$  

(5)

In the present method, the diffusive action is accounted for through modification of the particle strengths, via an evolution equation that is the discrete analog of (5). In order to ensure that this diffusion is conservative of the global circulation, the method of particle strength exchange is used, which replaces the Laplacian operator of (5) with a symmetric integral operator. Upon particle quadrature of the integral, the resulting equation for the particle strengths is

$$\frac{d\Gamma_p}{dt} = \nu \frac{\varepsilon^2}{\varepsilon^2} \sum_q (\Gamma_q - \Gamma_p) \eta_\varepsilon(\mathbf{x}_p - \mathbf{x}_q).$$

(6)

The kernel $\eta_\varepsilon$ can be constructed so that the integral operator approximates the Laplacian operator to any order of accuracy. In a given discrete time step, equation (6) is solved after the particle position equation (2) is evolved (i.e. a viscous splitting), resulting in a method for the direct numerical simulation of viscous incompressible flow.

B. Enforcing wall boundary conditions

The algorithm described thus far can be used to simulate unbounded viscous flows, but the inclusion of bounding surfaces requires further treatment, in order to enforce the conditions of no-through-flow and no-slip at the surfaces. In a vorticity-based method, such a task is not straightforward, but crucial nonetheless—particularly because the no-slip condition is the primary mechanism for vorticity production in flows of present interest.

Though there are several techniques for accounting for a no-slip surface in vortex methods, nearly all are based on the Lighthill creation mechanism. He focused on the production of vorticity in a small interval of time: At the beginning of the interval, the vorticity already present in the flow has a corresponding velocity that has the correct wall normal component, but in general, slips relative to the wall. Therefore, the vorticity created during the subsequent interval must be exactly that needed to cancel this spurious slip velocity.

The algorithm used by Koumoutsakos et al. involves a fractional step procedure that embodies this creation mechanism. In the first half-step, vortex particles move and exchange strength according to the unbounded flow algorithm above. In the second half-step, a vortex sheet is created at the wall, with a strength distribution that is determined by the no-through-flow condition (using a vortex panel method as in aerodynamics). By ‘displacing’ this vortex sheet from the wall into the flow, the no slip condition is also enforced under the sheet. This displacement is effected by the diffusion of the panel strengths onto existing particles adjacent
to the wall, through solution of the diffusion equation with the Neumann boundary condition

$$\frac{\partial \omega}{\nu \partial n} = -\frac{\gamma(s)}{\delta t},$$  \hspace{1cm} (7)

where $\gamma(s)$ is the local sheet circulation, $\delta t$ is the time step size and $n$ is the normal vector directed into the flow.

It is vital that the solution for the surface panel strengths be carried out with consideration of Kelvin’s theorem, that the sum of flow and body circulation must remain zero for all time. A rigid rotating body has circulation equal to $-2A_b \dot{\alpha}(t)$, where $A_b$ is the area of the two-dimensional body and $-\dot{\alpha}(t)$ is its angular velocity (the negative sign is included to identify $\dot{\alpha}$ with the rate of change of conventional angle of incidence). Thus the constraint on the flow circulation is

$$\frac{d\Gamma_{tot}}{dt} = 2A_b \frac{d\dot{\alpha}}{dt},$$  \hspace{1cm} (8)

where $\Gamma_{tot} = \sum_p \Gamma_p$, the sum of all particle strengths. The circulation of the flow is augmented at each time step by the diffusion of the surface vortex sheet. By integrating (5) over the flow domain, the rate of change of $\Gamma_{tot}$ is given by

$$\frac{d\Gamma_{tot}}{dt} = -\nu \int_C \frac{\partial \omega}{\partial n} \, ds,$$ \hspace{1cm} (9)

where the right-hand integral is taken over the body surface. Combining these relations, the constraint imposed on the surface circulation introduced in time interval $[t, t + \delta t]$ is

$$\int_C \gamma(s) \, ds = 2A_b \left( \dot{\alpha}(t + \delta t) - \dot{\alpha}(t) \right).$$ \hspace{1cm} (10)

Rather than impose (10) in this form, however, we use a slightly different form suggested by Ploumhans and Winckelmans:12

$$\int_C \gamma(s) \, ds = 2A_b \hat{\alpha}(t + \delta t) - \sum_p \Gamma_p^*(t),$$ \hspace{1cm} (11)

where $\Gamma_p^*(t)$ are the strengths of the particles at the end of the first (unbounded flow) half-step. This approach compensates for circulation leaks in other components of the algorithm, for example due to the interpolation of particle strength into the body during the particle re-initialization phase described below.

C. Other aspects

The particle evolution and strength exchange, coupled with the surface vortex sheet diffusion, constitute a complete algorithm for the solution of viscous, wall-bounded incompressible flows. In its practical implementation, the fast multipole method13 is used to accelerate the evaluation of the velocity field in (4) to $O(N)$, from its direct $O(N^2)$ form which is prohibitive of large simulations. A tree code is used to define a hierarchy of particle clusters that interact only through multipole expansions about their centers.

The flow map often contains local stagnation points that will cause some particles to cluster and others to disperse. Such behavior tends to degrade the long-term accuracy (and stability) of a particle-based simulation. Severe particle distortion is prevented in the present method by re-initializing the particle configuration every few time steps. In this procedure, the strengths of the old particles are interpolated onto new particle locations that are arranged on a Cartesian grid. We use the $M_3$ interpolation kernel—developed by Monaghan14 for smoothed particle hydrodynamics—which ensures the conservation of the first three discrete moments of vorticity. The new configuration is then trimmed of particles whose strengths, as well as those of all its neighbors within a viscous radius $C \sqrt{\nu \delta t}$, are smaller than some tolerance. This
Figure 1. Example of kinematics of a flapping wing. The leading edge is depicted by a small filled circle. Plunging: ‘—’; pitching: ‘—’. 

The technique limits the growth of the particle population with extraneous zero-strength particles that are merely passively convected.

The stencil of the $M'_t$ kernel requires a square array of sixteen new particles around an old particle, so its use for particles near the wall will tend to spread some circulation into the body. This circulation is lost when the new particles inside the body are eliminated. However, the circulation is regained in the subsequent time step, when the total vortex sheet circulation is enforced via equation (11), and the panel strengths diffused onto particles. This approach has been verified by Ploumhas and Winckelmans,\textsuperscript{12} who evaluated it by comparing it with a more complicated ‘body-aware’ interpolation scheme.

### III. Results

In this section we present the results from the computation of two different flapping motions. In general, the kinematics of the wing is described by a combination of transverse (‘plunging’) and angular (‘pitching’) motion. An example of such a combination is depicted in the full stroke exhibited in Figure 1, The plunging and pitching velocities of a half stroke of this motion are described mathematically by

\[
\frac{v(t)}{U} = \begin{cases} 
-1 & 0 \leq t < t_i \\
-\cos \frac{\pi(t - t_i)}{\Delta t} & t_i \leq t < t_i + \Delta t_i \\
1 & t_i + \Delta t_i \leq t < \frac{1}{2} T,
\end{cases}
\]

\[
\frac{\dot{\alpha}(t)}{\dot{\alpha}_0} = \begin{cases} 
0 & 0 \leq t < t_r \\
\frac{1}{2} \dot{\alpha}_0 \left[1 - \cos \frac{2\pi(t - t_r)}{\Delta t_r}\right] & t_r \leq t < t_r + \Delta t_r \\
0 & t_r + \Delta t_r \leq t < \frac{1}{2} T.
\end{cases}
\]

in which $U$ is the peak plunging motion and $T$ is the period of motion. Note that angle of incidence is measured clockwise from the negative $x$ axis (where upstroke corresponds to motion in the positive $y$
direction). Motion of this sort was imposed, for example, by Dickinson et al.\textsuperscript{15} on their mechanical wing apparatus. After an interval of constant plunging velocity during the downstroke, the wing decelerates and changes direction. The wing then accelerates on the upstroke to the same constant velocity, decelerates, then changes direction again. The wing angle of incidence remains constant at $\alpha_d$ for a portion of the downstroke, then the leading edge rotates clockwise (\textit{supinates}) during the transition from downstroke and upstroke. The wing maintains a new constant angle of incidence $\alpha_u$ for a portion of the upstroke, then begins to rotate counter-clockwise (\textit{pronates}) just prior to the downstroke.

The free parameters that describe this motion are: the Strouhal number $St = f c / U$ based on wing chord $c$ and frequency $f = 1 / T$; the extreme angles of incidence $\alpha_d$ and $\alpha_u$; the interval of direction change, $\Delta t_t$; the interval of angle of incidence change, $\Delta t_r$; and $\Delta t_{tag}$, the time by which the instant of direction change lags the instant of peak pitching. The other times are then given by $t_i = \frac{1}{2} T - \frac{1}{2} \Delta t_t$ and $t_r = \frac{1}{2} T - \frac{1}{2} \Delta t_r - \Delta t_{tag}$.

The peak pitching speed is given by $\dot{\alpha}_0 = 2 (\alpha_u - \alpha_d) / \Delta t_r$.

\subsection{A. Case 1}

In the first case, an elliptical wing with aspect ratio 5 and initial angle of incidence $\alpha = -40^\circ$ is prescribed with a pitching and plunging motion with Strouhal number $St = 0.25$. The intervals are $\Delta t_t / T = 0.667$, $\Delta t_r / T = 0.333$, and $\Delta t_{tag} / T = 0.08$, and the extreme angles of incidence are $\alpha_u = 40^\circ$ and $\alpha_d = -40^\circ$. The Reynolds number based on the peak plunging velocity and wing chord, $Re = U c / \nu$, is 550. The particle spacing is uniformly $\delta x / c = 0.005$ and the time step size used for the fourth-order Runge-Kutta method is $U \delta t / c = 0.005$. The elliptical wing is discretized with 284 panels, each with size determined by mapping a corresponding uniform panel from a circle of diameter $c$, which ensures that panels are most refined near the wing tips. The simulation begins with a distribution of around 1000 zero-strength particles within a viscous length scale of the wing. The vortex sheet established by the impulsive start is immediately diffused to the particles, and the time-stepping is subsequently begun.

The simulation of one stroke required a computational time of $\sim 9.5$\textit{hours} and two strokes required $\sim 38$ hours on a single 2.2GHz Intel Xeon processor; this processing time can be decreased by using larger particles as vortices travel far from the wing. At the end of the stroke the number of particles has grown to $2.6 \times 10^5$. Note that no attempt was made to find the optimal particle spacing and time step size for this problem; the values used are likely too conservative. The economy of the method is most clearly demonstrated by Figure 3, which depicts the boundary of particle coverage at the end of the stroke. Particles are distributed only as far as vorticity has spread by convection and diffusion. An equivalent simulation using a grid-based scheme would require a much larger domain, with an external boundary for the enforcement of the condition at infinity.

The vorticity at nine instants during the stroke is depicted in Figure 2. For reference, the initial configuration of the wing is depicted as a dashed curve in each plot. The vorticity contours are distributed between -30 and 30, with interval 1.875 and the zero contour omitted. Red contours denote positive values of vorticity. During the initial downstroke, opposite sign vortices are shed from the leading and trailing edges. As the wing supinates during this downstroke, the leading edge vortex is re-absorbed by the boundary layer on the upper surface. The trailing edge vortex couples with a new vortex shed during the ensuing upstroke, forming a vortex dipole that propagates downward. This process essentially repeats itself during the pronation period at the end of the upstroke—with vorticity of opposite sign—and a new dipole propagates upward.

The forces acting on the wing can be easily computed from the results of a vortex method simulation, by computing the rates of change of linear and angular momenta. The force that a vortical flow exerts on a solid body is easily computed from the impulse formula of classical fluid dynamics,\textsuperscript{16}

$$
F = -\rho \frac{d}{dt} \int y \times \omega \, d\gamma + \rho A_b \frac{dU}{dt} - 2 \rho A_b \frac{d}{dt} (x_b \times \Omega),
$$

(14)
where \( \rho \) is the density, \( A_b \) is the body area, \( \mathbf{x}_b \) is the body centroid, \( \mathbf{U} \) is the body velocity, and \( \Omega \) is the angular velocity of the body. In terms of particle quantities, the impulse components are given by
\[
I_x = \sum_p y_p \Gamma_p \\
I_y = - \sum_p x_p \Gamma_p
\]
Results for this case are shown in Figure 4. The pitching and plunging motions are nearly symmetric in this example. Thus, the normalized vertical force component, \( 2F_y/(\rho U^2) \), exhibits very little preference toward either direction; its mean value over the stroke is 0.21, due mostly to transient effects from the impulsive start. However, the horizontal component, \( 2F_x/(\rho U^2) \), has a mean value of -1.42, denoting a net thrust. The variation of this horizontal component about its mean is much less striking than the variation of the vertical force. The formation of new vortices during supination and pronation causes some horizontal disturbance, but once a vortex dipole is formed and the wing stops pitching, the force remains nearly constant.

Figure 2. Vorticity produced by a single stroke of a pitching/plunging wing for Case 1. The dashed ellipse corresponds to the initial configuration of the wing. Time is scaled by the period of motion.
Figure 3. Boundary of particle coverage at $t/T = 1.0$.

Note that the plane of plunging is arbitrary. For example, a two-winged insect in hover will generally adopt an approximately horizontal stroke plane, so that the stroke trajectory depicted in Figure 1 would actually be rotated by $-90^\circ$, thereby transforming the net ‘thrust’ into a net lift.

B. Case 2

In the second case, an elliptical wing with aspect ratio 5 and initial angle of incidence $\alpha = -30^\circ$ is prescribed with a pitching and plunging motion with Strouhal number $St = 0.25$. The intervals are $\Delta t_s/T = 0.25$, $\Delta t_r/T = 0.2$, and $\Delta t_{lag}/T = 0.025$, and the extreme angles of incidence are $\alpha_u = 30^\circ$ and $\alpha_d = -30^\circ$. The Reynolds number and all numerical parameters are identical to Case 1. The vorticity during one stroke is depicted in Figure 6. Vorticity contours are also identical to Figure 2. During the initial downstroke, opposite-sign vortices are shed from the leading and trailing edges. As the wing begins to supinate, a positive vortex is shed from each edge in response. The two leading edge vortices are of opposite sign and form a counter-rotating pair that travels downward as the wing begins its upstroke. The second trailing edge vortex couples with a third, negative vortex shed during the upstroke acceleration, and this pair also propagates downward. Similar shedding and dynamics occurs during the transition from upstroke to downstroke.

IV. Conclusion

A viscous vortex particle method has been described for the direct numerical simulation of the flow produced by an arbitrarily moving two-dimensional body. This method has the natural advantages that the correct velocity behavior at infinity is automatically accounted for and computational particles are only required in vortical regions of the flow. The application of this method to a pitching and plunging elliptical wing has
been presented.

The primary goal for this ongoing work is a tool for simulating the complex problem of insect flight aerodynamics. Insect wing motion is inherently three-dimensional, and involves significant deformation of the elastic wing structure from the anchor at the insect thorax to the wing tip. Wing flexure and three-dimensionality both play critical roles for the insect to generate the aerodynamic forces required to remain aloft and propel itself forward. In this spirit, we are currently extending the method to solve the flow produced by deforming three-dimensional surfaces. Many of the techniques required have been developed in previous work \(^8,17\) For example, implementing a three-dimensional version of the method involves, in addition to the analogs of techniques described here, accounting for vortex stretching and a careful treatment to maintain a divergence-free particle vorticity field. Also, note that, because of the grid-free nature of this method, there is no additional challenge posed by simulating multiple bodies in relative motion. This feature is especially attractive for problems involving multiple wing pairs, for example, dragonflies.

It is important to stress that, despite the title of this paper, the efficiency of the simulation tool used here can be significantly improved. A useful feature of the particle method is that the fast multipole computation of the velocity field \((4)\) via a tree code is amenable to parallelization. The computational time required on a single processor can be greatly reduced by performing the velocity field calculations, viscous diffusion, and panel strength solutions each in parallel. Such an approach will maximally exploit the natural efficiency of this method, and is also a component of our current effort.

Figure 4. Body force histories for two strokes of a wing for Case 1. Note that the rotational velocity \(\dot{\alpha}\) is measured in radians/time.
Figure 5. Breakdown of force components for Case 1. In each case, fluid impulse portion of force: ‘—–’; negative of body acceleration portion of force: ‘—’; total force: ‘—’.

References

Figure 6. Vorticity produced by a single stroke of a pitching/plunging wing. The dashed ellipse corresponds to the initial configuration of the wing. Time is scaled by the period of motion.