NUMERICAL INVESTIGATION OF A BIAS-FLOW PERFORATED LINER FOR DAMPING OF THERMOACOUSTIC INSTABILITIES

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ABSTRACT

Lean premixed prevaporized (LPP) gas turbine generators are naturally prone to thermoacoustic instabilities. Strategic placement of passive damping devices can provide simple, effective fixes for such unstable behavior. In this work, the thermoacoustic damping characteristics of a perforated liner with mean bias flow are examined. A recently-developed theoretical model, along with accompanying experimental investigation, has demonstrated that a bias-flow liner can very effectively absorb incident acoustic waves. Here, a modular simulation tool is utilized to examine the capability of the liner for stabilizing an unstable ducted flame. The simulation tool represents the acoustic interactions between duct elements in the form of transfer matrices, which can be modularly arranged for exploring a variety of configurations. An unstable thermoacoustic system is produced with a gain-delay flame model in a duct. The frequencies and growth rates of the linear model are examined. It is shown that, by tailoring the liner porosity and the bias flow, unstable modes of the thermoacoustic system can be stabilized. Furthermore, it is found that, for a double liner system, there is an optimal liner porosity for a given choice of bias flow, at which the modal decay rate is maximized.

INTRODUCTION

A lean premixed prevaporized (LPP) combustion process operates at a lower temperature, significantly reducing the production of undesirable pollutant emissions. One problematic outcome of running a LPP combustion process is that the system is more prone to combustion instabilities [12, 23]. The LPP combustion process is characterized by a greater sensitivity to fluctuations in the equivalence ratio \( \phi \) [21]. Flame heat release in gas turbine combustion chambers is generally unsteady. The unsteady heat release causes fluctuations in the local density, pressure and velocity which propagate away from the flame in the form of acoustic waves [11, 16]. These oscillations reflect off the chamber boundaries (compressor, turbine, etc) and propagate back towards the flame resulting in deviations in the fuel and air mixture and possibly amplifying the unsteady behavior of the heat release [21, 23]. If this interaction is in phase (i.e the Rayleigh criterion is satisfied) this process becomes self-excited and may cause the system to experience large amplitude oscillations, potentially causing structural damage.

In the early rockets, acoustic instabilities were also a major problem which eventually were overcome with extensive research on passive control. Control strategies typically consist of inclusion of baffles and/or geometric modifications [23]. However, unlike rocket motors, combustion instabilities in LPP gas turbine generators are excited by multiple means, such as rotating machinery, fuel mixing, and multiple flame mechanisms. Simple geometric modifications may not be sufficient to completely stabilize the system. Therefore, a more robust mechanism is needed to overcome these instabilities. One effective solution is the utilization of passive acoustic damping devices which remove energy from the acoustic field and thereby weaken the feedback cycle. One conventional damper is the Helmholtz resonator, which
is a cavity with a neck connected to the system through a small orifice. Near its resonance frequency, the resonator experiences large-amplitude acoustic oscillations in its neck. The oscillations in the neck dissipate acoustic energy by means of vortex shedding at the neck-duct interface and viscous effects of fluid and wall interaction [10].

A Helmholtz resonator, however, is only effective in the limited bandwidth near resonance. Also, the resonant oscillation in the neck is heavily dependent on the external pressure fluctuations, so the resonator fails to dissipate acoustic energy if placed near a pressure node. A damping system that may overcome some of these limitations is a perforated liner with bias flow, which has an array of orifices across the length and circumference of its cylindrical shell. Harmonic pressure differences across each orifice excite periodic vortex shedding, and shed vortices are convected away by the bias flow [20]. This process converts the acoustic energy into mechanical energy, which eventually is dissipated into heat. A liner, unlike a localized resonator, is distributed over a length of the duct, and consequently it is effective over a relatively larger range of frequencies and locations.

Perforated liners for their explicit acoustic damping behavior have been under study since the 1970s. Ffowcs Williams [1] examined an infinite plane perforated screen in the absence of a mean flow. Later, Leppington and Levine [2] refined Ffowcs Williams’ analysis with the help of integral equations and also studied a case of the perforated screen backed by a rigid wall. Practical perforated liners are not infinite, and Leppington [3] investigated a semi-infinite liner using a Wiener–Hopf approach. In the absence of a mean flow, the linear acoustic model of a liner contains no damping mechanisms. The nonlinear effects must be included to model sound absorption [4].

Bias-flow acoustic liners have different characteristic behavior than liners without such mean through-flow, because the mean flow provides a mechanism for linear absorption via the production and convection of vortical fluctuations. Hughes and Dowling [5] employed the vortex shedding model derived by Howe [20] to analyze damping provided by a perforated screen with mean bias-flow. Jing and Sun [6] analyzed this configuration experimentally and revealed the importance of the screen thickness. In their later work [7] they were able to numerically analyze the effects of screen thickness. These experimental and numerical investigations have been of transverse modes that impinge on the liner. In a thermoacoustic system with instabilities in the low frequency limit—for example, a gas turbine combustion chamber—the system experiences primarily longitudinal modes. A model recently developed by Eldredge and Dowling [17] predicts the absorptive properties of a bias-flow perforated liner in the presence of grazing acoustic waves. In the grazing-wave configuration, the liner absorbs acoustic energy most effectively when a downstream element (turbine blades, choked outlet, etc.) can reflect the energy back to the lined section.

When the porosity and bias flow are chosen properly, the liner can absorb 83 percent of incident acoustic energy over a broad frequency range. The liner model developed in [17] agreed well with accompanying isothermal experiments.

A natural question that arises from this previous work is whether the effectiveness of a bias-flow liner in absorbing incident sound can also be observed in the damping of an unstable thermoacoustic system. The purpose of the present work is to utilize the model developed by Eldredge and Dowling [17] to numerically examine the thermoacoustic stability characteristics in a heuristic combustion system. A particular goal of this research is to explore the robustness of damping to changes in liner geometry and combustor conditions. Another objective of this work is to develop a modular acoustic simulation tool to numerically investigate multiple liner system configurations with relative ease. Such a tool has been used successfully by other investigators [8, 9] to facilitate damper design strategies beyond empirical trial and error approaches. It is important to note that the present work relies on numerical modeling of the bias-flow liner in a simple ducted flame geometry. Thus, conclusions drawn are primarily meant to provide guidance for future experimental studies.

The modular simulation tool, discussed in the Methodology section, is based on plane wave theory and relates the propagating waves into and out of an acoustic source and/or sink with the use of transfer functions. A heuristic flame model, discussed in the Model section, is similar to the one used in [15], with an additional gain-delay model [21] to satisfy the Rayleigh criterion. The model for the perforated liner model is adapted from [17], in which its accuracy was verified experimentally. In the Results section the perforated liner is incorporated in a system with the heuristic gain-delay flame model and the frequencies and growth rates of the overall system are numerically evaluated.

**METHODOLOGY**

Acoustic perturbations in a duct consist of pressure, velocity and density perturbations. Because characteristic frequencies in the system of interest are below cut-on of higher order modes, we model behavior with plane acoustic waves, which satisfy a linear one dimensional wave equation. For a single frequency, a solution to the pressure perturbation in space and time is given by

\[
p'(x,t) = \exp(\imath \omega t) \left[ f \exp \left( \frac{-i \alpha x}{\bar{c} + \bar{u}} \right) + g \exp \left( \frac{i \alpha x}{\bar{c} - \bar{u}} \right) \right]
\]

(1)

where \( f \) and \( g \) are amplitudes of waves propagating downstream and upstream in the duct, respectively, and \( \alpha, \bar{c} \) and \( \bar{u} \) are the frequency, the mean speed of sound and the mean axial velocity inside the duct. From the definition of the speed of sound, density
fluctuations can be expressed in terms of the pressure perturbations given by equation (1) and the mean speed of sound,

$$\rho' = \rho' / c^2.$$  \hspace{1cm} (2)

Furthermore, the velocity perturbations can be expressed as follows

$$u' = \pm \frac{p'}{\rho c}.$$  \hspace{1cm} (3)

where $\bar{\rho}$ is the mean density. The positive sign is chosen for a forward-propagating wave and negative for a backward-propagating wave.

In a duct with plane waves, linearized mass and momentum conservation can be applied to examine the interaction of sound waves with system elements (e.g. flame, dampers, etc.), as depicted in figure 1. Upon substitution of the harmonic solution, equations (1)–(3), into the conservation equations, we get an equation set relating the upstream and downstream wave amplitudes. By representing this set of equations in a matrix form we arrive at a transfer matrix for an arbitrary system element,

$$\begin{bmatrix} f_2 \\ g_2 \end{bmatrix} = Y^{-1} X \begin{bmatrix} f_1 \\ g_1 \end{bmatrix}.$$  \hspace{1cm} (4)

In equation (4), subscripts 1 and 2 represent the upstream and downstream conditions, respectively. Again, $f$ and $g$ are forward and backward propagating wave amplitudes, while $Y$ and $X$ are matrices resulting from the application of conservation equations. For a duct with only one element, the linear equation can be closed by applying the upstream and downstream boundary conditions to acquire a homogeneous set,

$$\begin{bmatrix} R_d - 1 \end{bmatrix} Y^{-1} X \begin{bmatrix} R_u \\ 1 \end{bmatrix} = 0,$$  \hspace{1cm} (5)

where $R_u$ and $R_d$ are upstream and downstream reflection coefficients, respectively. Each reflection coefficient is the ratio of the reflected to incident wave amplitudes. Note, due to the homogeneous nature of this equation set, the wave amplitudes cannot be solved for. The only non-trivial solutions possible are the frequencies of the system. These frequencies are generally complex, and the imaginary part corresponds to exponential growth or decay (positive values denote decay). If the system consists of $n$ sequential elements, there will be $n \times 2$ transfer matrices representing each element in the system, ordered in reverse sequence as

$$\begin{bmatrix} R_d - 1 \end{bmatrix} (Y^{-1} X)_n \ldots (Y^{-1} X)_1 \begin{bmatrix} R_u \\ 1 \end{bmatrix} = 0$$  \hspace{1cm} (6)

One advantage of transfer matrices is that they can be easily modeled in a block diagram approach, connecting elements in a modular fashion. This approach allows the use of graphical software tools to perform the analysis. Figure 2 shows a schematic model for duct acoustics of one element in a simple straight duct.

In a system with entropy generation, the acoustic and entropy wave interaction modifies this approach slightly. Whenever there is heat addition entropy is generated. An entropy wave propagating downstream of the flame can always be related to the upstream density and temperature perturbations using the second law of thermodynamics. This interaction can be incorporated into the transfer matrix approach,

$$\begin{bmatrix} f_2 \\ g_2 \\ S \end{bmatrix} = Y^{-1} X \begin{bmatrix} f_1 \\ g_1 \\ \hat{Q} \end{bmatrix} + Y^{-1} \hat{C} \hat{Q}.$$  \hspace{1cm} (7)

Note that $Y$ is of dimension $3 \times 3$, $X$ is of dimension $3 \times 2$ and $C$ is of dimension $3 \times 1$, and $\hat{Q}$ represents heat addition fluctuations. Because the goal of the analysis is to solve for the frequency of acoustic oscillations, and not the entropy wave itself, the last row in the transfer matrix, which represents the entropy wave, can be ignored after the ducted elements have been considered. Once the entire elemental module network is in place the boundary conditions are used on the first two rows of the transfer matrix to solve the homogeneous system of equations, given by

$$\begin{bmatrix} R_{u1} & R_{u2} \end{bmatrix} (A + B) \begin{bmatrix} \frac{R_u}{2} \\ -\frac{R_d}{2} \end{bmatrix} = 0$$  \hspace{1cm} (8)
where \( A \) and \( B \) are the first two rows of \( Y^{-1}X \) and \( Y^{-1}C \) respectively, and the \( R_{d,1}^{\mu,d} \) represent the wave amplitudes at the boundaries, which are constrained to be in the ratios \( R_u = R_2^u/R_1^u \) and \( R_d = R_2^d/R_1^d \). Since the magnitude of the wave amplitudes are indeterminate, we are free to set one of the boundary amplitudes arbitrarily.

In this simple model, it is assumed that the flame is the only source of entropy fluctuations. Thus, all of the elements upstream of the flame can be represented with \( 2 \times 2 \) matrices while all the elements downstream of the flame are represented with \( 3 \times 3 \) matrices. However, the transfer matrix due to the flame (or, in general, due to an element that generates entropy) is a \( 3 \times 2 \) matrix, which leads to dimensional consistency throughout the matrix multiplication for all the elements.

**MODEL**

The goal of this work is to demonstrate the potential of a bias-flow acoustic liner for suppressing a thermoacoustic instability. No attempt is made here to develop or utilize a sophisticated model for a practical combustion system. Rather, we adopt the simpler approach of constructing a basic unstable thermoacoustic system, from a gain-delay flame model in a straight duct with simple boundary conditions. The stabilization behavior of the liner will be examined on this simple model.

**Heuristic Thermoacoustic Flame Model**

A heuristic linear flame model, adapted from [15], is outlined here. In figure 3, \( f \) and \( g \) are wave amplitudes, and the amplitude of the entropy wave is denoted by \( S \). Upstream of the flame zone is denoted by subscript 1, while downstream of the flame zone, denoted by subscript 2. Substitution of the harmonic form of acoustic perturbations into mass, momentum and energy conservation across the flame gives the transfer matrices \( Y, X \) and \( C \) in equation (7). The reader is referred to [15] for details.

The flame transfer system in equation (7) must be closed by relating \( \dot{Q} \) to the wave amplitudes requiring a flame model. A simple flame model is adapted from Dowling [15] and is given by

\[
\dot{Q} = \Delta h_{0,12} \left[ \frac{(1 + M_1)e_1/c_1^2 - (1 - M_1)e_2/c_1^2}{f_1} \right] \left[ \frac{f_1}{g_1} \right]
\]

where \( M_1 \) is the Mach number and \( c_1 \) the mean speed of sound upstream of the flame, \( x_f \) is the flame location, and \( \Delta h_{0,12} \) is the mean jump in stagnation enthalpy. This simple model does not exhibit unstable thermoacoustic behavior and hence some modifications are required. According to the Rayleigh criterion, a combustion system experiences acoustic instabilities when the pressure and unsteady heat release fluctuations are in phase. One approach is to incorporate a time delay—which physically exist in combustion systems (for example, the convection time for the fuel from the injector to the flame)—into the unsteady heat release and adjust in order to satisfy the Rayleigh criterion. With the addition of a time delay \( \tau_d \) the heat release model becomes

\[
\dot{Q}'(t,x) = m'(t - \tau_d,x)\rho_p (T_{02} - T_{01}) \tag{12}
\]

Substituting \( m'(t,x) = \rho_1 u'_1(t,x) + \rho_1'(t,x)\bar{u}_1 \) into equation (12) gives

\[
\dot{Q}'(t,x) = \left( \rho_1 u'_1(t - \tau_d,x) + \rho_1'(t - \tau_d,x)\bar{u}_1 \right) c_p (T_{02} - T_{01}) \tag{13}
\]

The unsteady heat addition with a time delay results in the following modified form of equation (9)

\[
\dot{Q} = \Delta h_{0,12} \exp\left(-i\omega\tau_d\right) \times \left[ \frac{(1 + M_1)e_1/c_1^2 - (1 - M_1)e_2/c_1^2}{f_1} \right] \left[ \frac{f_1}{g_1} \right] \tag{14}
\]

The two major contributors of unsteady heat release are the acoustic fluctuations at the fuel injector and at the base of the flame [21]. For a planar flame, since the heat is released uniformly the latter does not have any contribution. The injectors are usually located upstream of the flame, hence the fluctuations at the injector convect to the flame after some time, \( \tau_{conv} \). The convection time, \( \tau_{conv} \), is the distance the perturbations must travel divided by the mean velocity at which these perturbations are convected,

\[
\tau_{conv} = \frac{x_f - x_{inj}}{\bar{u}}. \tag{15}
\]

The current model contains a choked air inlet and a choked fuel injector. For a choked air inlet configuration the pressure perturbation leads the mass flow perturbation by 90 degrees. The current model assumes that the unsteady heat release is in phase
with the mass flow perturbation. Hence, the Rayleigh criterion is not satisfied.

In general, the heat release is not in phase with the fluctuations in the equivalence ratio, $\phi'$. The heat release for a planar flame lags $\phi'$ by the convective time delay $\tau_{\text{conv}}$. Also, unlike the previous assumption $\phi'$ is not in phase with oscillation in the mass flow, $\dot{m}_{\text{o}}'$. The oscillation in $\phi$ is given by

$$\frac{\dot{\phi}'}{\phi} = \frac{\dot{m}_{\text{f}}'/\bar{m}_{\text{f}} - \dot{m}_{\text{o}}'/\bar{m}_{\text{o}}}{1 + \dot{m}_{\text{o}}'/\bar{m}_{\text{o}}}.$$  \hspace{1cm} (16)

where, for a choked injector, $\dot{m}_{\text{f}}' \to 0$, indicating that the oscillations in $\phi$ lag $\dot{m}_{\text{f}}$ by 180 degrees. Hence, the phase of the unsteady heat release $\dot{Q}$ lags $\dot{m}_{\text{o}}'$ by a time delay of $\tau_d$ where

$$\tau_d = \tau_{\text{conv}} + \pi/\omega.$$ \hspace{1cm} (17)

The convective time delay $\tau_{\text{conv}}$ for a choked air inlet is given by [21]

$$\frac{\tau_{\text{conv}}}{T} = C_n = n - 3/4 \quad \text{where} \quad n = 1,2,3\ldots \hspace{1cm} (18)$$

The limits of the region are ±90 degrees phase difference between pressure and heat release oscillations. That is, the unstable region is given by [21],

$$C_n - 1/4 < \tau_{\text{conv}}/T < C_n + 1/4.$$ \hspace{1cm} (19)

**Acoustic Model of Perforated Liner with Bias Flow**

In this work we are interested in the damping provided by a perforated liner with bias flow. The configuration, shown in figure 4, consists of a circular duct of overall length $l$, circumference $C$ and cross-sectional area $S_p$. The perforated liner section has length $l_p = x_b - x_a$. The system is subject to an upstream axial flow $\bar{u}_a$ and a liner bias flow $v$. Note that both flows are small and have the same stagnation temperature associated with them, hence the mean speed of sound and density, $\tilde{c}$ and $\tilde{\rho}$ respectively, are uniform in the lined section. In a physical combustion chamber these properties do change; however, the effect of this variation is likely small and are not within the scope of this study. The downstream velocity is then given by $\bar{u}_d = \bar{u}_a + (C_l/S_p)v$.

The wave amplitudes $f_1$ and $f_2$ correspond to forward traveling waves entering and exiting the lined section, respectively, while $g_1$ and $g_2$ are amplitudes of waves traveling upstream. The perturbations in stagnation enthalpy, $\hat{B}$, and velocity, $\hat{u}$, are continuous across the upstream and downstream liner/duct interface to conserve mass and momentum.

These fluctuations are governed by linearized equations for mass and momentum. We can define composite variables $\psi^+ = \frac{1}{2}(1 + \bar{u})\hat{B} + (1 - \bar{u})\hat{u}$ and $\psi^- = \frac{1}{2}(1 - \bar{u})\hat{B} - (1 + \bar{u})\hat{u}$ and arrive at the following two coupled linear differential equations governing the behavior of $\hat{B}$ and $\hat{u}$ [17]:

$$\frac{d\psi^+}{dx} = -\frac{i\alpha l}{1 + \bar{u}}\psi^+ + \frac{1}{2}\frac{C_l}{S_p}\bar{u},$$ \hspace{1cm} (20)

$$\frac{d\psi^-}{dx} = \frac{i\alpha l}{1 - \bar{u}}\psi^- - \frac{1}{2}\frac{C_l}{S_p}\bar{u},$$ \hspace{1cm} (21)

where $\bar{u}$ are the amplitude of fluctuations in the liner throughflow. Continuity of $\hat{B}$ and $\hat{u}$ across the upstream and downstream interfaces gives the following boundary conditions,

$$\psi^+(x_a) = f_1\exp(-i\alpha x_a/(\tilde{c} + \bar{u}_a)),$$ \hspace{1cm} (22)

$$\psi^-(x_a) = g_1\exp(i\alpha x_a/(\tilde{c} - \bar{u}_a)),$$ \hspace{1cm} (23)

$$\psi^+(x_b) = f_2\exp(-i\alpha x_b/(\tilde{c} + \bar{u}_d)),$$ \hspace{1cm} (24)

$$\psi^-(x_b) = g_2\exp(i\alpha x_b/(\tilde{c} - \bar{u}_d)).$$ \hspace{1cm} (25)

The difference in stagnation enthalpy fluctuations across the liner can be related to $\bar{u}$ by employing the liner compliance relationship defined by $\partial B^+/\partial \bar{u} = \eta[B^+/\bar{u}]_{\text{inv}}$. Using this compliance with the momentum equation in the normal direction and the relation $\hat{B} = \psi^+ + \psi^-$ gives [17]

$$\bar{v}_1(x) = \frac{\eta\alpha}{i\omega}B_1(x) - \psi^+(x) - \psi^-(x)$$ \hspace{1cm} (26)
where $\hat{B}_1$ represents fluctuation in stagnation enthalpy outside of the liner and $\eta_1$ is the liner compliance. Note that $\hat{B}_1$ is determined by the configuration of the liner. In this work two configurations are investigated: a single liner exposed to a plenum (in which $\hat{B}_1 = 0$), and a double liner configuration (in which $\hat{B}_1$ is determined by acoustics between the liners, and $\hat{B}_2$ outside the second liner is zero). The second liner is assigned $\eta_2$ and $C_2$ as the compliance and the circumference respectively.

The total compliance of each liner is developed in [17] and is given by

$$\frac{1}{\eta} = \frac{\pi a^2}{\sigma} \frac{1}{K_a} + \frac{t}{\sigma}$$  \hspace{1cm} (27)

where $\sigma$ is the porosity of the perforated screen, and $a$ and $t$ are the orifice radius and the liner thickness, respectively. The Rayleigh conductivity, $K_a$, of a single orifice with bias flow was derived by Howe [20], and can be written generically as

$$K_a = 2a (\gamma + i \delta)$$  \hspace{1cm} (28)

where $\gamma$ and $\delta$ depend on the Strouhal number, $St = \omega \delta / U_c$, where $U_c$ is the mean convection velocity of vortical disturbances (approximately equal to the bulk orifice velocity, $\sqrt{\sigma}$). The functional forms are given in equation (3.14) of [20] and are plotted in figure 6.

The unstable modes focused on in this work are limited to $St \leq 0.1$. In the low frequency limit, $St \ll 1$, $\delta$ and $\gamma$ can be approximated by

$$\gamma \approx \frac{1}{3} St^2, \quad \delta \approx \frac{\pi}{4} St.$$  \hspace{1cm} (29)

Substituting equations (29) into equation (27) gives an expression for the liner compliance in the lower frequency limit (non-dimensionalized by the duct length, $l$):

$$\frac{\eta}{l} \approx \frac{2}{3 \pi} \frac{\sigma}{M_{h,1} t} \left( \frac{\hat{c}_1}{\hat{c}_2} \right)^2 \left( 1 + \frac{3 \pi}{8} \frac{t}{a} \right) \left( \frac{\omega \hat{c}_1}{\hat{c}_2} \right)^2 + i \frac{1}{2} \frac{\sigma}{M_{h,1} \hat{c}_2} \left( \frac{\omega \hat{c}_1}{\hat{c}_2} \right)$$  \hspace{1cm} (30)

where $\hat{c}_1$ and $\hat{c}_2$ are the speeds of sound on either side of the flame and $M_{h,1}$ is the orifice Mach number for the liner, $M_{h,1} = U_c / \hat{c}_2$.

It is important to note that the liner compliance is linearly proportional to the porosity in both the real and imaginary parts; furthermore, in the low frequency limit, it is proportional to the inverse square of the bias flow in the real part and is inversely proportional to the bias flow in the imaginary part. The effects of these relationships will be apparent during the optimization of the thermoacoustic/liner system in the next section.

Equations (20)–(26) form a closed set of two coupled linear differential equations governing the perforated liner system. This set of equations is easily solved using a Runge-Kutta method. To acquire frequency modes for a perforated liner it is vital to develop a technique which incorporates the solution for this set of equations in our current modular simulation tool. The technique is depicted schematically in figure 5, where $(Y^{-1}X)_{i=1}$ is the transfer matrix from a previous element which gives the upstream wave amplitudes $f_1$ and $g_1$ for the liner. The boundary conditions at the end of the liner, $\psi^+$ and $\psi^-$ at $x = x_b$, give the wave amplitudes downstream of the liner, $f_2$ and $g_2$, or $(Y^{-1}X)_i$, the transfer matrix for the perforated liner. This transfer matrix can then be incorporated in our network model to solve for modes of a system with a perforated liner.

**RESULTS**

The models described in the previous sections are implemented in the modular network to obtain numerical results for the following configurations. In the next subsection the effects of time-delay on the flame model are tailored to provide an unstable thermoacoustic system. A bias-flow liner is applied to this system in the following subsection, and the stability characteristics are explored for various choices of design parameters. In the final subsection the characteristics of a dual perforated liner system are briefly studied. The choices for liner and combustion
parameters are generally based on previous studies. Throughout the study, the duct of diameter 0.13l consists of a choked inlet at the upstream end and an open downstream end.

Effects of Time Delay on the Heuristic Flame Model

The thermodynamic properties of air are used and assumed to be constant across the flame, i.e. the gas constant $R = 287$ J/kg/K and the ratio of specific heats 1.4. The mean Mach number upstream of the flame is 0.046 with a mean stagnation temperature of 288 K (corresponding to a mean speed of sound, $\bar{c}_1$, of 340 m/s). The mean stagnation temperature jumps by a factor of 6 across the flame, from which the mean speed of sound is $\bar{c}_2 = 837$ m/s and the mean Mach number is 0.11 downstream of the flame. The flame is located at $x_f/l = 0.33$ and the fuel injector is located at $x_{inf}/l = 0.10$; the convection delay for the fuel to the flame is given by equation (15). This convective time is incorporated in $\tau_f$, the time delay, to satisfy the Rayleigh criterion. That is, the pressure perturbation decays with time. Figure 7 also shows that the inclusion of a sufficient time delay shifts some of the imaginary components of the frequency solutions below zero, resulting in an unstable system, in which the pressure oscillations grow with time. Both of these numerical results are consistent with the Rayleigh criterion.

Effects of a Perforated Liner on the Thermoacoustic System

In this section, the transfer matrix for the perforated liner is added to create a damped thermoacoustic system. The conditions upstream of the flame and the flame parameters are kept the same as in the previous section. Standard atmospheric pressure is assumed throughout. The mean bias flow through the liner is initially taken to be $\bar{M}_{h,1} = 0.03$ (this will be varied later). The liner is perforated by a regular lattice of holes of radius $3.98 \times 10^{-4}l$ and inter-orifice spacing $3.5 \times 10^{-3}l$, which is equivalent to a porosity of $\sigma = 0.040$. The thickness of the liner is $3.18 \times 10^{-3}l$. The center of the liner is in the middle of the duct with a liner length of $l_p = 0.19l$; that is, the liner upstream end is located at 0.405l and the downstream end at 0.595l.

Preliminary results of the liner’s effect are presented in figure 8. Plotted are the real and imaginary parts of the eigenfrequencies of the system, with the third mode having the lowest real part and the fifth having the largest. The results are somewhat promising in that the shifts of the imaginary frequencies are all positive. Aside from mode three (the leftmost)—which is stable to begin with—the liner falls short of stabilizing the system. However, several liner parameters may be practically varied to influence this behavior: most notably, the bias flow velocity, the porosity, the location and the length. It should be noted that there are practical constraints on these parameters, for example, generally the liner cannot have a porosity greater than 0.2. The bias flow Mach number should not exceed 0.2 and the length is subject to combustion chamber dimensional constraints.

For a single liner system the effects of varying $\sigma$, the porosity, and the bias Mach number, $M_{h,1}$, are displayed in figure 9. The fourth mode is selected for study here since it is the first unstable mode, as seen in figure 8. Figure 9(a) displays the effect on the imaginary eigenfrequency (decay rate) as $\sigma$ and $M_{h,1}$ are varied. Figure 9(b) displays the dependence of the real frequency component on porosity and bias flow. Figure 9(c) shows that, at low frequency, the liner compliance is linearly proportional to $\sigma$ and inversely related to $M_{h,1}$. Since the local energy loss is proportional to $\eta|\Delta B|^2$, where $\Delta B$ is the local enthalpy jump across the liner, damping is more effective for larger enthalpy fluctuations and higher compliance. This is clearly evident in figure 9(a), which shows that a lower $M_{h,1}$ and a higher $\sigma$ leads to a more stable system. Figure 9(b) shows a
slight shift in the real eigenfrequencies due to $\sigma$ and $M_{h,1}$, which is to be expected since changing system parameters do have an effect on the modes. Figure 9(c) demonstrates that a more stable system produces smaller enthalpy (or pressure) fluctuations in the lined section of the system (approximately between $x/l = 0.4$ and $x/l = 0.6$).

Other parameters such as the liner length and location can also be explored. In particular, an increase in liner length should have a positive influence on sound attenuation. Physically, a longer liner has more surface area exposed to acoustic fluctuations and more acoustic energy is dissipated. The effect of location on sound attenuation is not as simple. Since the absorption mechanism is based on enthalpy fluctuation, with greater difference leading to greater absorption, the liner is more effective at locations with higher enthalpy amplitudes. However, a change of position of the liner alters the enthalpy profile inside the duct.

Both of these parametric effects are observed in figure 10(a) and (b), respectively. In figure 10(a) the decay rate of the fourth mode is displayed for a bias flow Mach number of $M_{h,1} = 0.03$ and variable $\sigma$ and liner length. The three different liner lengths are 0.12$l$, 0.19$l$, and 0.3$l$. The rest of the parameters are the same as in the previous cases. It is clearly evident that a longer liner is more effective for stabilization. In figure 10(b) the location is varied for a liner with length 0.19$l$ and center at 0.5$l$ and 0.75$l$. The liner is more effective when closer to the flame, where enthalpy fluctuations are larger.

The fifth mode was more challenging to stabilize. The length of the liner was increased from 0.19$l$ to 0.53$l$ and the liner midpoint location shifted from 0.5$l$ to 0.63$l$. For a bias flow of $M_{h,1} = 0.01$ the system becomes stable for a porosity greater than $\approx 0.42$, which is impractically large. However, the system itself is only demonstrative, and damping characteristics in other systems will generally be different.

Figure 9. RESULTS OF OPTIMIZING THE POROSITY $\sigma$ AND THE BIAS MACH NUMBER $M_{h,1}$ FOR MODE 4 FOR A SINGLE LINER/FIRE SYSTEM. THE SOLID, DASHED, DOTTED, DASHED-DOTTED LINES ARE FOR $M_{h,1}$ OF 0.003, 0.012, 0.021, 0.03 RESPECTIVELY.

Figure 10. EFFECT OF VARYING LINER LENGTH AND LOCATION ON IMAGINARY EIGENFREQUENCY OF MODE 4. (a) SOLID, DASHED, AND DOTTED LINES ARE FOR LINER LENGTHS OF 0.12$l$, 0.19$l$, 0.3$l$. (b) SOLID AND DASHED ARE FOR LINER MIDPOINT LOCATIONS OF 0.5$l$, 0.75$l$. 
Figure 11. EFFECT OF VARYING POROSITY AND BIAS FLOW ON IMAGINARY EIGENFREQUENCY OF MODES 3 (IN (a)) AND 4 (IN (b) AND (c)) IN DUAL LINER SYSTEM. IN (a) AND (b) THE $M_{h,1}$ FOR SOLID, DASHED, DOTTED and DASHED-DOTTED ARE 0.03, 0.05, 0.07 AND 0.09 RESPECTIVELY, WHILE IN (c) THE $M_{h,1}$ FOR SOLID, DASHED, DOTTED AND DASHED-DOTTED ARE 0.003, 0.006, 0.009 AND 0.012 RESPECTIVELY.

Dual Liner Thermoacoustic System

In this section, a two-liner system is also tested numerically to explore some of its characteristics. The second liner, which surrounds and is coaxial with the first, has a diameter of $0.16l$ and is perforated by holes of radius $1.43 \times 10^{-3}l$ and spacing $1.80 \times 10^{-2}l$, corresponding to $\sigma_2 = 0.020$. The thickness of the outer liner is the same as the inner liner, $3.18 \times 10^{-3}l$. Eldredge and Dowling [17] found that a double liner does not improve the acoustic absorption, but enriches the parameter space for optimization. As in the single liner case, the porosity and bias flow (of the inner liner) are varied to examine their effect on the decay rate of unstable modes. These results are depicted in figure 11; mode 3 is depicted in figure 11(a) and mode 4 is shown in figures 11(b) and (c).

Once again, the third mode is stable, except for large liner porosities. In contrast, the fourth mode is unstable except for small value of bias flow ($M_{h,1} < 0.012$). As discussed previously, the acoustic liner compliance, which appears in the local energy loss $\eta |\Delta B'|^2$, is inversely related to bias flow in the low frequency limit, and therefore a lower bias flow results in a higher compliance and therefore larger acoustic energy absorption.

A phenomenon that is unique to the dual liner system is a maximum in the imaginary part of the dual liner eigenfrequency for varying $\sigma_1$, clearly evident in figures 11(a)-(c). This maximum shifts as the bias flow is varied. In other words, an optimal $\sigma$ exists for a given choice of $M_{h,1}$. Such an optimization relationship was also found in Eldredge and Dowling [17], though the ‘objective function’ in this previous work (maximum absorption of acoustic energy) was different. It is interesting to note that, for a stabilized mode, the optimal $\sigma$ increases with increasing bias flow—as seen in figures 11(a) and (c)—while for an unstable mode, the optimal $\sigma$ decreases with increasing bias flow, as figures 11(b) shows. This behavior is easier to see in figure 12, which depicts the locus of $\sigma$ and $M_{h,1}$ for peak imaginary eigenfrequencies for mode 4.

The fifth mode, as in the single liner case, is more challenging to stabilize. Using the same length and location as for the single liner case (fifth mode only), but a bias flow of $M_{h,1} = 0.03$, stability was achieved by choosing impractically large $\sigma_1 \approx 0.58$. Again, it should be noted that the conclusions drawn in this work are specific to this simple heuristic system. Other systems, with more sophisticated flame models and more combustor elements,
CONCLUSION

In this work, a modular simulation tool has been developed to examine the interaction of plane acoustic waves with a perforated liner system, to explore passive damping of instabilities. The simulation tool focuses on representing these interactions in the form of transfer matrices, enabling a modular approach. A heuristic gain-delay flame model is represented in a transfer matrix. Similarly, a transfer matrix for a perforated liner with bias flow is developed, based on an acoustic model that has been experimentally validated in previous work. This damping model has been used to explore the stabilization of thermoacoustic instabilities generated by the heuristic flame model.

The effect of liner parameters on the suppression of instabilities has been explored. Varying porosity and the bias flow, stability was achieved for both a single perforated liner and a double perforated liner. In the double perforated liner a maximum in effective stability is observed suggesting that an optimal operation under certain condition exists. The results clearly show that a perforated liner with a bias flow has promise for thermoacoustic damping applications. The results obtained in this work will also be used to design future laboratory-scale experiments, for the purposes of validation as well as for developing new strategies for damping of combustion dynamics in commercial gas turbines.

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