

# Steady-State Analysis of Continuous Adaptation Systems in Hearing Aids

M. G. Siqueira, A. Alwan, R. Speece

Department of Electrical Engineering  
University of California, Los Angeles  
Los Angeles, CA 90095

## ABSTRACT

Acoustic feedback is a problem in hearing aids that contain a substantial amount of gain, hearing aids that are used in conjunction with vented or open molds, and in-the-ear hearing aids. Acoustic feedback is both annoying and reduces the maximum usable gain of hearing-aid devices. This paper studies analytically the steady-state convergence behavior of LMS-based adaptive algorithms when operating in continuous adaptation to reduce acoustic feedback. A bias is found in the adaptive filter's estimate of the hearing-aid feedback path. A method for reducing this bias and producing an improved estimate of the feedback path is analyzed. It is shown that by the use of delays in the forward path of the hearing aid plant, it is possible to reduce the bias considerably.

## 1. Introduction

A major complaint of hearing-aid users is acoustic feedback which is perceived as whistling or howling (at oscillation) or distortion (at sub-oscillatory intervals). This feedback occurs, typically at high gains, because of leakage from the receiver to the microphone. Acoustic feedback suppression in hearing-aids is important since it can increase the maximum insertion gain of the aid. The ability to achieve target insertion gain leads to better utilization of the speech bandwidth and, hence, improved speech intelligibility for the hearing-aid user. The acoustic path transfer function can vary significantly depending on the acoustic environment [1]. Hence, effective acoustic feedback cancellers must be adaptive.

Both continuous and non-continuous adaptation systems for feedback reduction have been proposed. Non-continuous adaptation systems periodically use a training sequence, such as white noise, for adaptation of the feedback canceler coefficients. Kates [2] proposed such a system that adapts to the cancellation path when howling is detected, while Maxwell and Zurek [3] propose a system that adapts only when the input signal level is low. Continuous adaptation systems, on the other hand, adjust the adaptive filter coefficients continuously and do not have to detect silence or 'howling' intervals. The problem with continuous adaptation systems is their closed-loop nature; the adaptive filters used in these systems do not behave as in open-loop systems that have been analyzed in the literature [4] because the input to the adaptive filter is derived directly from the error signal. Delays have been proposed ([1], [3], [5]) to decorrelate the error signal from the input to the adaptive filter. The choice of delays, however, was justified empirically through simulations or real-time evaluations with hearing aids but no mathematical justification has been provided.

This paper is a first step toward an analytical approach for evaluating the performance of the LMS adaptive filter [4] when correlated inputs are used in continuous adaptation systems. A mathematical

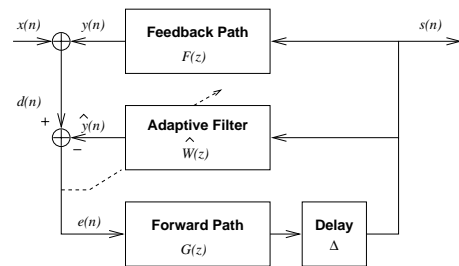


Figure 1: Hearing-aid Plant: Processing plant  $G(z)$ , Acousto-Electric Feedback Path  $F(z)$ , and Adaptive Filter  $\hat{W}(z)$

analysis of the steady-state convergence is presented. It is shown that when the input signal to the adaptive filter is correlated, a bias is introduced in the adaptive filter taps estimate, causing a non-optimal acoustic cancellation. The use of delays in the forward path, suggested previously in [5], is then analytically justified. Simulations with correlated and uncorrelated signals follow.

## 2. The hearing-aid model

In digital hearing-aids, the hearing-aid processing plant consists of an automatic gain control (AGC) to limit the dynamic range of the input signal, a frequency-shaping filter, that correlates with the subjects' audiogram, and a user-adjusted gain (or volume) control. In this paper, the hearing-aid forward path  $G(z)$  is modeled as  $G(z) = G_c z^{-1}$  [5] in Figure 1, without an AGC mechanism or a frequency-shaping filter.

The acoustic feedback path of a hearing-aid accounts for the leakage of sound from the hearing-aid receiver in the ear canal, through and around the hearing-aid ear mold, to the hearing-aid microphone. The acoustic feedback path model also includes transducers' characteristics. The feedback model,  $F(z)$  in Figure 1, was estimated based on measurements for the feedback path of three human subjects and a KEMAR mannequin [6]. In order to identify the acoustic feedback path, white noise was sent to the hearing-aid receiver while the forward path of the hearing-aid was disconnected. The signal sent to the receiver and the microphone response were simultaneously recorded on a DAT. Using these measurements, a 15-pole/14-zero linear model of the acoustic feedback path was determined using the Steiglitz-McBride (SM) algorithm [7]. The feedback path was modeled under both static and time-varying conditions. For the purposes of analysis and simulation, the first 101 samples of the impulse response of the SM model, shown in Figure 2, were used to construct a 100<sup>th</sup> order FIR model of the feedback path.

### 3. Convergence analysis using continuous adaptation

Adaptive filters based on the Least Mean Squares (LMS) algorithm are popular because of their computational efficiency. The convergence properties of this class of algorithms are analyzed to determine whether a gradient-based adaptive algorithm could effectively estimate the hearing aid feedback path transfer function  $F(z)$  shown in Figure 1. The closed loop configuration shown in Figure 1 was used in the feedback reduction scheme. In order for the LMS algorithm to converge, it is necessary for the mean of the adaptive filter tap weights difference vector to converge to zero in the steady state. The following analysis will show that if the LMS algorithm is used in the plant in Figure 1, bias will occur which prevents the filter's complete convergence. The adaptive filter tap weights difference vector is defined as

$$\epsilon(n) = \hat{\mathbf{w}}(n) - \mathbf{f} \quad (1)$$

where  $\mathbf{f}^T = [f(0) \ f(1) \ \dots \ f(N)]$  and  $f(n) = \mathcal{Z}^{-1}[F(z)]$ .  $\hat{\mathbf{w}}(n)$  is the adaptive filter tap weights vector.

The steady-state mean of the adaptive filter tap weights error vector for the LMS algorithm can be evaluated by manipulating the adaptive filter update equation. The adaptive filter tap weights vector is updated by subtracting a scaled estimate of the gradient of the cost function from the current adaptive filter tap weights vector, where  $e(n)$  is the error signal. The filter tap weights vector  $\hat{\mathbf{w}}(n)$  is updated using

$$\hat{\mathbf{w}}(n+1) = \hat{\mathbf{w}}(n) + \mu \mathbf{s}(n)e(n) \quad (2)$$

where, for an  $N^{\text{th}}$  order adaptive filter, the adaptive filter input signal vector and filter tap weight vector are  $\mathbf{s}^T(n) = [s(n) \ s(n-1) \ \dots \ s(n-N)]$  and  $\hat{\mathbf{w}}^T(n) = [\hat{w}_0(n) \ \hat{w}_1(n) \ \dots \ \hat{w}_N(n)]$ , and the adaptive filter error signal is

$$e(n) = d(n) - \mathbf{s}^T(n)\hat{\mathbf{w}}(n) \quad (3)$$

where  $d(n)$  is the adaptive filter reference signal as shown in Figure 1.

Subtracting  $\mathbf{f}$  from both sides of (2), substituting the error given by (3) into (2) and using the definition given by (1), an iterative equation for the adaptive filter tap weights difference vector is found.

$$\epsilon(n+1) = \epsilon(n) + \mu d(n)\mathbf{s}(n) - \mu \mathbf{s}(n)\mathbf{s}^T(n)\hat{\mathbf{w}}(n) \quad (4)$$

The definition in (1) is used to further modify (4).

$$\begin{aligned} \epsilon(n+1) &= \epsilon(n) - \mu \mathbf{s}(n)\mathbf{s}^T(n)\epsilon(n) + \mu d(n)\mathbf{s}(n) \\ &\quad - \mu \mathbf{s}(n)y(n) \end{aligned} \quad (5)$$

where  $y(n) = \mathbf{s}^T(n)\mathbf{f}$  is the output of the feedback path. Using  $d(n) = x(n) + y(n)$  (see Figure 1),

$$\epsilon(n+1) = \epsilon(n) - \mu \mathbf{s}(n)\mathbf{s}^T(n)\epsilon(n) + \mu x(n)\mathbf{s}(n) \quad (6)$$

By using the Direct-Averaging Method [4] that assumes small values for the convergence factor  $\mu$ , and taking the expected value of the resulting equation, it is possible to derive the following equation for the mean values of the taps vector

$$E[\epsilon(n+1)] = [\mathbf{I} - \mu \mathbf{R}(n)]E[\epsilon(n)] + \mu \mathbf{p}_1(n) \quad (7)$$

where  $\mathbf{R}(n) = E[\mathbf{s}(n)\mathbf{s}^T(n)]$  and  $\mathbf{p}_1(n) = E[\mathbf{s}(n)x(n)]$ . The second term on the right of (7) does not allow  $E[\epsilon(n)]$  to converge to zero. In steady-state, it is possible to show that

$$E[\epsilon] = \mathbf{R}^{-1}\mathbf{p}_1 \quad (8)$$

In the above equation, we have omitted the time indices to denote

stationary behavior as  $n \rightarrow \infty$ .

If  $x(n)$  and  $s(n)$  are correlated, the cross-correlation vector  $\mathbf{p}_1(n)$  is non-zero and the mean of the adaptive filter taps error vector will not converge to zero. Since it is not reasonable to assume that  $x(n)$  and  $s(n)$  are not correlated, the use of the LMS adaptive algorithm in Figure 1 will lead to a biased solution. This result is equally valid for the Normalized Least Mean Squares (NLMS) algorithm.

### 4. Decorrelating the hearing-aid input and output signals

One method for decorrelating the signals  $x(n)$  and  $s(n)$  is by using delays in the forward path, as shown in Figure 1. This section investigates the effects of such delays on steady-state performance.

Determining quantitative relations for the bias as a function of the input signal statistics, the delay ( $\Delta$ ) and the forward path gain ( $G_c$ ) is extremely complicated for arbitrary signals. This section analyzes the case where the input is a colored sequence derived from a single pole ( $\rho$ ) and white noise (First-Order Markov Process) to provide insight into the effect of the delays on the bias. Simulations will extend the obtained results to more general cases with speech-shaped noise as the input.

#### 4.1. Bias reduction by use of delays in the forward path

In this section, we derive equations for the cross-correlation vector  $\mathbf{p}_1$  and the autocorrelation matrix  $\mathbf{R}$  in the case that a delay is introduced in the forward path (as shown in Figure 1) and the input  $x(n)$  a First-Order Markov process.

The variables in this analysis will use the superscript  $f$  for denoting the case when delays are introduced in the forward path. At the end of this section, an example with a single tap adaptive filter will be shown.

**Calculation of  $\mathbf{p}_1^f$**  The reference signal  $d(n)$  can be written as (see Fig. 1)

$$d^f(n) = x(n) + \mathbf{f}^T \mathbf{s}^f(n) \quad (9)$$

The a-priori error for the adaptive filter shown in Figure 1 is

$$e^f(n) = d^f(n) - \mathbf{s}^{fT} \hat{\mathbf{w}}^f(n). \quad (10)$$

Substituting equation (9) into (10) we obtain

$$e^f(n) = x(n) + \mathbf{f}^T \mathbf{s}^f(n) - \hat{\mathbf{w}}^{fT}(n)\mathbf{s}^f(n) \quad (11)$$

From Figure 1, and assuming that  $G(z) = G_c z^{-1}$

$$\mathbf{s}^f(n) = G_c e^f(n - \Delta - 1) \quad (12)$$

Substituting the above equation and definition (1) into (11), it is possible to obtain

$$e^f(n) = x(n) - G_c e^{fT}(n)e^f(n - \Delta - 1) \quad (13)$$

Using equation (12), the vector  $\mathbf{p}_1^f$  can be written as

$$\mathbf{p}_1^f = E\{G_c e^f(n - \Delta)x(n)\} = G_c \mathbf{r}_{xe}^f(\Delta + 1) \quad (14)$$

The  $i$ -th element of the vector  $\mathbf{p}_1^f$ ,  $i = 0, \dots, N$ , is denoted by  $p_{1,i}^f$  and can be written as

$$p_{1,i}^f = G_c E\{x(n)e^f(n - i - \Delta - 1)\} = G_c r_{xe}^f(i + \Delta + 1) \quad (15)$$

Thus, for computing the elements of the vector  $\mathbf{p}_1^f$ , it is necessary to compute the cross-correlation of the signals  $x(n)$  and  $e(n)$ .

Assuming that  $x(n)$  is a first-order Markov process with a pole equal to  $\rho$  and derived from white Gaussian noise  $g(n)$  with variance  $\sigma_g^2$ , the transfer function  $\frac{X(z)}{G(z)} = \mathcal{Z}[x(n)]/\mathcal{Z}[g(n)]$  can be written

$$H_1(z) = \frac{X(z)}{G(z)} = \frac{1}{1 - \rho z^{-1}} \quad (16)$$

Using the above transfer function and equation (13), it is possible to determine the transfer function  $\frac{E^f(z)}{X(z)} = \mathcal{Z}[e^f(n)]/\mathcal{Z}[x(n)]$

$$H_2(z) = \frac{E^f(z)}{X(z)} = \frac{1}{1 + G_c \varepsilon^f T \mathbf{T}(z^{-1}) z^{-\Delta-1}} \quad (17)$$

where  $\mathbf{T}(z) = [1 \ z \ \dots \ z^N]^T$  and  $\varepsilon^f = E\{e^f\}$ . It is now possible to compute  $r_{xe}^f(n)$  by

$$r_{xe}^f(n) = \sigma_g^2 \mathcal{Z}^{-1}\{H_1(z)H_1(z^{-1})H_2(z^{-1})\} \quad (18)$$

By using the Residue Theorem [8], it is possible to calculate the Inverse Z-Transform in the above equation

$$\begin{aligned} r_{xe}^f(n) &= \frac{\sigma_g^2}{1 - \rho^2} \frac{\rho^n}{1 + G_c \varepsilon^f T \mathbf{T}(\rho) \rho^{\Delta+1}} \\ &= \frac{\sigma_x^2 \rho^n}{1 + G_c \varepsilon^f T \mathbf{T}(\rho) \rho^{\Delta+1}} \end{aligned} \quad (19)$$

where  $\sigma_x^2$  is the variance of the input signal. The  $i$ -th element of the vector  $\mathbf{p}_1^f$  can now be computed as

$$p_{1,i}^f = \frac{G_c \sigma_x^2 \rho^{i+\Delta+1}}{1 + G_c \varepsilon^f T \mathbf{T}(\rho) \rho^{\Delta+1}} \quad (21)$$

**Autocorrelation Sequence of  $s^f(n)$**  The autocorrelation of  $s^f(n)$  can be calculated as a function of the autocorrelation of  $e(n)$  as follows

$$r_{ss}^f(n) = G_c^2 r_{ee}^f(n) \quad (22)$$

The autocorrelation of  $e^f(n)$  can be calculated by using equation (13). It is possible to show that

$$r_{ee}^f(n) = r_{xe}^f(n) - G_c \varepsilon^f T \mathbf{r}_{ee}^f(n - \Delta - 1) \quad (23)$$

where  $\mathbf{r}_{ee}^f(n - \Delta - 1) = [r_{ee}^f(n - \Delta - 1) r_{ee}^f(n - \Delta - 2) \dots r_{ee}^f(n - \Delta - N - 1)]^T$ . The above equation can be used for recursively calculating the autocorrelation sequence for  $e^f(n)$ . The autocorrelation of  $s^f(n)$  can be calculated from  $r_{ee}^f(n)$  by using relation (22).

**Bias for a Single-Tap Case** In the case of a single tap, equations (8), (23), (22) and (21) can be used for determining the bias magnitude for  $N = 0$

$$\varepsilon_0^f = \frac{\rho^{\Delta+1}}{G_c} \quad (24)$$

Computer simulations were performed to evaluate the analytic estimate of the convergence error for this case. In the simulations, the gain  $G_c$  was set to 2 and 4, the Markov process constant  $\rho$  was set to 0.9, and delays ( $\Delta$ ) between 0 and 40 samples were used. The simulated and predicted results (from equation (24)) are shown in Figure 3. The dashed lines represent the simulated values and the solid lines represent values obtained from equation (24).

The performance criterion used to evaluate the simulations was defined to estimate the extent to which the adaptive filter approximates the feedback path impulse response. It was defined as follows

$$\mathcal{C}(n) = \frac{\|\hat{\mathbf{w}}(n) - \mathbf{f}\|^2}{\|\mathbf{f}\|^2} \quad (25)$$

From Figure 3, it can be seen that the magnitude of the convergence metric  $\mathcal{C}(n)$  in steady-state is reduced with increasing delays  $\Delta$ .

For higher gains, the magnitude of the convergence metric  $\mathcal{C}(n)$  in steady-state is also reduced. The difference between the predicted and simulated values for forward path delays closer to 40 samples is due to the lack of precision in the averaging process because of slower convergence for high delays. Simulations in the next sections will show that these results hold for more general cases.

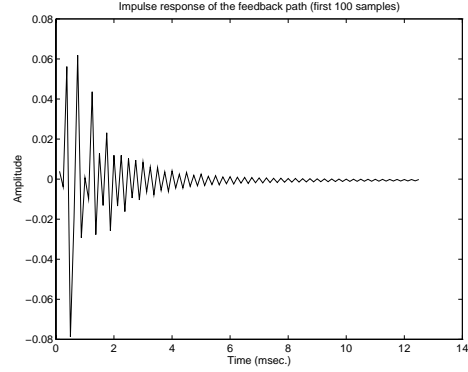


Figure 2: Impulse response of the hearing-aid feedback path for a human subject fitted with a hearing aid

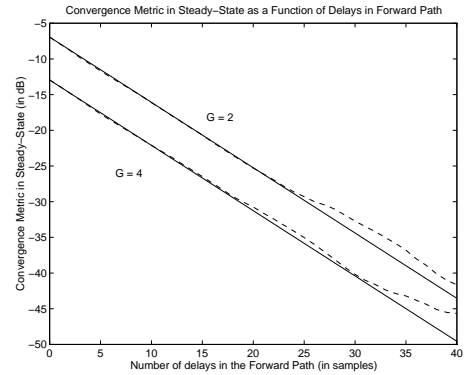


Figure 3: Steady-State Convergence Metric  $\mathcal{C}(n)$  as a Function of Forward Delay for  $G_c = 2$  and  $G_c = 4$ . Continuous lines represent predicted values and dashed lines represent simulation results.

## 5. Simulations

Computer simulations were conducted to evaluate the effectiveness of the NLMS algorithm in estimating the feedback path with a forward path delay to decorrelate the hearing-aid input and output signals. As in the previous section, the acoustic feedback path model used a 100<sup>th</sup> order FIR linear filter, and a forward path transfer function  $G(z) = G_c z^{-1}$ ; no frequency-shaping filter or AGC were assumed. A 100<sup>th</sup> order NLMS algorithm was used for the adaptive filter. The performance of the feedback cancellation scheme was evaluated for various values of the forward gain ( $G_c$ ) and convergence factor ( $\mu$ ).

### 5.1. Simulations using noise signals

Simulations were performed using white noise speech-shaped noise as the hearing-aid input signal  $x(n)$ . Speech-shaped noise was created by passing white Gaussian noise through a 12-pole frequency

shaping filter to model the long-term average speech spectrum. Simulations were carried out to verify that the adaptive filter is biased when correlated hearing-aid input signals are used. Figure 4 shows the convergence of the adaptive filter for both a white noise input signal and the colored noise input signal described above. Both the white noise signal and the colored noise signal had variance equal to 66.3 dB. It is expected that when a white noise input signal is used, the hearing-aid input and output signals will be reasonably well decorrelated, thus reducing the adaptive filter bias. On the other hand, when the correlated noise input signal is used and no delays are present in the forward path, a bias is expected. Evidence of this behavior can be seen in Figure 4. Also, the same figure shows that when the input is correlated noise and a delay of 10 samples is included in the forward path the adaptive filter approximates well the feedback path, with  $\mathcal{C}(n) \leq -10$  dB in steady state. All the simulations shown in Figure 4 used the Normalized LMS Algorithm with  $\mu = 0.1$ .

## 5.2. The dependence of convergence on forward path gain

The second objective of the simulations was to evaluate the steady-state convergence of the adaptive filter for various values of added forward path delay  $\Delta$ . Figure 5 shows the average convergence metric in the steady-state over a range of forward path delays for values of forward path gain  $G_c$  equal to 3, 7 and 12. Figure 5 shows that the best steady-state convergence is attained when the forward path delay is at least 10 samples (1.25 ms at 8 kHz sampling rate). Figure 5 also indicates that the adaptive filter is more accurate in estimating the acoustic feedback path impulse response when higher gains are used.

## 6. Summary and conclusions

Unlike most adaptive filtering applications [4], the LMS algorithm (and variants such as NLMS) in continuous feedback reduction systems for hearing aids has a bias in its estimate when correlated inputs, such as speech, are used. This bias is due to non-zero cross-correlation of the input and output signals in the hearing aid plant. Analysis in the case of a first-order Markov process as the input signal was presented, and equations that allow prediction of the bias were derived. The analysis was also carried out for the case that the forward path is delayed to decorrelate the error signal and the input signal to the adaptive filter. The analysis showed how delays in the forward path decrease the bias in the adaptive filter's estimate.

Simulations with white noise and speech-shaped noise followed. It was shown that a significant reduction in the adaptive filter taps deviation can be obtained when the input signal is speech-shaped noise and delays are used in the forward path. As the forward path gain  $G_c$  increased, better estimates were obtained. In addition, simulations showed that the magnitude of the bias does not further decrease with forward path delays greater than 1 ms. In the future, we will extend our analysis to delays placed in the cancellation path and examine the effects of delays on the system's stability.

## References

- [1] D. Bustamante, T. Worrall, and M. Williamson, "Measurement of adaptive suppression of acoustic feedback in hearing aids," *Proc. 1989 IEEE ICASSP*, pp. 2017–2020, 1989.
- [2] J. Kates, "Feedback cancellation in hearing aids: Results from a computer simulation," *IEEE Transactions on Signal Processing*, vol. 39, pp. 553–562, 1991.
- [3] J. Maxwell and P. Zurek, "Reducing acoustic feedback in hearing aids," *IEEE Transactions on Speech and Audio Processing*, vol. 4, pp. 304–313, July 1995.
- [4] S. Haykin, *Adaptive Signal Processing*. Upper Saddle River, New Jersey: Prentice-Hall, 1996.
- [5] P. Estermann and A. Kaelin, "Feedback cancellation in hearing aids: Results from using frequency-domain adaptive filters," *Proc. 1994 IEEE ISCAS*, pp. 257–260, 1994.
- [6] M. Siqueira, R. Speece, E. Petsalis, A. Alwan, S. Soli, and S. Gao, "Subband adaptive filtering applied to acoustic feedback reduction in hearing aids," *Proceedings of the Asilomar Conference on Signals, Systems, and Computers*, Nov. 1996.
- [7] P. Regalia, *Adaptive IIR Filtering in Signal Processing and Control*. New York: Dekker, 1994.
- [8] A. Oppenheim and R. Schaffer, *Discrete-Time Signal Processing*. Englewood-Cliffs: Prentice-Hall, 1989.

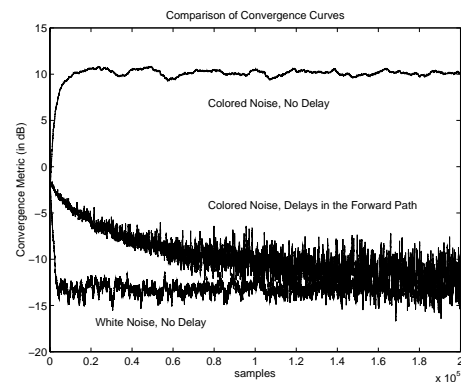


Figure 4: Adaptive filter convergence using white and colored noises as input to the hearing-aid with and without a delay (10 samples) and forward gain  $G_c = 7$ .

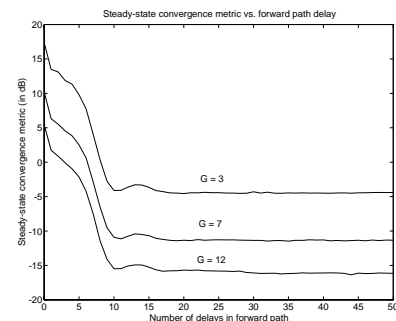


Figure 5: Steady-state convergence vs forward path delay for  $G_c=3,7,12$ . Using the colored noise input signal and  $\mu=0.01$

[1] D. Bustamante, T. Worrall, and M. Williamson, "Measurement of adaptive suppression of acoustic feedback in hearing aids," *Proc. 1989 IEEE ICASSP*, pp. 2017–2020, 1989.